Ideological Infection

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Abstract

Political parties have grown increasingly polarized in recent decades. Furthermore, this polarization has spilled over into common-values issues that we would not expect to be subject to ideological disagreements. We develop a model to study why and how this ideological infection occurs. In the model, two parties bargain over an ideological and a common-values issue. Although they have different ideal points on the ideological dimension, they agree on optimal policies for the common-values issue. In this setting, the majority party can leverage the common-values dimension to obtain favorable policies on the ideological dimension. We show that in a changing world this can cause ideological infection, and the equilibrium policy to be inefficient on the common values dimension. This inefficiency can be induced by the proposer, in which case the policy undershoots the parties' shared ideal point, or the veto-player, in which case the policy overshoots the optimal.

Introduction

Two important observations often characterize accounts of American politics. First, political parties are growing increasingly polarized. Looking at the US Congress, measures of polarization were quite low until the mid 1970s, but have seen a steep increase since then (Barber et al., 2015). Second, while political conflict between the parties has remained organized along classic dimensions of polarization, other issues 'have been absorbed into it' (Barber et al. (2015), p. 23). Indeed, Lee (2005) describes how congressmen exhibit partisan divisions on issues ranging from good government, to disaster relief, to transportation programs, on which we would expect the preferences of 'both parties and all voters [to be] located at a single point' (Stokes (1963), p. 372). Thus, political parties now appear to be polarized on virtually all dimensions, including those that seem to have no ideological connotation. As a consequence, in recent decades we have witnessed a stark decrease in the ability of Congress's to legislative efficiently, even on these common-values issues over which parties have no underlying ideological disagreement (Layman et al., 2006).

In this paper, we build on these observations and the intuition that these two developments are fundamentally related. We develop a model showing how, why, and when, parties' dynamic incentives may cause increasing polarization to spillover from dimensions of genuine conflict to common-values issues, in turn producing inefficient policies. Thus, political conflict can become multidimensional and parties appear as if they are divided on both ideological and common-values dimensions. Substantively, our results imply that the key consequence of this "ideological infection" is that 'public policy does not adjust to changing economic and demographic circumstances' (Barber et al. (2015), p. 41). In the most severe form of this inefficiency, Congress appears unwilling to address even the most pressing issues facing the country, instead choosing to 'kick the can down the road (...) and govern by (artificial) crises' (Barber et al. (2015), p. 41).

In our model, two parties, R and L, repeatedly bargain over both an ideological issue, e.g., economic redistribution or gun control, and a common-values issue, e.g., infrastructure spending or crisis relief. The parties have different ideal points on the ideological issue, but agree on the optimal common-values policy. Initially, the status quo on the conflict dimension lies between the ideal policies of the two players, i.e., in the gridlock interval, and the status quo on the common values dimension is inefficient. In each period R makes a proposal, which consists of a policy on the ideological and common-values issue, and L votes in favor or against the proposal. On both dimensions, policies remain in place until a new policy is passed. Thus, the status quo in each period is determined by the bargaining outcome of the previous period.

The key feature of our model is that players' ideal policies on both dimensions may change between periods. Consistent with our earlier observations, we assume the ideal points of the parties on the ideological dimension become more polarized each period. This may, for example, stem from the parties expecting their constituents will grow more and more divided, continuing the trend of previous decades. As a result, parties anticipate they will have to appeal to more extreme preferences in order to maintain support of their respective voter base in the future (Peltzman, 1984). Additionally, the optimal common-values dimension policy evolves over time. For example, a crisis may be expected to worsen and thus require bolder intervention in the future to be adequately addressed; infrastructure may keep deteriorating, necessitating greater investment tomorrow; or anticipated technological progress may alter the optimal investment in green technologies in the future.

The ability to bargain over multiple issues creates new strategic incentives for the parties. When parties bargain over multiple policy dimensions, proposals on unrelated issues are often bundled together in the same bill (Krutz, 2001; Hanson, 2014; Krutz, 2001; Clinton and Lapinski, 2006; Kardasheva, 2013; Hazama and Iba, 2017; Meßerschmidt, 2021). In our setting, combining the ideological dimension with the commonly beneficial policy change on the crisis issue gives the proposer leverage to pass divisive policies that would otherwise fail, a tactic commonly used in practice (Krutz, 2000; Casas et al., 2020).

In a static world, this form of issue linkage generates no detrimental spillovers from the ideological issue to the common-values issue. The efficient policy on the dimension of agreement also maximizes the ideological concessions the proposer can obtain. As such, in equilibrium, there is no impediment to the parties implementing the commonly beneficial policy, e.g., the crisis is immediately addressed or necessary investments in infrastructure are made.

In a dynamic setting, this is not always true. As discussed above, the ideal policy on the common-values dimension moves away from the initial status quo over time. Consequently, inaction (or inefficient action) on this dimension becomes more costly for both parties over time and party R, the proposer, faces a trade-off. R can use the efficient policy to buy ideological concessions today, or pay the cost of inaction today and leave more inefficiency on the table to buy additional concessions in the future. In other words, the proposer's dynamic payoff is maximized by doing too little on the common values dimension today and *undershooting* the efficient policy to preserve leverage in the future. If policy undershoots we may observe too little investment in infrastructure, or a crisis that is only partially addressed. On the other hand, the dynamic payoff of L, the veto player, is maximized by doing too much today and *overshooting* the efficient policy to tie the proposer's hands in the future. Here, we may observe the parties over-investing in infrastructure, or policies aimed not only at addressing the current crisis but also preventing future ones.

Despite these contrasting dynamic preferences the parties still have strong incentives to implement the efficient policy on the common-value dimension. This is because inefficient policies are costly for *both* the proposer R and the veto player L. We show that increasing polarization is crucial for dynamic incentives to dominate and inefficiency to emerge. In particular, ideological infection of the common-values dimension may be avoided if the rate of growth in polarization is slow enough. Under rapidly increasing polarization, instead, ideological infection is inevitable.

Slowly increasing ideological polarization coupled with rapid change on the dimension of shared agreement gives R enough leverage to obtain its static optimal ideological policy each period, even if the common-values policy is always set at the efficient level. As such, if the optimal common-values policy changes fast enough relative to ideological polarization, then policy is always efficient. Furthermore, even when ideological infection emerges inefficiency can only be sustained for a finite amount of time, because eventually polarization is growing very slow relative to changes on the common-values issue.

If polarization instead increases rapidly, then ideological infection always emerges. Thus, even though parties have ideological disagreements only on one issue, rapidly increasing polarization on this dimension implies that in equilibrium political conflict inevitably becomes multidimensional. The parties then appear as if they are divided on both the ideological and the common values dimension. Furthermore, this inefficiency on the common-values dimension can be sustained for an arbitrarily long period of time.

In equilibrium, inefficiency under rapid polarization may exhibit undershooting or overshooting. If the initial status quo is sufficiently good for the veto player then L is very unwilling to grant R ideological concessions unless R reduces its future leverage. In turn, R overshoots the optimal common-values policy and moves policy closer to tomorrow's optimum, thus tying its own hands for the future in order to be rewarded with ideological gains today.

In contrast, when the initial status quo is sufficiently bad for the veto player, the proposer undershoots on the common-values policy today to maintain greater bargaining leverage tomorrow. Naive intuition may suggest this form of inefficiency arises because R can obtain its first-period ideal point on the ideological dimension without moving policy all the way to the efficient common-values policy today, and can therefore save some inefficiency to improve its bargaining leverage tomorrow. In other words, undershooting should only emerge when the proposer does not 'need' leverage today. However, this is not always true in our setting. Indeed, we find that the proposer sometimes undershoots the optimal common-values policy, even if doing so does not provide enough leverage to shift today's ideological policy all the way to the proposer's preferred outcome in the first period. Thus, the proposer chooses to incur immediate costs on *both* the common-values and the ideological dimension.

The reason is that movement of the ideal common-values policy creates a compounding effect in the cost of inefficiency. In the model, this is captured by concave preferences. In practice, suppose that a crisis is only partially addressed today. As the crisis worsens over time, the cost imposed by this residual inefficiency becomes more and more significant. Similarly, the cost of having only partially repaired infrastructures is larger as the infrastructure continues to deteriorate. Thus, although undershooting the common-values dimension policy is costly for both parties in the first period, it imposes an even larger cost in the second period if it goes unaddressed. Consequently, undershooting implies that the proposer forgoes some ideological concessions today, but is able to obtain even greater concessions tomorrow.

The speed of polarization relative to change on the common-values dimension also significantly impacts observed long-run outcomes. If polarization increases slowly then, as discussed, R is able to obtain its optimal ideological policy every period. Thus, we should observe large rapid policy changes on the ideological dimension. On the other hand, if polarization rapidly increases then the players become relatively more stubborn over time and policy changes become smaller.

Our model makes a number of simplifying assumptions to clearly highlight the mechanism driving ideological infection. However, our main insights extend beyond the stark setting of the baseline model.

First, while the baseline model assumes that the same party has proposal power in all periods, in an extension we study the effect of turnover in power. We show, perhaps surprisingly, that turnover does not eliminate inefficiency. Furthermore, we show that turnover can sometimes generate inefficiency even when the equilibrium would be efficient in the baseline with persistent power. Second, although our model assumes parties can perfectly anticipate where their future preferences, we show that the mere possibility that ideological polarization will increase rapidly is enough to generate inefficiency on the common-values dimension. Finally, because our objective is to study the effects of increasing polarization, we assume both parties become (weakly) more extreme over time. However, allowing for more general movements largely confirms the intuitions of the baseline. Rapidly increasing polarization leads to inevitable ideological infection. Conversely, inefficiency never emerges when polarization decreases too quickly.

Contribution to the Literature

Our research falls within the extensive political economy literature that explores bargaining with an endogenous status quo.[1] While our work shares similarities with previous studies that uncover

[1] See Eraslan et al. (2022) for an extensive review of this literature.

inefficiencies generated by dynamic incentives, our focus is unique in examining the consequences of increasing polarization over time and the possibility of conflict spillovers from issues of disagreement to pure common-values. Moreover, the mechanisms that generate inefficiency in our setting differ from those in existing literature, where inefficiencies stem from motives such as insurance against turnover, insurance against the possibility of developing future conflict, the possibility of being in a worse bargaining position in the future, or frictions in the bargaining process. In contrast, our model generates inefficiency due to the multidimensionality of the policy space combined with a compounding effect that increases inefficiency costs over time.[2] In the following discussion, we provide a detailed overview of related works and highlight the differences.

In a world with turnover, Buisseret and Bernhardt (2017) find that a far-right proposer may propose a leftist policy to constrain policymaking in the event a far-left proposer holds power in the future. Under evolving preferences, Austen-Smith et al. (2019) shows that, when parties fear that increased polarization in the future will make it more difficult to agree on policy changes, statically inefficient policies may be implemented because they are easier to repeal tomorrow. Several other works find that bargaining parties may pursue statically inefficient policies as a form of insurance against future fluctuations in power and/or the possibility of disagreements over this dimension intensifying in the future (Riboni and Ruge-Murcia, 2008; Zápal, 2011; Dziuda and Loeper, 2016; Buisseret and Bernhardt, 2017; Dziuda and Loeper, 2018; Austen-Smith et al., 2019).

Our paper highlights that inefficiency on common-values issues can arise in a multidimensional world even when the proposer is guaranteed to remain in power indefinitely, and the parties are certain they will never disagree on this dimension. Therefore, unlike existing literature, there is no insurance motive driving our results. Instead, inefficiency emerges due to the changing cost of residual inefficiency over time. With concave preferences and shifting ideal policies on common-values issues, a policy that is inefficient today becomes even more costly tomorrow. Thus, even if the proposer could secure ideological concessions by implementing the efficient policy today, leaving some

[2]Previous papers in this literature that incorporate multiple policy dimensions focus on issues of existence (Duggan and Kalandrakis, 2012) and indeterminacy (Anesi and Duggan, 2018). Chen and Eraslan (2017) also analyzes dynamic bargaining with multiple policy dimensions, but assumes parties can only address one issue at a time. In contrast, we specifically focus on the effects of bundling different dimensions.

residual inefficiency on the table can yield even greater concessions in the future. This dynamic creates incentives for both the proposer and veto player to distort policy.[3] As ideological polarization increases rapidly, these dynamic incentives overpower the static push for efficiency, leading to ideological infection. Therefore, we contribute to the literature by showing that inefficiencies may be even more severe than previous results show, conflict can spillover onto unrelated dimensions on which parties never face any disagreement.[4]

The emergence of inefficient agreements in our model also relates to commitment problems in bargaining models of conflict. Previous papers have explored how rapid shifts in power may render peaceful agreements infeasible and lead to war (e.g., Fearon, 1995; Powell, 2006).[5] The basic intuition is that a powerful country, fearing a future loss of bargaining strength, may initiate a statically inefficient war to preempt such a shift. Our results are more nuanced. Sometimes inefficiency emerges for a related logic: the proposer does not need bargaining leverage today but anticipates needing it in the future. Under other conditions, however, the proposer may prefer pursuing an inefficient policy even if it reduces its bargaining leverage today and incurs a cost on both the ideological and common-values dimensions. Once again, the compounding effect described earlier sustains inefficiency in our setting. This is highlighted by noting that if players' preferences over the common-values dimension are linear, or the optimal policy remains unchanged over time, then the equilibrium is always efficient.

In a different context, some existing works find inefficiency due to frictions in the bargaining processes. Acharya and Ortner (2013) and Lee (2020) find that dynamic incentives may produce inefficiencies in a world where policy issues/dimensions are not all available at the same time. For

[3]Other works study the effects of bundling different dimensions, but consider a one-shot interaction or assume bargaining concludes once an agreement is reached (e.g., Fershtman, 1990; Jackson and Moselle, 2002; Chen and Eraslan, 2013; Câmara and Eguia, 2017). Consequently, the inefficiencies we find due to evolving preferences and an endogenous status quo do not arise in these models.

[4]This distinguishes our work from papers that find strategic polarization on a single dimension that can exhibit conflict (Dziuda and Loeper, 2018). Penn (2009) allows for multiple dimensions and characterizes how continuing policies distort preferences. However, proposals are exogenous in her model, which focuses on voting behavior. In contrast endogenous proposals are a crucial determinant of preferences our model.

[5]Of course, our framework differs from the conventional context of war in several ways. Most prominently, a power shift in a conflict setting typically improves the outside option for one player at the expense of the other. In contrast, the payoff from the initial no-agreement status quo decreases for both players under increasing polarization.

example, Lee (2020) finds that policy bundling can create incentives to delay today in order to leverage a bipartisan policy with ideological policies in the future. However, this result rests on the assumption that some conflict policy dimensions are only available in the future. If all dimensions were available immediately, as in our model, there would be no delay in his setting.[6] Our paper complements these works, highlighting how evolving preferences may sustain inefficiency even absent exogenous constraints in the availability of policy dimensions.

Closer to our paper, Callander and Martin (2017) studies a bargaining model where policies have an ideological and a quality component. Quality decays over time but can be restored. Our results show that inefficiency is sustained by the evolution of the player's ideal points on the two dimensions. In Callander and Martin (2017), ideological polarization does not increase over time. Consequently, the proposer never underinvests in quality for leverage. Further, by assumption parties cannot overinvest today to avoid future decay. Thus, inefficiency never emerges in equilibrium.

Finally, our work is also related to studies that analyze multidimensional bargaining where players can make transfers to each other (e.g., Austen-Smith and Banks, 1988; Diermeier and Merlo, 2000). Similar to a transfer, our proposer can use the common values dimension for favorable policy on the partisan dimension. However, this differs from a transfer because both players benefit from moving policy to the common values ideal point, therefore inefficiency is costly for the proposer as well. Thus, leaving leverage for the future on the common values dimension is always Pareto inefficient. Other studies analyze how bargaining players divide a budget between private benefits and public goods (e.g., Battaglini and Coate, 2007; Volden and Wiseman, 2007). In such settings, a proposer may have incentives to underprovide on the public good dimension to extract maximum private transfers for itself. In this context, Bowen et al. (2014) show that an endogenous status quo, e.g., mandatory public good spending, will actually help *correct* the inefficiencies that would emerge in a static setting. This contrasts with our setup, where the endogenous status quo is precisely what generates inefficiencies.

[6] Furthermore, Lee (2020) considers a discrete policy space, where a single alternative to the status quo is available. As such, in his setting there is no scope for veto-player induced inefficiency. On the other hand, our model abstracts from elections, whereas Lee (2020) explicitly incorporates voters.

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The Model

Our baseline model considers a two-period game that is purposefully stark to clarify the logic behind results. We show that inefficiency can only be avoided if ideological polarization increases sufficiently slowly. We later extend our analysis to an infinite horizon and demonstrate that rapid increases in polarization inevitably leads to ideological infection, while the problem is always resolved in the long run with slow polarization.

Players and policies. Two parties L and R interact over two periods, $t \in \{1, 2\}$. The policy space $\mathcal{P}$ is composed of an ideological dimension $X \subset \mathbb{R}$, and a common values dimension $Y \subset \mathbb{R}$, where X and Y are both non-degenerate compact intervals and $\mathcal{P} = X \times Y$.[7] In every period t the parties bargain to determine a policy outcome $p_t \in \mathcal{P}$.

Preferences. The stage utility to party i in period t from a policy outcome $p_t = (x, y)$ is $u_{it}(x) + v_t(y)$, where $u_{it}(x) = -(x - \hat{x}^i_t)^2$ and $v_t(y) = -(y - \hat{y}_t)^2$. Party i's payoff in the dynamic game is given by

$$\sum_{t \in \{1,2\}} u_{it}(x) + v_t(y),$$

where we assume no discounting for simplicity in the two-period case.

Party i's statically optimal policy in period t is given by its ideal point $(\hat{x}^i_t, \hat{y}_t) \in \mathcal{P}$. Thus, the Y dimension features common values because in every period t the parties receive the same payoff $v_t(y)$ from a policy y.[8] On the other hand, the X dimension exhibits ideological conflict because

[7]We assume X and Y are sufficiently large for the two-period model to avoid corner solutions.

[8]We assume that parties share exactly the same ideal policy on the dimension of agreement in order to sharpen our results and characterization. However, our intuitions apply more broadly to cases in which parties face some conflict on this dimension, but nonetheless agree on what is the optimal direction of reform (i.e., even though their ideal policy may differ, the status quo is outside of the gridlock interval).

parties have different ideal policies. We assume that party R's preferred ideological policy is always to the right of party L in each period, $\hat{x}_t^L < \hat{x}_t^R$.

Importantly, ideal policies are indexed by the time period t and may change over time. We assume that each party becomes more extreme in the second period, $\hat{x}_2^L \leq \hat{x}_1^L$ and $\hat{x}_1^R \leq \hat{x}_2^R$. Thus, polarization increases over time. As discussed above, this can capture situations in which parties anticipate an increasing polarization among their respective voter bases. In such cases, parties would need to adapt to these developments. We capture this adaptation in a simplified form by shifting their primitive ideal points. Similarly, $\hat{y}_1$ and $\hat{y}_2$ determine the parties' shared ideal policy on the common values dimension in each period. For illustration purposes, we assume the ideal policy on the common-values issue is monotonically increasing, so $\hat{y}_1 \leq \hat{y}_2$.

The sequences of ideal policies are common knowledge. Thus, the evolution of ideal points is deterministic and the parties in our model face no uncertainty. Assuming no uncertainty is not necessary for our results. However, it allows us to isolate how spillovers from the ideological dimension to the common values dimension creates inefficiency, rather than inefficiency being due to the proposer attempting to insure against extreme shocks. Figure 1 depicts one example of this evolution process.

Political environment. We assume that R is the proposer in each period and L the veto player. Thus, at the start of each period t party R makes a proposal $(x_t, y_t) \in \mathcal{P}$, which consists of a policy on the ideological issue, $x_t \in X$, and a policy on the common values issue, $y_t \in Y$. Next, party L decides whether to accept or reject the proposal. If the proposal is accepted then the policy outcome in period t is $p_t = (x_t, y_t)$. If the proposal is rejected then the policy outcome in period t is $p_t = q_t$, where $q_t \equiv (x_t^q, y_t^q) \in \mathcal{P}$ is the status quo in period t. Notice that we assume proposals on the two dimensions are bundled together. In our analysis below, we show tha this is without loss of generality: even if given the choice to present two separate proposals, R would always prefer to bundle instead.

The policy outcome in the current period becomes the status quo in the subsequent period, i.e.,

$q_t = p_{t-1}$. The status quo at the beginning of the game is exogenously set at $q_1 = (x_1^q, y_1^q)$. We assume that x_1^q is in the static gridlock interval, $x_1^q \in (\hat{x}_1^L, \hat{x}_1^R)$, and policy is initially inefficient on the common values dimension, specifically $y_1^q < \hat{y}_1$.

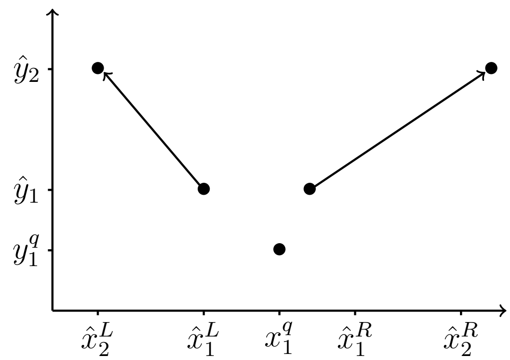

Figure 1: Evolution of ideal points, example

In concluding this section, it is important to acknowledge the simplifying assumptions that we impose in our baseline model. To facilitate our presentation, we assume parties can perfectly anticipate their future preferences. Additionally, we impose a specific direction for the evolution of their respective bliss points, assuming that they must move in opposite directions and become more extreme over time. We also assume that R is the proposer in each period, without any turnover. Furthermore, we focus on quadratic-loss preferences, rather than considering general concave functions. In the Appendix, we confirm the robustness of our qualitative findings to relaxing each of these assumptions. The results of these analyses are briefly discussed in a robustness section below.

A Definition of Efficiency

It is useful to start by establishing some terminology. Given our focus on the common-values dimension, we refer to a policy p_t in period t as "efficient" if the outcome on the Y dimension is at $\hat{y}_t$. Otherwise, we say that policy is inefficient.

Definition 1. *A policy outcome p_t in period t is efficient if it sets the common-values policy at $\hat{y}_t$. Otherwise, p_t is inefficient.*

Notice that our definition of inefficiency also implies the policy outcome is not Pareto efficient (statically or dynamically).[9] Throughout the paper, we use the term **ideological infection** to refer to the emergence of an inefficient policy outcome in equilibrium. This terminology is used to emphasize that inefficiency arises due to spillovers from the dimension of conflict to the common-values issue.

Indeed, if the two policy dimensions cannot be bundled together into a single proposal then there is always efficiency in equilibrium. Specifically, assume that after any proposal (x_t, y_t) L votes to accept or reject the x_t policy and separately votes to accept or reject the y_t policy. Thus, L can pass both policies, $p_t = (x_t, y_t)$, one policy, $p_t \in \{(x_t, y_t^q), (x_t^q, y_t)\}$, or neither policy, $p_t = (x_t^q, y_t^q)$.

Remark 1. *If bundling is not allowed then policy outcomes are efficient in every period.*

Without bundling L can always reject a policy worse than the status quo on the ideological dimension, and accept any proposal that improves the common-values policy. Consequently, R has no leverage to move policy away from the initial ideological status quo x_1^q and no incentive to implement an inefficient common-values policy. As such, policy outcomes are efficient. Thus, if inefficiency ever arises in equilibrium then it must be due to policy bundling and spillovers from the ideological dimension. Moreover, any policy changes on the ideological dimension must be due to the ability to bundle.

Importantly, we note that the proposer does benefit from an institution that allows bundling and, thus, would not to commit to removing this power.

Remark 2. *R's equilibrium payoff is greater in the game with bundling than in the game without.*

[9]Specifically, Pareto efficiency requires $p_t \in [\hat{x}_t^L, \hat{x}_t^R] \times \{\hat{y}_t\}$, i.e., efficiency on Y and policy in the gridlock interval on X. Notice that in our definition of efficiency we therefore sidestep the issue that even with efficiency on the common values dimension we may have Pareto inefficiency on the conflict issue (or vice versa).

Analysis

Moving to the analysis, our solution concept is subgame perfect equilibrium and we proceed by backwards induction. In the second period of the game the players only consider their static payoffs. Thus, L only accepts a proposal if it statically prefers the proposed bundle to the status quo. Formally, L accepts any policy (x, y) such that

$$u_{L2}(x) + v_2(y) \geq u_{L2}(x_2^q) + v_2(y_2^q). \tag{1}$$

In this multidimensional world, L is only willing to accept an ideological policy away from its bliss point if the proposal improves on the common-values status quo. Thus, if the inherited status quo is inefficient, $y_2^q \neq \hat{y}_2$, then R can extract concessions on the conflict dimension by proposing a bundle that moves the policy on the common-values dimension closer to $\hat{y}_2$.

In equilibrium, R chooses its proposal to maximize $u_{R2}(x) + v_2(y)$ subject to (1). Let $\overline{x}(x_2^q, y_2^q)$ be the upper solution that solves (1) at equality. This yields:

Lemma 1. *In the second period R proposes $y_2^* = \hat{y}_2$ and $x_2^*(x^q, y^q) = \min\left\{\hat{x}_2^R, \bar{x}(x^q, y^q)\right\}$.*

Proposing $y = \hat{y}_2$ maximizes L's payoff from the proposal on the common-values dimension, and thus maximizes L's willingness to accept a worse payoff on the ideological dimension. In turn, the efficient policy $y = \hat{y}_2$ not only maximizes R's payoff on the common-values dimension, but also maximizes the leverage R has to move policy towards its ideal policy $\hat{x}_2^R$. Therefore, the equilibrium policy outcome is always efficient. It is important to note that this result highlights that inefficiency never emerges in a static world. The possibility to bundle different dimensions does not create detrimental spillovers absent dynamic motives.

Notice that R's leverage in the second period increases the further y_2^q is from $\hat{y}_2$. From equation 1, we can see that as y_2^q moves further away from $\hat{y}_2$, the utility $v_2(y_2^q)$ decreases, allowing R to extract more ideological concessions. Consequently, the proposer's equilibrium payoff in the second period increases as the initial status quo becomes more inefficient, while the veto player's payoff decreases. As we demonstrate below, these dynamic considerations form the foundation of ideological infection.

Efficiency in an Unchanging World

The above discussion highlights how the players' dynamic motive may generate incentives to distort policy in the first period. However, Proposition 1 demonstrates these incentives are not strong enough to generate ideological infection if $\hat{y}$ remains fixed over time.

Proposition 1. *If $\hat{y}_1 = \hat{y}_2 = \hat{y}$ then the equilibrium policy outcome is always efficient, $y_1^* = y_2^* = \hat{y}$.*

If there is no movement on the common-values dimension, then the policy that minimizes the proposer's leverage in the future is the static optimum $\hat{y}$, which is also the statically efficient policy. Thus, the veto player's dynamically preferred common-values policy is $\hat{y}$. However, for R the efficient policy today removes any ability to change policy in the future, because L will veto any further movement to the right on the ideological dimension. Therefore, R has dynamic incentives to adopt an inefficient policy on the common-values dimension today, $y_t < \hat{y}$, to retain leverage for the future and pull policy further to the right on the ideological dimension.

However, any ideological cost R can impose on L tomorrow must be because the inefficiency on the common-values dimension is at least as costly today. In equilibrium, the best R can do is make L indifferent between accepting the proposal and keeping the status quo, forcing L to absorb the entire cost of the common-values inefficiency. However, when $\hat{y}_1 = \hat{y}_2$, this inefficiency imposes the exact same cost on *both* the proposer and the veto player today. Moreover, L's cost from allowing policy to move further to the right is higher than R's gain when utilities are concave, and this is exacerbated by increasing polarization. Thus, the proposer's cost of inefficiency today on the common-values dimension is always higher than the future gain on the ideological dimension. Therefore, the parties' induced preferences on the common-values dimension still coincide, and there is no ideological infection. However, as we show below, this no longer holds in a changing world.

Ideological Infection in a Changing World

We now go back to our original assumption, whereby the optimal common-values policy evolves over time, $\hat{y}_1 < \hat{y}_2$.

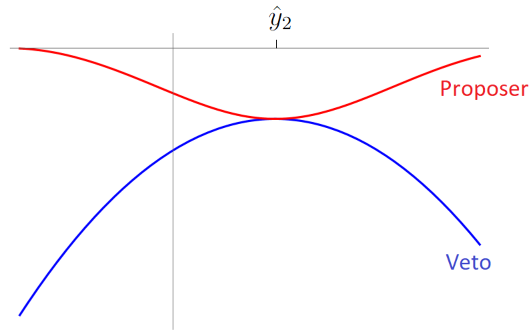

Figure 2: Second-period equilibrium payoffs as a function of y_2^q.

Let $U_q = u_{L1}(x_1^q) + v_1(y_1^q) + u(x_2^*(x^q, y^q))$ denote L's dynamic utility from keeping the status quo. Thus, in the first period L accepts a proposal (x, y) if

$$u_{L1}(x) + v_1(y) + u(x_2^*(x, y)) \geq U_q,$$

and rejects otherwise. Facing this constraint from L, in the first period party R chooses (x, y) to solve the following maximization problem:

$$\max_{x,y} \; u_{R1}(x) + v_1(y) + u_{R2}(x_2^*(x, y)) \tag{2}$$
$$\text{s.t. } u_{L1}(x) + v_1(y) + u_{L2}(x_2^*(x, y)) \geq U_q$$

Building on the above discussion, we see that each party faces a trade-off when bargaining in the first period. While both parties prefer an efficient common-values policy today, their incentives to influence future policy outcomes can lead to a divergence in their preferred policies. Equation 1 shows that as y_2^q moves further from $\hat{y}_2$, $\overline{x}(x_2^q, y_2^q)$ increases. As a result, the proposer's second-period equilibrium payoff increases when the status-quo common-values policy inherited from the first period moves away from $\hat{y}_2$, as shown in Figure 2. Conversely, the veto player's continuation value increases as y_1 moves closer to $\hat{y}_2$. As a consequence, L prefers to *overshoot* the efficient common-values policy, $\hat{y}_1 < y_1 < \hat{y}_2$, while R prefers to *undershoot* it $y_1 < \hat{y}_1 < \hat{y}_2$.

Proposition 1 demonstrates that these dynamic considerations never dominate when the optimal

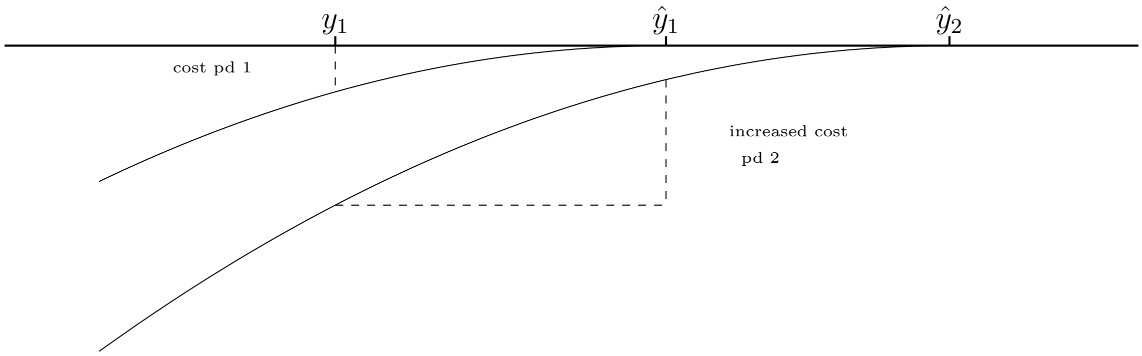

Figure 3: Common-values utility in first and second period.

common-values policy remains fixed across periods, as any cost R imposes on L to gain leverage tomorrow imposes the same cost on both L and R today. We now instead show that efficiency no longer prevails when $\hat{y}_1 \neq \hat{y}_2$.

In particular, if $\hat{y}_1 < \hat{y}_2$, then concavity implies that for any $y_1 < \frac{\hat{y}_1+\hat{y}_2}{2}$ we have

$$|v_2(y_1) - v_2(\hat{y}_1)| > |v_1(y_1) - v_1(\hat{y}_1)|. \tag{3}$$

For example, suppose an inefficient common-values policy $y_1 < \hat{y}_1$ is implemented in the first period, which moves too little in the optimal direction. This policy imposes costs on both parties in the first period and increases the cost to the veto player for maintaining the status quo in the second period, allowing R to pull x_2 closer to $\hat{x}_2^R$. Condition 3 indicates that, when $\hat{y}$ changes over time, this increased cost on the veto player is greater than the cost that *both* parties pay for inefficiency in the first period (as shown in Figure 3).

This wedge in the cost of residual inefficiency today and tomorrow creates the possibility for R to benefit from an inefficient policy outcome, which can lead to ideological infection. In particular, the emergence of infection in equilibrium, as well as the form of resulting inefficiency, depends on the rate at which ideological polarization increases relative to the evolution of the common-values dimension, as well as how favorable the initial status quo is to the veto player.

Slowly Increasing Ideological Polarization

First, we consider the case where polarization on the ideological dimension increases slowly relative to the movement on the common value dimension. Specifically, assume

$$u_{L_2}(\hat{x}_2^R) - u_{L_2}(\hat{x}_1^R) \geq v_2(\hat{y}_1) - v_2(\hat{y}_2). \tag{4}$$

Condition 4 states that the change in L's utility from R's ideologically preferred policies is greater than the change in utility on the common-values dimension from the optimal policy.Proposition 2 characterizes the first-period equilibrium policy outcome in this case, where ideological polarization increases faster than the evolution of the common-values dimension.

Proposition 2. *Assume that $u_{L_2}(\hat{x}_2^R) - u_{L_2}(\hat{x}_1^R) \geq v_2(\hat{y}_1) - v_2(\hat{y}_2)$. There exists a unique $\tilde{U}$ such that*

- *If $U_q < \tilde{U}$, then the equilibrium policy outcome is efficient, $y_1^* = \hat{y}_1$, and $x_1^* = \hat{x}_1^R$*
- *If $U_q > \tilde{U}$, then the equilibrium policy outcome overshoots, $y_1^* > \hat{y}_1$, and $x_1^* < \hat{x}_1^R$*

Condition 4 implies that even if the efficient common-value policy is implemented in the first period, R still has enough leverage to get its ideologically preferred bundle approved in the second, even if the ideological policy component is at $\hat{x}_1^R$. Rapid movement on the common-values dimension, such as a rapid worsening of the crisis or deteriorating infrastructure, makes even a fully efficient policy today very inefficient tomorrow. Therefore, the proposer can pull the second-period ideological policy all the way to $\hat{x}_2^R$, and has no incentive to undershoot on the common-values dimension.

Consequently, if the initial status quo is sufficiently bad for the veto player (U_q is low) then, given 4, the proposer can get its static optimum accepted in both periods. The proposer has no reason to pay the cost of inefficiency and in equilibrium $y_1^* = \hat{y}_1$. If instead U_q is sufficiently high, the veto player has relatively high bargaining power in the first period. In turn, the proposer is forced to overshoot on the common-values dimension, $y_1 > \hat{y}_1$, to obtain ideological concessions. Setting

$y_1 > \hat{y}_1$ reduces R's future bargaining power and improves the veto player's expected second-period payoff (see Figure 2). The veto player is thus willing to reward R by compromising on the ideological dimension today.

Rapidly Increasing Ideological Polarization

We now consider the case where polarization on the ideological dimension increases rapidly:

$$u_{L_2}(\hat{x}_2^R) - u_{L_2}(\hat{x}_1^R) < v_2(\hat{y}_1) - v_2(\hat{y}_2). \quad (5)$$

Under Condition 5 if R obtains its statically optimum $(\hat{x}_1^R, \hat{y}_1)$ in the first period, then it does not have enough leverage to get its preferred policy in the second. In this case, the parties' induced preferences over the common-values dimension always differ.The proposer's optimal policy implements $y_1 < \hat{y}_1$ to increase future leverage, while the veto player's optimal policy sets $y_1 > \hat{y}_1$ (Figure 4). Thus, the ideological dimension infects the common-values dimension, and inefficiency always arises in equilibrium.

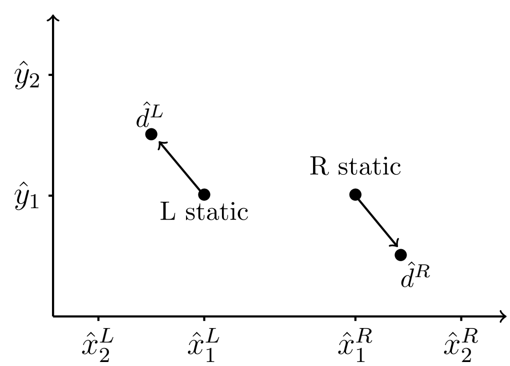

Figure 4: Dynamic ideal points ($\hat{d}^L$ and $\hat{d}^R$) under rapidly increasing polarization.

Proposition 3 characterizes equilibrium outcomes in the first period when polarization increases rapidly.

Proposition 3. *Assume $u_{L_2}(\hat{x}_2^R) - u_{L_2}(\hat{x}_1^R) < v_2(\hat{y}_1) - v_2(\hat{y}_2)$. There exists unique $\underline{U} < \overline{U}$ such that*

- *If $U_q < \underline{U}$, then in equilibrium $y_1^* < \hat{y}_1$, and $x_1^* > \hat{x}_1^R$*

- *If* $U_q \in (\underline{U}, \overline{U})$*, then in equilibrium* $y_1^* < \hat{y}_1$*, and* $x_1^* < \hat{x}_1^R$
- *If* $U_q > \overline{U}$*, then in equilibrium* $y_1^* \in (\hat{y}_1, \hat{y}_d^L)$*, and* $x_1^* < \hat{x}_1^R$

Suppose $U_q > \overline{U}$, meaning the initial status quo is very favorable to the veto player (depicted in the right-most panel of Figure 5). In this case, R has little leverage in the first period and is unable to pull the conflict-dimension policy to its first-period ideal, $x_1^* < \hat{x}_1^R$. Thus, the proposer finds it profitable to bind themselves in the future to obtain larger concessions today. The equilibrium is characterized by a veto-player-induced inefficiency, where the common-values policy overshoots the first-period ideal and moves the status quo too much in the optimal direction, $y_1^* \in (\hat{y}_1, \hat{y}_d^L)$.

If instead $U_q < \underline{U}$, then the initial status quo is very unfavorable to the veto player, as depicted in the left-most panel of Figure 5, and R is unconstrained by L. This means that R can obtain a favorable policy on the conflict dimension and still preserve its future leverage. In equilibrium, the common values policy undershoots the first-period optimum with $y_1^* < \hat{y}_1$, and the proposer is able to pass an extreme ideological policy with $x_1^* \in (\hat{x}_1^R, \hat{x}_2^R)$.

Finally, consider the case where the status quo is not too favorable or unfavorable to either party ($U_q \in (\underline{U}, \overline{U})$), depicted in the middle panel of Figure 5. Here, R undershoots on the common values dimension, $y_1^* < \hat{y}_1$, *and* proposes an ideological policy to the left of both its first and second period ideals, $x_1^* < \hat{x}_1^R$. Interestingly, notice that in this case the proposer *could* get its first-period ideologically preferred policy accepted. However, this would require satisfying the veto player by overshooting on the common values dimension, i.e., bundling $\hat{x}_1^R$ with $y_1 > \hat{y}_1$. Recall that, given concavity, any residual inefficiency the players inherit from the first period is more costly in the second, and thus allows the proposer to buy *even more* concessions than could be obtained by implementing the efficient $\hat{y}_1$ immediately. Thus, R chooses an inefficient policy in the first period, even though it incurs an immediate cost on both the common-values and ideological dimensions. In this case, undershooting does not occur because the proposer does not need leverage today. Rather, the evolution of the common-values optimal policy creates a compounding effect in the cost of inefficiency, which makes it advantageous for the proposer to save additional bargaining leverage for the future.

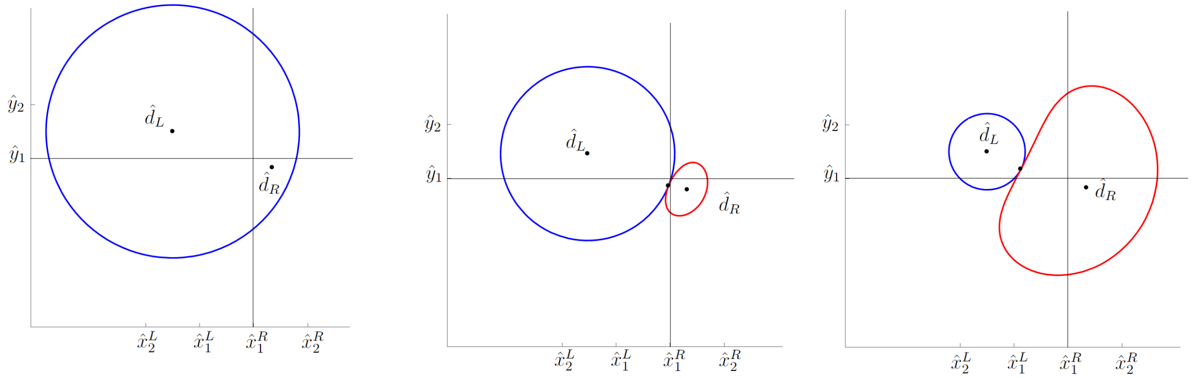

Figure 5: Dynamic indifference curves (blue for the veto player, red for the proposer). The left-most panel considers $U_q < \underline{U}$. In the middle panel we have $U_q \in (\underline{U}, \overline{U})$. In the right-most panel we set $U_q > \overline{U}$.

Relative Extremism and Inefficiency

Our analysis so far demonstrates that a sufficient condition for ideological infection to emerge is that the players grow highly polarized in the second period. Here, we study how the extremism of the veto player and proposer respectively influences the likelihood and magnitude of the inefficiency in equilibrium.

Recall that outcomes are always inefficient when

$$u_{L2}(\hat{x}_2^R) - u_{L2}(\hat{x}_1^R) < v_2(\hat{y}_1) - v_2(\hat{y}_2). \tag{6}$$

Increasing $\hat{x}_2^R$ decreases the left-hand side of 6 and makes it easier for the conditions for inefficiency to hold. L becoming more extreme also decreases the left-hand side of (6). However, increasing $\hat{x}_2^R$ decreases the left-hand side of (6) at a faster rate. Thus, although either way of increasing polarization can lead to ideological infection, proposer extremism has more significant impact than veto player extremism. Furthermore, if R does not become more extreme, $\hat{x}_2^R = \hat{x}_1^R$, then condition (6) never holds and policy is efficient.

The evolution of ideological polarization also affects the degree of inefficiency in the rapid polarization case. However, the direction of this effect is contingent on whether the proposer or veto player becomes increasingly more extreme, as well as whether the status quo is favorable for the veto player.

If R's second-period ideal point becomes more extreme, then it is important for R to maintain leverage for the future. Consequently, increasing $\hat{x}_2^R$ incentivizes R to propose a policy that is further from $\hat{y}_2$. The left panel of Figure 6 depicts a numerical example. Increasing $\hat{x}_2^R$ decreases y_1^*, whether the original first-period status quo is bad or good for the veto player. However, this implies opposite effects on the magnitude of the inefficiency in the first period. When the first-period status quo is good for the veto player, the equilibrium common-values policy always overshoots the static optimum, so increasing $\hat{x}_2^R$ moves y_1^* towards $\hat{y}_1$ and decreases inefficiency. In contrast, in the bad status quo case there is undershooting in equilibrium, thus increasing $\hat{x}_2^R$ moves y_1^* away from $\hat{y}_1$ and increases inefficiency.

The effect of L's increasing extremism is more subtle. Decreasing $\hat{x}_2^L$ impacts y_1^* by changing the second-period constraint, through $\overline{x}$, but it also alters the constraint in the first period. When the initial status quo is bad for the veto player, the first-period constraint is not binding and thus only the effect through the second-period constraint matters. This is depicted in the blue curve of the right-panel in Figure 6. Moving $\hat{x}_2^L$ to the left makes L more stubborn in the second period, and thus bargaining leverage is relatively less effective. Consequently, undershooting becomes a less profitable strategy for the proposer. The equilibrium policy moves closer to $\hat{y}_1$ and inefficiency decreases. If instead the status quo is sufficiently good for L, then L's participation constraint is binding for the proposer in the first period. Here, making $\hat{x}_2^L$ more extreme also decreases L's payoff from the status quo in the first period, which improves R's ability to pass policies. These first and second-period constraint effects can be competing, and generate a non-monotonicity. When $\hat{x}_2^L$ is sufficiently low the first-period constraint effect dominates, and further decreasing $\hat{x}_2^L$ allows R to overshoot less while still satisfying L's constraint. When $\hat{x}_2^L$ is larger, the second-period constraint effect dominates and moving $\hat{x}_2^L$ to the left forces R to overshoot more to compensate. An example is depicted by the yellow curve in left panel of Figure 6.

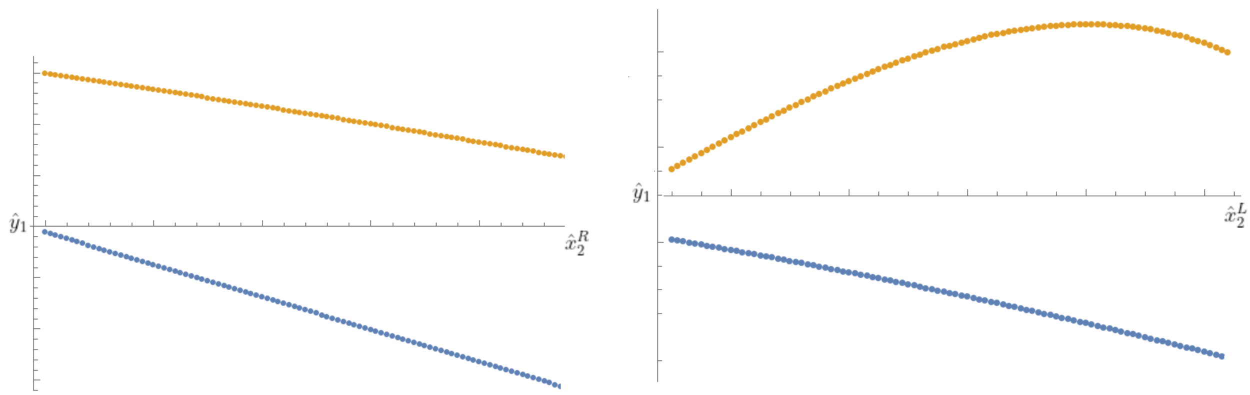

Figure 6: Numerical exercise studying the effect of increasing extremism on y_1^*. The left panel depicts effect of changing $\hat{x}_2^R$ and the right panel the effect of $\hat{x}_2^L$. The blue curve represents the case where the status-quo payoff to L is low, so his participation constraint is not binding. The yellow curve depicts the case where the payoff to L from the status quo is high enough that the constraint is binding.

Long-run Inefficiency

We now extend our analysis to an infinite-horizon game in order to study when increasing polarization leads to inefficiency over the long run. Bargaining proceeds as before, but unfolds over an infinite number of periods, $t = 1, 2, ...$

Recall that the policy dimensions X and Y are both compact. Denote $\overline{Y} = \max Y$ and $\overline{X} = \max X$ as the (upper) boundaries of the policy space on the two dimensions. Additionally, we specify the evolution of ideal points as $\hat{y}_t = \min\{\gamma t, \bar{Y}\}$ and $\hat{x}_t^R = -\hat{x}_t^L = \min\{t^\eta, \bar{X}\}$, with $\gamma > 0$ and $\eta > 0$. Thus, the common-values ideal point increases linearly in time, while the evolution of ideal points on the conflict dimension may be concave or convex. The speed of these evolution processes are parameterized by γ and η. Boundedness of the policy space simplifies existence and characterization of equilibrium behavior, but by taking $\overline{X}$ and $\overline{Y}$ large we can study long-run outcomes that are not sensitive to the ideal points eventually hitting the boundary.

We focus on pure strategy stationary Markov perfect equilibria (henceforth referred to as "equilibria"). Additionally, we assume that L votes to accept whenever indifferent and R proposes a policy that passes in each period, which is without loss of generality.

In our model, a (pure) strategy for R is a function π that determines proposals, and a strategy for L is a function α that determines acceptance decisions. The restriction to Markov strategies requires π depends only on the current status quo, (x_t^q, y_t^q), and the parties' period t ideal points, $(\hat{x}_t^R, \hat{y}_t)$ and $(\hat{x}_t^L, \hat{y}_t)$, thus, $\pi : \mathcal{P}^3 \to \mathcal{P}$. Likewise, for L α is a mapping from ideal points, the status quo, and a proposed policy to an accept ($\alpha = 1$) or reject ($\alpha = 0$) decision, $\alpha : \mathcal{P}^4 \to \{0, 1\}$.

Given a strategy profile $\sigma = (\pi, \alpha)$ we can define continuation payoffs for each player as $V_t^i(x^q, y^q)$, for $i \in \{L, R\}$ (suppressing dependence on σ). Although σ is stationary, we still index continuation payoffs and strategies by t to capture the changing ideal points of the players and reduce notation.

With this notation in hand, we say that a strategy profile $\sigma^* = (\pi^*, \alpha^*)$ constitutes an equilibrium if for all ideal points $(\hat{x}^i, \hat{y})$ for $i \in \{L, R\}$, every status quo q, and all t, the following conditions hold:

- for any proposal (x, y), L accepts, $\alpha(x, y, q) = 1$, if and only if

$$u_{Lt}(x) + v_t(y) + \delta V_t^L(x, y) \geq u_{Lt}(x^q) + v_t(y^q) + \delta V_t^L(x^q, y^q).$$

- R's proposal, $\pi(q)$, solves

$$\begin{aligned} \max_{(x,y)\in\mathcal{P}} \quad & u_{Rt}(x) + v_t(y) + \delta V_t^R(x, y) \\ \text{s.t.} \quad & u_{Lt}(x) + v_t(y) + \delta V_t^L(x, y) \geq u_{Lt}(x^q) + v_t(y^q) + \delta V_t^L(x^q, y^q) \end{aligned}$$

Let $\bar{t}$ be the first period such that $\hat{y}_t = \overline{Y}$ and $\overline{x}_t^R = \overline{X}$. Corollary 1 characterizes equilibrium behavior when $t \geq \bar{t}$. In this case, ideal policies have stopped evolving and the equilibrium policy is at the efficient outcome every period.

Corollary 1. *If $t \geq \bar{t}$ then in equilibrium R proposes $y_t^* = \hat{y}_t = \overline{Y}$.*

As in the two-period model, when the common-values dimension does not evolve, the cost of inefficiency today and tomorrow is the same.[10] Thus, there is no room for either the proposer or the veto player to strategically exploit inefficiency. At $t = \bar{t}$, R uses the optimal common-values policy to maximize ideological concessions and the policy on both dimensions is stuck in all subsequent periods.

We now characterize how policy outcomes evolve leading up to $\bar{t}$. We first focus on the case where polarization increases slowly in the long run relative to changes on the common-values dimension.

Proposition 4. *If $\eta < 1$ then there exists a $\hat{t} < \bar{t}$ such that the equilibrium policy outcome is $x_t = \hat{x}_t^R$ and $y_t = \hat{y}_t$ in every period $t \geq \hat{t}$. Furthermore, for γ sufficiently large we have $\hat{t} = 1$.*

Proposition 4 confirms our insights from the baseline model. When polarization on the ideological dimension evolves slowly (i.e., the evolution is concave), the parties always reach the efficient common-values policy before the 'forced' end date $\bar{t}$. For any period $t > \hat{t}$, R proposes a bundle combining the commonly beneficial policy $\hat{y}_t$ with its preferred ideological policy in period t. Period $\hat{t}$ is thus the first period where the growth in polarization has slowed down enough relative to the common-values dimension that even by proposing the statically efficient Y-policy in each period the proposer has enough leverage to implement his ideal point on the conflict dimension. Furthermore, when the evolution of the common-values dimension is sufficiently rapid the parties reach efficiency immediately.

Next, we consider the case of rapidly growing polarization:

Proposition 5. *If $\eta > 1$ then there exists $\tilde{t}$ such that $y_t^* \neq \hat{y}_t$ in every period $t \in [\tilde{t}, \bar{t}_Y)$. If γ is sufficiently small then $\tilde{t} = 1$.*

As in the two-period model, when polarization on the ideological dimension grows fast enough, the policy outcome is inefficient in every period prior to the 'end date' $\bar{t}_Y$. Notice that this holds for any $\bar{t}_Y$, indicating that rapidly growing polarization can sustain inefficiency on the path of play for an arbitrarily long period of time.

[10]A similar result can be shown if only the ideal point on the common-values dimension has stopped evolving.

Robustness

Here, we discuss the robustness of our results to relaxing several of the assumptions imposed in the baseline model. Formal analysis is presented in Appendix C. For simplicity, we return to our two-period setting. Further, we assume the initial status quo is sufficiently unfavorable for L that R is unconstrained in the first period. Recall that, in this case, the baseline model predicts undershooting under rapidly increasing polarization and efficiency under slow polarization. Thus, this case creates the starkest comparison for understanding the conditions that lead to inefficiency.

Turnover. Up to this point, we assumed that party R is always the proposer, thus emphasizing that inefficiency and ideological infection in our setting do not stem from fear of the other party taking power. We now turn our attention to the implications of turnover for inefficiency. Specifically, we assume R is the proposer in period 1 and remains so with probability ρ in period 2, while L becomes the proposer with probability $1-\rho$.

We demonstrate that policy outcomes are still almost always inefficient when polarization increases rapidly.

Proposition 6. *Assume $u_{R2}(\hat{x}_{L2}) < \max\{u_{R2}(\hat{x}_{R1}), u_{L2}(\hat{x}_{R1})\} + v_2(\hat{y}_1)$. There exists a unique $\tilde{\rho} \in (0,1)$ such that $|y_1^* - \hat{y}_1|$ is decreasing in ρ for $\rho < \tilde{\rho}$ and increasing in π for $\pi > \tilde{\pi}$. Furthermore, $y_1^* < \hat{y}_1$ for $\rho > \tilde{\rho}$ and $y_1^* > \hat{y}_1$ for $\rho < \tilde{\rho}$.*

If ρ is high then R is confident of remaining the proposer tomorrow and therefore undershoots in the first period. However, as ρ decreases, R becomes less confident of remaining in power and moves y towards $\hat{y}_1$ to offset the downside of keeping leverage in case L becomes the proposer. Eventually, $\rho < \tilde{\rho}$ and the probability of remaining proposer is sufficiently low that R begins to overshoot as insurance against L becoming the proposer tomorrow. Thus, there is a unique value of ρ for which the proposer's incentives to under and overshoot exactly compensate each other, resulting in an efficient equilibrium. For all other values, inefficiency persists. This finding suggests that during times of rapidly increasing polarization, electoral uncertainty over who will hold power tomorrow can partially mitigate ideological infection, but cannot completely eliminate such distortions.

Furthermore, Corollary 2 illustrates that, under some parameter values, turnover can actually create inefficiency:

Corollary 2. *Suppose that* $u_{R_2}(\hat{x}_1^R) > u_{L_2}(\hat{x}_1^R)$ *and* $u_{R2}(\hat{x}_{L2}) \in [u_{L2}(\hat{x}_{R1})+v_2(\hat{y}_1), u_{R2}(\hat{x}_{R1})+v_2(\hat{y}_1)]$. *Then,* $y_1^* = \hat{y}_1$ *when* $\rho = 1$, *but* $y_1^* \neq \hat{y}_1$ *for any* $\rho < 1$.

When R is sure to remain in power in the second period, efficiency emerges if and only if by setting $y_1 = \hat{y}_1$ (and $x_1 = \hat{x}_1^R$) R still maintains enough leverage to obtain its static optimum in the second period. This condition ensures that any marginal movement away from $\hat{y}_1$ has no impact on the outcome of the second-period bargaining. However, when $\rho < 1$ for this marginal effect to be insignificant L must also be able to pass its optimal bundle after inheriting the status quo at $(\hat{x}_1^R, \hat{y}_1)$, should L come to power in the second period. When $uR_2(\hat{x}_1^R) > uL_2(\hat{x}_1^R)$, this second condition is more binding. As a result, introducing turnover can generate inefficiency under certain conditions.

Stochastic Preferences. In our analysis thus far, we have assumed the players can perfectly anticipate how their respective ideological preferences will evolve over time. This assumption is useful isolate the mechanism behind our results and abstract from potential insurance motives that could introduce distortions when players are confronted with uncertainty, which is a common occurrence in the existing literature.

Suppose the players anticipate that ideological polarization will (weakly) increase in the second period, but do not know by how much. Formally, $\hat{x}_2^R = \hat{x}_1^R + \epsilon$ and $\hat{x}_2^L = \hat{x}_1^L - \epsilon$, where ϵ is drawn from a distribution with support $[0, \bar{\epsilon}]$. First, we note that equilibrium outcomes are never inefficient if the optimal common-value dimension remains constant over time, $\hat{y}_1 = \hat{y}_2$. Consider instead $\hat{y}_2 > \hat{y}_1$. To begin, suppose that $\bar{\epsilon}$ is such that when $\hat{x}_2^R = \hat{x}_1^R + \bar{\epsilon}$ and $\hat{x}_2^L = \hat{x}_1^L - \bar{\epsilon}$, condition 5 is satisfied. In this case, although the parties cannot perfectly predict their ideological preferences, it is possible they will become so polarized that inefficiency in the first period could yield gains for the proposer in the second.[11] This generates incentives for the proposer to distort policy in the

[11] Recall that condition 5 indicates that if R implements his static optimum in the first period, $(\hat{x}_1^R, \hat{y}_1)$, he will not have enough leverage to get his static optimum in the second.

first period, resulting in $y_1^* \neq \hat{y}_1$. On the other hand, if condition 5 is not met at $\hat{x}_2^R = \hat{x}_1^R + \bar{\epsilon}$ and $\hat{x}_2^L = \hat{x}_1^L - \bar{\epsilon}$, then R chooses $x_1^* = \hat{x}_1^R$, $y_1^* = \hat{y}_1$, and is able to obtain its ideal point in the second period regardless of the shock.

Therefore, even when parties face uncertainty over their future preferences, the mere possibility of rapid ideological polarization is enough to generate inefficiency on the common-values dimension. Moreover, similar results would hold if players also faced uncertainty over the evolution of the optimal policy on the y dimension.

Different Weights. Our previous discussions highlight that the reason for inefficiency in equilibrium is the gap between the cost that the proposer must pay in the first period and the cost imposed on the veto player in the second. In the baseline model, this gap arises from the evolution of the optimal common-values policy, which, when combined with concave preferences, results in residual inefficiency from the first period being more costly in the second. Alternatively, we can generate this wedge by assuming that the proposer and veto player attach different weights to the common-values dimension.

Suppose that the common-values ideal policy remains unchanged over time, but R places weight θ on this dimension, $\theta v(y)$. If R places greater weight than L, $\theta > 1$ then the equilibrium outcome is efficient as in the baseline. Additionally, if there is no change in the players' ideological preferences then R simply moves policy to its preferred ideal point on both dimensions. Thus, increasing polarization remains important for inefficiency to emerge. In particular, if θ is sufficiently low and the parties become more extreme then the proposer still implements an inefficient policy $y_1 < \hat{y}_1$ to maintain future bargaining leverage. This inefficiency is advantageous because, due to the different weights assigned to the common-values dimension, it is more costly for the veto player in the second period than it was for the proposer in the first.

Risk Aversion. To obtain sharper characterization of the results, in the baseline model we assume quadratic-loss utility functions for the players on each dimension. However, the mechanism that generates inefficiency only requires the players have some risk-aversion. Specifically, the proof that policy outcomes are inefficient when U_q is sufficiently low only requires that utility functions are single-peaked and everywhere differentiable, i.e., utility is concave near the ideal point.

However, a minimal amount of risk-aversion is necessary for inefficiency. When $v_t(y) = -|y - \hat{y}_t|$ we show that the equilibrium policy is always efficient. Thus, this analysis confirms an intuition already described in details in the previous analysis section. The mechanism underlying our results is the property that concave preferences generate the compounding effect, whereby (under evolving ideal points) a residual inefficiency unit inherited from the first period being more costly in the second.

General Movements. Building on our motivating observation, in the baseline model we assumed both parties become more extreme over time. We now consider what happens if ideal points can move in any direction.

In this case, there is still no inefficiency whenever the common-values ideal policy remains constant. Additionally, we find that the proposer must become more extreme in the second period to sustain inefficiency in the first. That is, if the right-wing party's ideal point shifts to the left, the efficient policy $\hat{y}_1$ is always implemented in the first period. Alternatively, if both parties' ideal points shift to the right, then a sufficient condition for inefficiency is that the right-wing party's ideal point moves faster, which increases polarization. Conversely, if the left-wing party's movement is faster and polarization decreases, then inefficiency may only emerge if the movement is not too large. Therefore, as in the baseline model, rapidly increasing polarization leads to inevitable ideological infection. Conversely, when polarization decreases too quickly, inefficiency never emerges.

Conclusion

This paper analyzes a model where two parties bargain over two issues, one with ideological conflict and one where they agree on the correct policy, and today's policy outcome becomes tomorrow's status quo. In equilibrium, the proposer can use the common values dimension to extract concessions on the ideological one. When parties are forward looking and their preferences evolve over time, this may generate inefficiencies in equilibrium. Even though the parties agree on what is the optimal policy on the common values dimension, they sometimes fail to implement it. Crucially, this form of ideological infection is inescapable if polarization increases rapidly over time.

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A Proofs for Baseline Model

We split problem 2 into two cases, depending on if $x_2^*(x,y) = \overline{x}(x,y)$ or $x_2^*(x,y) = \hat{x}_2^R$.

First, for (x,y) such that $x_2^* = \overline{x}(x,y)$ we can write the proposer's problem as

$$\begin{aligned} \max_{x,y} \; & u_{R1}(x) + v_1(y) + u_{R2}\Big(\overline{x}_2(x,y)\Big) \\ & \text{s.t. } u_{L1}(x) + v_1(y) + u_{L2}(x) + v_2(y) \geq U_q \\ & u_{L2}(x) + v_2(y) \geq u_{L2}(\hat{x}_2^R) \end{aligned}$$

System 7 yields the following KKT conditions for this problem:

$$u'_{R1}(x) + \frac{\partial \overline{x}_2}{\partial x} u'_{R2}\Big(\overline{x}(x,y)\Big) + \lambda_1 \left[u'_{L1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{L2}\Big(\overline{x}(x,y)\Big) \right] + \lambda_2 u'_{L2}(x) = 0 \tag{7a}$$

$$v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{R2}\Big(\overline{x}(x,y)\Big) + \lambda_1 \left[v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{L2}\Big(\overline{x}(x,y)\Big) \right] + \lambda_2 v'_2(y) = 0 \tag{7b}$$

$$\lambda_1 \left[u_{L1}(x) + v_1(y) + u_{L2}\Big(\overline{x}(x,y)\Big) - U_q \right] = 0 \tag{7c}$$

$$\lambda_2 \left[u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) \right] = 0 \tag{7d}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{7e}$$

Alternatively, for (x,y) such that $x_2^*(x,y) = \hat{x}_2^R$ the proposer's problem is

$$\begin{aligned} \max_{x,y} \; & u_{R1}(x) + v_1(y) \\ & \text{s.t. } u_{L1}(x) + v_1(y) + u_{L2}(\hat{x}_2^R) \geq U_q \\ & u_{L2}(x) + v_2(y) \leq u_{L2}(\hat{x}_2^R) \end{aligned}$$

The KKT conditions for this problem yields the system 8:

$$u'_{R1}(x) + \lambda_1 u'_{L1}(x) - \lambda_2 u'_{L2}(x) = 0 \tag{8a}$$

$$v'_1(y) + \lambda_1 v'_1(y) - \lambda_2 v'_2(y) = 0 \tag{8b}$$

$$\lambda_1 \left[u_{L1}(x) + v_1(y) + u_{L2}(\hat{x}_2^R) - U_q \right] = 0 \tag{8c}$$

$$\lambda_2 \left[u_{L2}(\hat{x}_2^R) - u_{L2}(x) - v_2(y) \right] = 0 \tag{8d}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{8e}$$

Lemma A.1. *Any (x, y) such that $\overline{x}(x, y) > \hat{x}_2^R$ and $y \neq \hat{y}_1$ does not solve 2.*

Proof. Assume (x, y) is optimal. Since $\overline{x}(x, y)$ is continuous in y there exists y' closer to $\hat{y}_1$ such that $\overline{x}(x, y') > \hat{x}_2^R$. Because $u_{L1}(x) + v_1(y') + u_{L2}(\hat{x}_2^R) > u_{L1}(x) + v_1(y) + u_{L2}(\hat{x}_2^R)$ it must be that (x, y') also satisfies the constraint. Moreover, evaluating the objective function at (x, y') and (x, y) yields $u_{R1}(x) + v_1(y') + u_{R2}(\hat{x}_2^R) > u_{R1}(x) + v_1(y) + u_{R2}(\hat{x}_2^R)$, contradicting that (x, y) solves 2. □

Proposition 1. *If $\hat{y}_1 = \hat{y}_2 = \hat{y}$ then $y_1^* = y_2^* = \hat{y}$.*

Proof. Since $\hat{y}_1 = \hat{y}_2$ let $v(y) = v_1(y) = v_2(y)$. To start, note that by A.1 if (x, y) is optimal with $y \neq \hat{y}_1$ then we must have $\overline{x}(x, y) \leq \hat{x}_2^R$. Thus, if (x, y) is optimal then it must solve system 7.

If $\lambda_2 > 0$ then $\overline{x}(x, y) = \hat{x}_2^R$ and 7b reduces to $v'(y)(1 + \lambda_1 + \lambda_2) = 0$ which holds iff $y = \hat{y}$. Moving forward, assume $\lambda_2 = 0$.

Assuming $\lambda_1 = 0$, any equilibrium pair (x, y) must satisfy:

$$u'_{R1} + \frac{\partial \overline{x}}{\partial x} u'_{R2}(\overline{x}(x, y)) = 0 \tag{9}$$

$$v'(y) + \frac{\partial \overline{x}}{\partial y} u'_{R2}(\overline{x}(x, y)) = 0. \tag{10}$$

Assuming $y \neq \hat{y}$[12] and substituting in $\frac{\partial \overline{x}}{\partial x} = \frac{u'_{L2}(x)}{u'_{L2}(\overline{x})}$ and $\frac{\partial \overline{x}}{\partial y} = \frac{v'(y)}{u'_{L2}(\overline{x})}$, the above reduce to

[12]Notice that at $y = \hat{y}$, 10 is always satisfied.

$$u'_{R1} + \frac{u'_{L2}(x)}{u'_{L2}(\overline{x})} u'_{R2}(\overline{x}(x,y)) = 0 \tag{11}$$

$$v'(y) + \frac{v'(y)}{u'_{L2}(\overline{x})} u'_{R2}(\overline{x}(x,y)) = 0. \tag{12}$$

By 12, in equilibrium we must have

$$\frac{u'_{R2}(\overline{x})}{u'_{L2}(\overline{x})} = -1, \tag{13}$$

and by 11

$$\frac{u'_{R2}(\overline{x})}{u'_{L2}(\overline{x})} = -\frac{u'_{R2}(x)}{u'_{L2}(x)}. \tag{14}$$

Combining the above, we obtain

$$\frac{u'_{R1}(x)}{u'_{L2}(x)} = 1, \tag{15}$$

which never holds.

Next, assume the veto player's first period constraint is binding, so $\lambda_1 > 0$. Consequently, for the KKT conditions to hold requires that $u_{L2}(x) + v(y) = u_{L2}(\hat{x}_2^R)$, which implies that $\overline{x}(x,y) = \hat{x}_2^R$. We can also rewrite the condition (7b) as $v'(y)\Big(1 + \frac{u'_{R2}(\overline{x}(x,y))}{u'_{L2}(\overline{x}(x,y))} + \lambda_1\Big) = 0$. Because $\overline{x}(x,y) = \hat{x}_{R2}$ this yields $u'_{R2}(\overline{x}(x,y)) = u'_{R2}(\hat{x}_2^R) = 0$. Thus, (7b) simplifies to $v'(y)(1+\lambda_1) = 0$. If $y \neq \hat{y}_1$ then $v'(y) \neq 0$, thus for this condition to hold requires $1 + \lambda_1 = 0$. However, $\lambda_1 > 0$, a contradiction. Thus, there cannot exist a solution to system (7) with $y \neq \hat{y}$. □

Lemma A.2. *If $\lambda_1 > 0$ and $\lambda_2 = 0$ then there is at most one value of U_q such that $y_1^* = \hat{y}_1$ solves 7.*

Proof. By 7 if $y = \hat{y}_1$ then x must solve

$$u_{L1}(x) + u_{L2}\Big(\overline{x}(x, \hat{y}_1)\Big) = U_q \tag{16}$$

The LHS of the is strictly decreasing in x for $x > \hat{x}_1^L$, thus, there must be a unique $x' > \hat{x}_1^L$ that solves this equality.

Solving for λ_1 and rearranging x must also solve

$$\frac{u'_{R2}(\overline{x}(x, \hat{y}_1))}{u'_{L2}(\overline{x}(x, \hat{y}_1))} = \frac{u'_{R1}(x)}{u'_{L1}(x)} \tag{17}$$

Since u_i is quadratic it is straightforward to show that there exists a unique x'' that solves 17. Notice that neither side of 17 depends on U_q whereas RHS 16 is strictly increasing in U_q. Consequently, $x' = x''$ for at most one values of the status quo U_q. □

Lemma A.3. *If $u_{L2}(\hat{x}_2^R) < u_{L1}(\hat{x}_1^R) + v_2(\hat{y}_1)$ then $x_2^*(x, y) = \overline{x}(x, y)$.*

Proof. Note that $x_2^*(x, y) \neq \overline{x}(x, y)$ if and only if $\overline{x}_2(x, y) > \hat{x}_2^R$. Lemma A.1 implies that if there is a solution with $\overline{x}_2(x, y) > \hat{x}_2^R$ then $y = \hat{y}_1$. Thus, if $\overline{x}(x, y) > \hat{x}_2^R$ then $u_{L2}(\hat{x}_2^R) > u_{L1}(x) + v_2(\hat{y}_1)$. If $u_{L2}(\hat{x}_2^R) \leq u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$ then $u_{L2}(\hat{x}_2^R) < u_{L2}(x) + v_2(\hat{y}_1)$ for all $x \in (\hat{x}_2^L, \hat{x}_1^R)$. Additionally, it cannot be optimal for R to choose $x > \hat{x}_1^R$ s.t. $u_{L2}(\hat{x}_2^R) > u_{L2}(x) + v_2(\hat{y}_1)$ since R can deviate to x' closer to $\hat{x}_1^R$ and improve the first-period payoff for L and R without changing second period payoffs. Thus, if $u_{L2}(\hat{x}_2^R) \leq u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$, then it cannot be optimal to choose x, y such that $\overline{x}(x, y) > \hat{x}_2^R$. □

Lemma A.4. *Assume $u_{L2}(\hat{x}_2^R) < u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$. There exists $\overline{U}$ such that if $U_q > \overline{U}$ then $y_1 > \hat{y}_1$. Otherwise, if $U_q < \overline{U}$ then $y_1 < \hat{y}_1$.*

Proof. By Lemma A.3, we can focus on (x, y) such that $x_2^*(x, y) = \overline{x}(x, y)$. Thus, if (x, y) is optimal there must be a λ_1, λ_2 such that $(x, y, \lambda_1, \lambda_2)$ solves 7.

To start, note that if U_q is sufficiently low then the first-period veto player constraint does not bind, thus, $\lambda_1 = 0$. In this case, the KKT conditions become

$$u'_{R1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{R2}\Big(\overline{x}(x,y)\Big) + \lambda_2 u'_{L2}(x) = 0 \tag{18}$$

$$v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{R2}\Big(\overline{x}(x,y)\Big) + \lambda_2 v'_2(y) = 0 \tag{19}$$

$$\lambda_2 \Bigg[u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) \Bigg] = 0 \tag{20}$$

$$\lambda_2 \geq 0 \tag{21}$$

First, consider $u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) > 0$ so that $\lambda_2 = 0$. Assume $y \in [\hat{y}_1, \hat{y}_2)$. This implies $v'_1(y) \leq 0$, $\frac{\partial \overline{x}}{\partial y} < 0$ and $u'_{R2}(\overline{x}(x,y)) \geq 0$. Thus, a necessary condition for the second equality to hold is that $\overline{x}(x,y) = \hat{x}_2^R$, which contradicts $u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) > 0$.

Second, consider $u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) = 0$ so $\overline{x}(x,y) = \hat{x}_2^R$. Thus, for the first condition to hold we must have $u'_{R1}(x) + \lambda_2 u'_{L2}(x) = 0$, which implies $u'_{R1}(x) \geq 0$ which implies $x \leq \hat{x}_1^R$. Thus, $u_{L2}(x) + v_2(y) \geq u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$. But by assumption $u_{L2}(\hat{x}_2^R) < u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$, which contradicts that $u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) = 0$.

Next, let $x_1^q = \hat{x}_1^L$ and $y_1^q = \hat{y}_1$, thus, the first period constraint is binding. This yields $U_q = u_{L2}(\hat{x}_1^L) + v_2(\hat{y}_1)$. For a contradiction assume that $y < \hat{y}_1$ is optimal. To satisfy first period constraint we need $u_{L1}(x) + v_1(y) + u_{L2}(x) + v_2(y) = u_{L2}(\hat{x}_1^L) + v_2(\hat{y}_1)$. If $y < \hat{y}_1$ a necessary condition for this to be satisfied is that $x < \hat{x}_1^L$. In this case, $u_{R1}(x) < u_{R1}(\hat{x}_1^L)$ and $v_1(y) < v_1(\hat{y}_1)$ a necessary condition for such a proposal to be optimal for R is that $\overline{x}(x,y) > \overline{x}(\hat{x}_1^L, \hat{y}_1)$. This inequality implies that $u_{L2}(x) + v_2(y) < u_{L2}(\hat{x}_1^L) + v_2(\hat{y}_1)$. However, then for the veto player to accept (x, y) requires $u_{L1}(x) + v_1(y) > 0$, which can never hold.

The theorem of the maximum yields that the correspondence of equilibrium proposals is upper hemi-continuous in U_q.[13] By Lemma A.2 there is at most one values of U_q such that $y_1^* = \hat{y}_1$. Thus, there must be a unique $\overline{U}_q$ such that in equilibrium $y_1^* < \hat{y}_1$ for $U_q < \overline{U}_q$ and $y_1^* > \hat{y}_1$ for $U_q > \overline{U}_q$.

□

[13]Moreover, Theorem I.3.1 from Mas-Colell (1985) yields that the equilibrium proposal is unique for almost all parameters.

Lemma A.5. *Assume* $u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1) \leq u_{L2}(\hat{x}_2^R)$ *and the veto player's constraint binds. Any* (x, y) *such that* $\overline{x}_2(x, y) = \hat{x}_2^R$ *with* $y \neq \hat{y}_1$ *does not solve 2.*

Proof. By Lemma A.1 if (x, y) is optimal with $y \neq \hat{y}_1$ then it solves system 7. If $\overline{x}_2(x, y) = \hat{x}_2^R$ then the system reduces to

$$u'_{R1}(x) + \lambda_1(u'_{L1}(x) + u'_{L2}(x)) + \lambda_2 u'_{L2}(x) = 0 \tag{22}$$

$$v'_1(y) + \lambda_1(v'_1(y) + v'_2(y)) + \lambda_2 v'_2(y) = 0 \tag{23}$$

$$u_{L1}(x) + v_1(y) + u_{L2}(\hat{x}_2^R) - U_1 = 0 \tag{24}$$

$$u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) = 0 \tag{25}$$

If $y < \hat{y}_1$ then the LHS of 23 is strictly positive, so this cannot be optimal. Next, assume $y > \hat{y}_1$. If $x < \hat{x}_1^R$ then $u_{L2}(\hat{x}_2^R) = u_{L2}(x) + v_2(y) > u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$, which implies that R can get $\hat{x}_2^R$ passed tomorrow if $(\hat{x}_1^R, \hat{y}_1)$ is implemented today. Further, notice that $u_{L1}(x) + v_1(\hat{y}_1) + u_{L2}(\hat{x}_2^R) > u_{L1}(x) + v_1(y_1) + u_{L2}(\hat{x}_2^R)$, therefore $(x, \hat{y}_1)$ is passable today, and preferred by the proposer to the conjectured (x, y). Finally, supposes instead $x \geq \hat{x}_1^R$. Then LHS 22 is strictly less than 0, and so this cannot be optimal. □

Lemma A.6. *Assume* $u_{L2}(\hat{x}_2^R) \geq u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$. *Let* x' *be the upper solution to* $u_{L1}(x) + u_{L2}(\hat{x}_2^R) - U_q = 0$. *Let* x'' *be the solution to* $u_{L2}(\hat{x}_2^R) - u_{L2}(x) - v_2(\hat{y}_1) = 0$. *There exists* U'_q *such that* $x' > x''$ *if and only if* $U_q < U'_q$.

Proof. At $U_q = u_{L1}(\hat{x}_1^R) + u_{L2}(\hat{x}^2)$ we have $x' = \hat{x}_1^R \geq x''$. The solution to $u_{L1}(x) + u_{L2}(\hat{x}_2^R) - U_q = 0$ is strictly decreasing in U_q. The claim then follows from the observation that x'' is not a function of U_q. □

Lemma A.7. *Assume* $U_q < \overline{U}_q$. *There exists* $\underline{U}_q < \overline{U}_q$ *such that* $x > \hat{x}_1^R$ *for* $U < \underline{U}_q$ *and* $x < \hat{x}_1^R$ *for* $U > \underline{U}_q$.

Proof. Set $x = \hat{x}_1^R$. Then need y such that

$$u_{L1}(\hat{x}_1^R) + v_1(y) + u_{L2}(\overline{x}(\hat{x}_1^R, y)) = U_q$$

and y must solve

$$\frac{u'_{R2}(\overline{x}(\hat{x}_1^R, y))}{u'_{R2}(\overline{x}(\hat{x}_1^R, y))} \frac{u'_{L1}(\hat{x}_1^R)}{u'_{L1}(\hat{x}_1^R) + u'_{L2}(\hat{x}_1^R)} = \frac{v'_1(y) + \frac{u'_{R2}(\overline{x}(\hat{x}_1^R, y))}{u'_{R2}(\overline{x}(\hat{x}_1^R, y))} v'_2(y)}{v'_1(y) + v'_2(y)}$$
$$\Leftrightarrow u'_{R2}(\overline{x}(\hat{x}_1^R, y)) \frac{u'_{L1}(\hat{x}_1^R)}{u'_{L1}(\hat{x}_1^R) + u'_{L2}(\hat{x}_1^R)} = u'_{R2}(\overline{x}(\hat{x}_1^R, y))$$

If $u'_{R2}(\overline{x}) \neq 0$ then the above reduces to $\frac{u'_{L1}(\hat{x}_1^R)}{u'_{L1}(\hat{x}_1^R) + u'_{L2}(\hat{x}_1^R)} = 1$, which never holds. Thus, this only holds for the unique $y < \hat{y}_2$ such that $\overline{x}(\hat{x}_1^R, y) = \hat{x}_2^R$, which sets both sides equal to 0 and does not depend on U_q. Hence, there is at most one U_q such that $x_1^* = \hat{x}_1^R$. □

Proposition 2. *Assume that* $u_{L_2}(\hat{x}_2^R) - u_{L_2}(\hat{x}_1^R) \geq v_2(\hat{y}_1) - v_2(\hat{y}_2)$. *There exists a unique* $\tilde{U}$ *such that*

- *If* $U_q < \tilde{U}$, *then the equilibrium policy outcome is efficient,* $y_1^* = \hat{y}_1$, *and* $x_1^* = \hat{x}_1^R$
- *If* $U_q > \tilde{U}$, *then the equilibrium policy outcome overshoots,* $y_1^* > \hat{y}_1$, *and* $x_1^* < \hat{x}_1^R$

Proof. For $U_q \leq u_{L1}(\hat{x}_1^R) + u_{L2}(\hat{x}_2^R)$ clearly $(x, y) = (\hat{x}_1^R, \hat{y}_1)$ is optimal. For the remainder of the proof, assume $U_q > u_{L1}(\hat{x}_1^R) + u_{L2}(\hat{x}_2^R)$.

To start, assume $U_q < U'_q$. Thus, by Lemma A.6 $x'' < x'$. We show that $y < \hat{y}_1$ cannot be optimal. For a contradiction, assume (x, y) such that $y < \hat{y}_1$ is optimal. First, if $x > x'$ then $u_{L2}(x) + v_2(y) < u_{L2}(x') + v_2(\hat{y}_1) \leq u_{L2}(\hat{x}_2^R)$. Therefore, $u_{L2}(x_2^*(x, y)) = u_{L2}(x_2^*(x', \hat{y}_1)) = u_{L2}(\hat{x}_2^R)$. However, L's utility in the first period from (x, y) is strictly less than from $(x', \hat{y}_1)$, thus $u_{L1}(x) + v_1(y) + u_{L2}(\hat{x}_2^R) < U_q$. Second, if $x < x'$ then R can deviate to $y = \hat{y}_1$ and $x = x'$, which L accepts and this strictly improves R's first-period payoff and weakly improves the second-period payoff. Thus, $y < \hat{y}_1$ cannot be optimal.

At $U_q = U'_q$ if $y = \hat{y}_1$ then $x = x'$ and $\overline{x}(x', \hat{y}_1) = \hat{x}_2^R$. Thus, if this is optimal it must solve

7. However, for this to hold requires $\lambda_1 = \lambda_2 = 0$. But this implies that a necessary condition is $u'_{R1}(x) = 0$, which fails at x'. However, by the argument in the previous paragraph $y = \hat{y}_1$ must be better than $y < \hat{y}_1$. Thus, at U'_q we must have $y_1^* > \hat{y}_1$. Moreover, by Lemma A.2 we can have $y_1^* = \hat{y}_1$ for at most one values of U_q. However, the arguments from Lemma A.4 imply that for U_q sufficiently high $y_1^* > \hat{y}_1$. Thus, we must have $y_1^* > \hat{y}_1$ for all $U_q \geq U'_q$. □

Proposition 3. *Assume $u_{L_2}(\hat{x}_2^R) - u_{L_2}(\hat{x}_1^R) < v_2(\hat{y}_1) - v_2(\hat{y}_2)$. There exists unique $\underline{U} < \overline{U}$ such that*

- *If $U_q < \underline{U}$, then in equilibrium $y_1^* < \hat{y}_1$, and $x_1^* > \hat{x}_1^R$*
- *If $U_q \in (\underline{U}, \overline{U})$, then in equilibrium $y_1^* < \hat{y}_1$, and $x_1^* < \hat{x}_1^R$*
- *If $U_q > \overline{U}$, then in equilibrium $y_1^* \in (\hat{y}_1, \hat{y}_d^L)$, and $x_1^* < \hat{x}_1^R$*

Proof. The result follows from Lemmas A.4 and A.7. □

B Proofs for Long-run Outcomes

Recall that $\max X = \overline{X}$, $\max Y = \overline{Y}$, and define $\min X = \underline{X}$, $\min Y = \underline{Y}$. Similar to $\bar{t}_Y$, define $\bar{t}_X$ be the first period such that $\hat{x}_t^R = \overline{X}$ and $\hat{x}_t^L = \underline{X}$ and define $\bar{t} = \max\{\bar{t}_X, \bar{t}_Y\}$.

Lemma B.1. *Any subgame starting at $t \geq \bar{t}$ has an equilibrium in which R proposes $y_t^* = \hat{y}_t = \overline{Y}$ and $x_t^* = \min\{\overline{X}, \overline{x}(x_t^q, y_t^q)\}$. Moreover, every equilibrium yields the same continuation payoff.*

Proof. For all $t \geq \bar{t}$ we have $v_t(y) = v_{\bar{t}}(y) = -(y - \overline{Y})^2 \equiv v(y), u_{Lt}(x) = u_{L\bar{t}}(x) = -(x - \underline{X})^2$, and $u_{Rt}(x) = u_{R\bar{t}}(x) = -(x - \overline{X})^2$. Conjecture that in every period L accepts any (x, y) such that $u_{L\bar{t}}(x) + v(y) \geq u_{L\bar{t}}(x_t^q) + v(y_t^q)$ and R proposes (x, y) to solve

$$\max_{x,y} \; u_{R\bar{t}}(x) + v(y)$$
$$\text{s.t. } u_{L\bar{t}}(x) + v(y) \geq u_{L\bar{t}}(x_t^q) + v(y_t^q),$$

which yields $y^* = \overline{Y}$ and $x^* = \min\{\overline{X}, \overline{x}(x^q, y^q)\}$. We show this is an equilibrium.

If L accepts this yields

$$\frac{u_{L\bar{t}}(x^*) + v(Y)}{1-\delta}$$

If L rejects then the status quo tomorrow remains (x^q, y^q) and tomorrow R again proposes (x^*, y^*), which under the conjectured strategy profile L accepts. Thus, L's payoff from rejecting is

$$u_{L\bar{t}}(x^q) + v(y^q) + \delta \frac{u_{L\bar{t}}(x^*) + v(\overline{Y})}{1-\delta},$$

which is weakly less than its payoff from accepting by construction of (x^*, y^*).

Now consider the proposer. If R does not deviate from the proposed strategy this yields a payoff

$$u_{R\bar{t}}(x^*) + v(y^*) + \delta \frac{u_{R\bar{t}}(x^*) + v(y^*)}{1-\delta}$$

If R deviates and proposes any (x, y) such that $u_{L\bar{t}}(x) + v(y) < u_{L\bar{t}}(x^q) + v(y^q)$ then L rejects, the status quo remains in place, and tomorrow R proposes (x^*, y^*) which is accepted. Thus, such a deviation yields

$$u_{R\bar{t}}(x^q) + v(y^q) + \delta \frac{u_{R\bar{t}}(x^*) + v(y^*)}{1-\delta},$$

which is weakly less than its payoff from not deviating by construction of (x^*, y^*).

Finally, consider a deviation to $(x', y') \neq (x^*, y^*)$ such that $u_{L\bar{t}}(x') + v(y') \geq u_{L\bar{t}}(x^q) + v(y^q)$. If $u_{L\bar{t}}(x') + v(y') = u_{L\bar{t}}(x^q) + v(y^q)$ then the constraint tomorrow is the exact same today and R proposes x^*, y^* tomorrow which is accepted and remains in place forever after. However, by construction the utility today from (x', y') must be weakly worse than the utility from $(x^*, y*)$, thus, (x', y') is not profitable. If $u_{L\bar{t}}(x') + v(y') > u_{L\bar{t}}(x^q) + v(y^q)$ then (x', y') must yield a worse utility today to R and makes the constraint tomorrow more binding which under the conjectured strategy yields a worse payoff in every subsequent period.

We now prove uniqueness of continuation values. To start, note that $(\overline{X}, \overline{Y})$ is R's unconstrained maximizer. If (x^q, y^q) is such that $\overline{x}(x^q, y^q) \geq \overline{X}$ then the equilibrium is unique, as R can deviate to the strategy propose $(\overline{X}, \overline{Y})$ every period, and L's equilibrium acceptance condition implies that L must accept. Next, assume $\overline{x}(x^q, y^q) < \overline{X}$. There cannot be an equilibrium in which R ever proposes and passes $(\overline{X}, \overline{Y})$, as this implies that at some point R passes a policy which is worse than the status quo for L. But then either every policy on the path is worse for L than the status quo, in which case L can deviate from the start by initially rejecting every period. Or some policy is better for L on the path, in which case L can profitably deviate by rejecting all future offers. Thus, in any equilibrium L's constraint must be binding every period, which implies that L's payoff from accepting any offer (x, y) is $\frac{u_{L\bar{t}}(x)+v(y)}{1-\delta}$. Constraint set in any period is $A(x^q, y^q) = \{(x, y) \in \mathcal{P} | u_{L\bar{t}}(x) + v(y) = u_{L\bar{t}}(x^q) + v(y^q)\}$, which is a continuous correspondence. Thus, the dynamic programming problem for R has a unique solution (Stokey et al., 1989).

□

Proposition 4. *There exists an equilibrium and every equilibrium yields the same continuation values.*

Proof. Given the assumed evolution of ideal points, the ideal points hit the boundary of $\mathcal{P}$ at some time $\bar{t} < \infty$. Lemma B.1 yields unique continuation values $V_{\bar{t}}^L(x, y)$ and $V_{\bar{t}}^R(x, y)$ starting in period $\bar{t}$. Thus, define the finite horizon $\bar{t}$ with payoffs set at these continuation payoffs then the equilibria of this finite game corresponds to those of the infinite horizon model. Again this finite game clearly must have an equilibrium must exist and all equilibria must yield the continuation payoffs. □

Proposition 4. *If $\eta < 1$ then there exists a $\hat{t} < \bar{t}$ such that the equilibrium policy outcome is $x_t = \hat{x}_t^R$ and $y_t = \hat{y}_t$ in every period $t \geq \hat{t}$. Furthermore, for γ sufficiently large we have $\hat{t} = 1$.*

Proof. First, we show that if t is sufficiently large then in equilibrium R proposes $x_t^* = \hat{x}_t^R$ and $y_t^* = \hat{y}_t$ whenever the status quo is such that $u_{L_t}(\hat{x}_t^R) + v_t(\hat{y}_t) \geq u_{L_t}(x_t^q) + v_t(y_t^q)$ and L accepts the proposal. Clearly R has no incentive to deviate given the proposed strategies. Next, consider L. L is willing to accept the proposal if rejecting today leads to R making the same form of proposal tomorrow, since given the strategies L accepts which thus yields the same dynamic utility to L and by construction it is statically optimal to accept. Thus, we need to show that if t is sufficiently large and $u_{L_t}(\hat{x}_t^R) + v_t(\hat{y}_t) \geq u_{L_t}(x_t^q) + v_t(y_t^q)$ then $u_{L_{t+1}}(\hat{x}_{t+1}^R) + v_{t+1}(\hat{y}_{t+1}) \geq u_{L_{t+1}}(x_t^q) + v_{t+1}(y_t^q)$.

Writing the condition as $u_{L_t}(\hat{x}_t^R) + v_t(\hat{y}_t) - u_{L_t}(x_t^q) - v_t(y_t^q) \geq 0$ we have that the same condition will hold in the next period if the LHS of the condition is increasing in t. Differentiating yields

$$2\eta t^{n-1}(x^q + t^n) - 2(y^q - \gamma t) - 8\eta t^{2n-1}.$$

Rearranging this is positive if and only if

$$y^q < \frac{1}{\gamma}\eta t^{\eta-1}\Big[3t^\eta - x^q\Big] + \gamma t \tag{26}$$

As $t \to \infty$ the RHS of (26) goes to infinity when $\eta < 1$ and $\gamma > 0$. Thus, as $y^q \leq \overline{Y}$ for all t there exists a t' such that for all $t \geq t'$ the statement holds.

This implies that R's unconstrained optimal policy in any period $t \geq t'$ is $(\hat{x}_t^R, \hat{y}^t)$. Thus, if R does not get its unconstrained optimal in a period then R must be choosing (x, y) to make L indifferent between accepting and rejecting. Therefore, if there is an equilibrium such that for all t party R does not propose $(\hat{x}_t^R, \hat{y}_t)$ then L's dynamic payoff is given by

$$\sum_{t=0}^{\infty} u_{L_t}(x^q) + v_t(y^q).$$

However, by $\eta < 1$ there is some period t'' such that starting in t'' the dynamic payoff to L from getting $(\hat{x}_t^R, \hat{y}_t)$ every period is greater than this dynamic payoff from the status quo, which contradicts that this is an equilibrium. Finally, note that for γ sufficiently large 26 holds for $t = 1$. □

Proposition 5. *If $\eta > 1$ then there exists $\tilde{t}$ such that $y_t^* \neq \hat{y}_t$ in every period $t \in [\tilde{t}, \bar{t}_Y)$. If γ is* *sufficiently small then* $\tilde{t} = 1$.

Proof. R's problem in period t is

$$\max_{(x,y)} \; u_{Rt}(x) + v_t(y) + \delta V^R_{t+1}(x,y)$$
$$\text{s.t. } u_{Lt}(x) + v_t(y) + \delta V^L_{t+1}(x,y) \geq u_{Lt}(x^q_t) + v_t(y^q_t) + \delta V^L_{t+1}(x^q_t, y^q_t).$$

Considering the modified finite $\bar{t}$ period game defined in Lemma B.1 we can use the Lagrangian to analyze solutions to this problem, note that the continutation payoffs are given by the finite sum of differentiable functions. This gives us the following system

$$u'_{Rt}(x) + \delta\frac{\partial V^R_{t+1}}{\partial x} + \lambda\Big[u'_{Lt}(x) + \delta\frac{\partial V^L_{t+1}}{\partial x}\Big] = 0$$
$$v'_t(y) + \delta\frac{\partial V^R_{t+1}}{\partial y} + \lambda\Big[v'_t(y) + \delta\frac{\partial V^L_{t+1}}{\partial y}\Big] = 0$$
$$\lambda\Big[u_{Lt}(x) + v_t(y) + \delta V^L_{t+1}(x,y) - u_{Lt}(x^q_t) - v_t(y^q_t) - \delta V^L_{t+1}(x^q_t, y^q_t)\Big] = 0$$

We show that we must have $y \neq \hat{y}$ for almost all values of the status quo. First, let $\lambda = 0$. In this case, if $y = \hat{y}_t$ then the second equality becomes $\delta\frac{\partial V^R_{t+1}}{\partial y}|_{y=\hat{y}} = 0$. However, $\frac{\partial V^R_{t+1}}{\partial y}|_{y=\hat{y}} = 0$ requires $\lambda^*_{t+1} = 0$ and thus the proposer is unconstrained in the next period. By a similar argument to the two period case, if the proposer is unconstrained in the next period and this period we must also have $x^*_t = \hat{x}^R_t$. First, note that if the proposer is unconstrained in all subsequent periods then we must have $(x^*_t, y^*_t) = (\hat{x}^R_t, \hat{y}_t)$ in each subsequent period, however, the limiting argument from equation 26 contradicts this for $\eta > 1$. Thus, we must have that the constraint binds in some period after t. Let the first period the constraint binds be period $t+2$ (if it is $t' > t+2$ we can shift the argument up to start at period $t'-2$). Note that for (x^*_{t+1}, y^*_{t+1}) to be an unconstrained maximizer in $t+1$ implies $x^*_{t+1} > \hat{x}^R_{t+1}$ and $y^*_{t+1} < \hat{y}_{t+1}$. Thus, the dynamic payoff to L of (x^*_{t+1}, y^*_{t+1}) is less than its dynamic payoff to $(\hat{x}^R_{t+1}, \hat{y}_{t+1})$. But then this would imply that $(\hat{x}^R_{t+1}, \hat{y}_{t+1})$ is passable in period $t+1$. For $\eta > 1$ and t sufficiently large, however, this is impossible given that we know the t period outcome must be $(\hat{x}^R_t, \hat{y}_t)$.

Next, assume $\lambda > 0$. For the above inequalities to hold at $y = \hat{y}$ requires

$$\frac{u'_{Rt}(x) + \delta\frac{\partial V^R_{t+1}}{\partial x}}{u'_{Lt}(x) + \delta\frac{\partial V^L_{t+1}}{\partial x}} = \frac{\partial V^R_{t+1}/\partial y}{\partial V^L_{t+1}/\partial y}$$

$$u_{Lt}(x) + \delta V^L_{t+1}(x, y) = u_{Lt}(x^*_{t-1}(x^q_1, y^q_1)) + v_t(y^*_{t-1}(x^q_1, y^q_1)) + \delta V^L_{t+1}\Big(x^*_{t-1}(x^q_1, y^q_1), y^*_{t-1}(x^q_1, y^q_1)\Big).$$

The LHS of the second inequality is decreasing in x, thus there is at most one x which solves it. However, the first equality does not depend on the initial status quo and thus any perturbation of the initial status quo only changes the solution to the second equality. Define $g(x) = \Big(u'_{Rt}(x) + \delta\frac{\partial V^R_{t+1}}{\partial x}\Big)\partial V^L_{t+1}/\partial y - \Big(u'_{Lt}(x) + \delta\frac{\partial V^L_{t+1}}{\partial x}\Big)\partial V^R_{t+1}/\partial y$. V^i_{t+1} is defined by the sum of polynomials, and the resulting derivatives are also polynomials in x. Thus, $g(x)$ is a polynomial and by the Fundamental Theorem of Algebra g has a finite number of roots. Therefore, both equalities can hold only for a measure zero set of initial values of the status quo.

Finally, if γ is sufficiently small then the proposer is constrained every period, thus, $\lambda > 0$ and so there must be inefficiency starting in period 1.

□

C Proofs for Robustness Extensions

Turnover

Proposition 6. *Assume $u_{R2}(\hat{x}_{L2}) < \max\{u_{R2}(\hat{x}_{R1}), u_{L2}(\hat{x}_{R1}\} + v_2(\hat{y}_1)$. There exists $\tilde{\rho} \in (0, 1)$ such that $|y^*_1 - \hat{y}_1|$ is decreasing in ρ for $\rho < \tilde{\rho}$ and increasing in ρ for $\rho > \tilde{\rho}$. Furthermore, $y^*_1 < \hat{y}_1$ for $\rho > \tilde{\rho}$ and $y^*_1 > \hat{y}_1$ for $\rho < \tilde{\rho}$.*

Proof. If player R is the proposer in the second period then R chooses $x^*_{R2}(x^q_2, y^q_2) = \min\{\hat{x}_{R2}, \overline{x}_{L2}\}$ and if player L is the proposer in the second period then L chooses $x^*_{L2}(x^q_2, y^q_2) = \max\{\hat{x}_{L2}, \underline{x}_{R2}\}$, where $\overline{x}_{L2}(x^q_2, y^q_2) = \overline{x}(x^q_2, y^q_2)$ and $\underline{x}_{R2}(x^q_2, y^q_2)$ is defined analogously as the lower solution to L's indifference condition in the second period.

Thus, player R chooses (x, y) in the first period to solve

$$\max_{(x,y)} u_{R1}(x) + v_1(y) + \rho u_{R2}(x^*_{R2}(x, y)) + (1 - \rho)u_{R2}(x^*_{L2}(x, y)).$$

We can break this maximization problem into several cases depending on (x, y).

For (x, y) such that $u_{R2}(x) + v_2(y) \geq u_{R2}(\hat{x}_{L2})$ and $u_{L2}(x) + v_2(y) \geq u_{L2}(\hat{x}_{R2})$ we can write the above maximization problem as

$$\max_{(x,y)} u_{R1}(x) + v_1(y) + \rho u_{R2}(\overline{x}_{L2}(x, y)) + (1 - \rho)\Big(u_{R2}(x) + v_2(y)\Big).$$

Similar arguments as in the baseline show that there cannot be a solution with $y^* = \hat{y}_1$.

For (x, y) such that $u_{R2}(\hat{x}_{L2}) \geq u_{R2}(x) + v_2(y)$ and $u_{L2}(\hat{x}_{R2}) \geq u_{L2}(x) + v_2(y)$ we have

$$\max_{(x,y)} u_{R1}(x) + v_1(y) + \rho u_{R2}(\hat{x}_{R2}) + (1 - \rho)u_{R2}(\hat{x}_{L2}).$$

Clearly, if there is an interior solution in this case it must be that $x_1 = \hat{x}_{R1}$ and $y_1 = \hat{y}_1$. Which is only possible if $u_{R2}(\hat{x}_{L2}) \geq u_{R2}(\hat{x}_{R1}) + v_2(\hat{y}_1)$ and $u_{L2}(\hat{x}_{R2}) \geq u_{L2}(\hat{x}_{R1}) + v_2(\hat{y}_1)$.

For (x, y) such that $u_{R2}(\hat{x}_{L2}) \geq u_{R2}(x) + v_2(y)$ and $u_{L2}(\hat{x}_{R2}) \leq u_{L2}(x) + v_2(y)$ we have

$$\max_{(x,y)} u_{R1}(x) + v_1(y) + \rho u_{R2}(\overline{x}_{L2}) + (1 - \rho)u_{R2}(\hat{x}_{L2}).$$

In this case, the optimality conditions look similar to the baseline, but with a weight of ρ on some terms, and the same arguments yield that $y^*_1 \neq \hat{y}_1$ for almost all parameters.

Finally, (x, y) such that $u_{R2}(\hat{x}_{L2}) \geq u_{R2}(x) + v_2(y)$ and $u_{L2}(\hat{x}_{R2}) \leq u_{L2}(x) + v_2(y)$ we have

$$\max_{(x,y)} u_{R1}(x) + v_1(y) + \rho u_{R2}(\hat{x}_{L2}) + (1 - \rho)\Big(u_{R2}(x) + v_2(y)\Big).$$

Any interior solution must solve

$$u'_{R1}(x) + (1-\rho)u'_{R2}(x) = 0$$
$$v'(y) + (1-\rho)v'_2(y) = 0,$$

which clearly cannot be satisfied if $y = \hat{y}_1$.

Thus, a necessary condition for $y^* = \hat{y}_1$ is that $u_{R2}(\hat{x}_{L2}) \geq u_{R2}(x) + v_2(y)$ and $u_{L2}(\hat{x}_{R2}) \geq u_{L2}(x) + v_2(y)$.

Next, we characterize how the degree of inefficiency changes with ρ. Assume R's optimal choice does not involve either player getting its ideal point in the second period. Letting $a = -x$ and $b = -y$ we can write R's maximization problem as

$$\max_{(x,y)} u_{R1}(-a) + v_1(-b) + \rho u_{R2}(\overline{x}_{L2}(a,b)) + (1-\rho)\Big(u_{R2}(-a) + v_2(-b)\Big).$$

Taking cross-partials of the objective function we have

$$\frac{\partial^2}{\partial a \partial b} = \rho\left(\frac{\partial^2 \overline{x}_{L2}}{\partial a \partial b}u'_{R2}(\overline{x}_{L2}(a,b)) + \left(\frac{\partial \overline{x}_{L2}}{\partial a}\right)\left(\frac{\partial \overline{x}_{L2}}{\partial b}\right)u''_{R2}(\overline{x}_L(a,b))\right)$$
$$\frac{\partial^2}{\partial a \partial \rho} = \frac{\partial \overline{x}_{L2}}{\partial a}u'_{R2}(\overline{x}_{R2}(a,b)) + u'_{R2}(-a)$$
$$\frac{\partial^2}{\partial b \partial \rho} = \frac{\partial \overline{x}_{L2}}{\partial b}u'_{R2}(\overline{x}_{L2}(a,b)) + v'_2(-b)$$

We have $\frac{\partial^2 \overline{x}_{L2}}{\partial a \partial b} > 0, \frac{\partial \overline{x}_{L2}}{\partial a} < 0$, and $\frac{\partial \overline{x}_{L2}}{\partial b} > 0$, which yields, $\frac{\partial^2}{\partial a \partial b} > 0$ and $\frac{\partial^2}{\partial b \partial \rho} > 0$. Finally, $u'_{R2}(\overline{x}) < u'_{R2}(x) = u'_{R2}(-a)$ and $\frac{\partial \overline{x}_{L2}}{\partial a} \in (-1, 0)$, thus $\frac{\partial^2}{\partial a \partial \rho} > 0$. The desired result then follows from the standard theorems on monotone comparative statics, e.g., Milgrom and Shannon (1994). At $\rho = 1$ the problem is the same as the baseline so we must have $y_1^* < \hat{y}_1$. At $\rho = 0$ there is no incentive to undershoot, and the previous argument implies that we cannot have $y_1^* = \hat{y}_1$ so it must be that $y_1^* > \hat{y}_1$. □

Stochastic Shocks

Given a realization ϵ in the second period R chooses $y_2^* = \hat{y}_2$ and $x_2^* = \min\{\hat{x}_1^R + \epsilon, \overline{x}(x, y; \epsilon)\}$, where $\overline{x}(x, y; \epsilon)$ solves

$$-(z - (\hat{x}_1^L - \epsilon))^2 = -(x - (\hat{x}_1^L - \epsilon))^2 - (y - \hat{y}_2)^2.$$

Let $\epsilon^*(x, y)$ be the $\epsilon > 0$ that solves $\overline{x}_2^R(x, y; \epsilon) = \hat{x}_1^R + \epsilon$. We can now write R's problem in the first period as

$$\max_{(x,y)} \; u_{R1}(x) + v_1(y) + \int_{\min\{\overline{\epsilon}, \epsilon^*(x,y)\}}^{\overline{\epsilon}} u_{R2}(\overline{x}(x, y; \epsilon)) f(\epsilon) d\epsilon,$$

where f is the pdf of the distribution of ϵ. Note that at ϵ^* we have $u_{R2}(\overline{x}_2^R(x, y; \epsilon^*)) = u_{R2}(\hat{x}_1^R + \epsilon^*) = 0$. Thus, applying Leibniz rule, the first order conditions are given by

$$u'_{R1}(x) + \int \frac{\partial \overline{x}}{\partial x} u'_{R2}(\overline{x}(x, y; \epsilon)) f(\epsilon) d\epsilon = 0 \tag{27}$$

$$v'_1(y) + \int \frac{\partial \overline{x}}{\partial y} u'_{R2}(\overline{x}(x, y; \epsilon)) f(\epsilon) d\epsilon = 0. \tag{28}$$

If $\hat{y}_1 = \hat{y}_2$ then $\frac{\partial \overline{x}}{\partial y} = 0$ iff $y = \hat{y}_1$. Thus, the second condition can only be satisfied if $y = \hat{y}_1$ and there must be efficiency. Now assume $\hat{y}_2 > \hat{y}_1$. If $\overline{\epsilon} < \epsilon^*(\hat{x}_1^R, \hat{y}_1)$ then R chooses $x^* = \hat{x}_1^R$ and $y^* = \hat{y}_1$, as this gives R is first-period optimal and R is able to get its second period optimal for any realization of ϵ.

Next, assume $\overline{\epsilon} > \epsilon^*(\hat{x}_1^R, \hat{y}_1)$. In this case, R always chooses x, y such that $\overline{\epsilon} \geq \epsilon^*(x, y)$. Otherwise, if $\overline{\epsilon} < \epsilon^*(x, y)$ then for any $(x, y) \neq (\hat{x}_1^R, \hat{y}_1)$ it could deviate closer to $(\hat{x}_1^R, \hat{y}_1)$ and improve its first-period payoff without changing its second-period payoff. Note that we must have $\overline{\epsilon} > \epsilon^*(x, y)$. If $\overline{\epsilon} = \epsilon^*(x, y)$ then the the first order conditions become $u'_{R1}(x) = 0$ and $v'_1(y) = 0$ thus $(x, y) = (\hat{x}_1^R, \hat{y}_1)$, but this contradicts $\overline{\epsilon} > \epsilon^*(\hat{x}_1^R, \hat{y}_1)$. Finally, if $\overline{\epsilon} > \epsilon^*(x, y)$ then the term under the integral is non-zero. Since $u'_{R2}(\overline{x}) > 0$ and $\frac{\partial \overline{x}}{\partial y} < 0$ the second condition can never hold at $y = \hat{y}_1$, thus, the equilibrium policy must be inefficient.

Different Weights

Suppose that there is no change in the common-values ideal policy, and $\hat{y}_2 = \hat{y}_2 = \hat{y}$. Further, let R's per-period utility on the common-values dimension be $\theta v_t(y)$, where instead L's utility remains $v_t(y)$. If $\hat{x}_{R1} = \hat{x}_{R2}$ then clearly R maximizes its utility by choosing $x = \hat{x}_R$ and $y = \hat{y}$. Now assume $\hat{x}_{R1} < \hat{x}_{R2}$. Notice that if $y = \hat{y}$ then R cannot get its ideal point in the second period because $u_{L2}(\hat{x}_2^R) + \theta v(\hat{y}) < u_{L2}(x) + \theta v(\hat{y})$ for all $x < \hat{x}_2^R$. Thus, if R gets its ideal point in the second period then we must have inefficiency. But then, it is easy to see that R can never get its ideal point in the second period. Choosing a first-period policy that moves the second-period outcome marginally far from $\hat{x}_2^R$ is costless in period 2, but beneficial in period 1 as it improves efficiency.

So consider the case where R does not get its second period ideal point. The term $\overline{x}(x, y)$ is defined the same as before. R's problem is to solve

$$\max_{x,y} u_{R1}(x) + \theta v(y) + u_{R2}(\overline{x}(x, y))$$

This yields first-order conditions

$$\begin{aligned} u'_{R1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{R2}(\overline{x}(x, y)) &= 0 \\ \theta v'(y) + \frac{\partial \overline{x}}{\partial y} u'_{R2}(\overline{x}(x, y)) &= 0, \end{aligned}$$

which we can rewrite as

$$\begin{aligned} u'_{R1}(x) + \frac{u'_{L2}(x)}{u'_{L2}(\overline{x}(x, y))} u'_{R2}(\overline{x}(x, y)) &= 0 \\ v'(y)\Big(\theta + \frac{u'_{R2}(\overline{x}(x, y))}{u'_{L2}(\overline{x}(x, y))}\Big) &= 0, \end{aligned}$$

First, we show that if $\theta \geq 1$ then there must be efficiency. If there is an equilibrium with $y \neq \hat{y}$

then for the above conditions to hold requires

$$\frac{u'_{R1}(x)}{u'_{L2}(x)} = -\frac{u'_{R2}(\overline{x}(x,y))}{u'_{L2}(\overline{x}(x,y)))}$$
$$\theta + \frac{u'_{R2}(\overline{x}(x,y))}{u'_{L2}(\overline{x}(x,y)))} = 0.$$

Thus, $\frac{u'_{R1}(x)}{u'_{L2}(x)} = \theta$. For $\frac{u'_{R1}(x)}{u'_{L2}(x)} > 0$ to hold we must have $x > \hat{x}_{R1}$. However, this implies $|x - \hat{x}_{R1}| < |x - \hat{x}_{L2}|$ and thus $\frac{u'_{R1}(x)}{u'_{L2}(x)} < 1 < \theta$, a contradiction. Next, we show that if θ is sufficiently small then $y^* \neq \hat{y}$. Assume $y = \hat{y}$ is optimal. Then $\overline{x}(x, \hat{y}) = x$ and the first condition becomes $u'_{R1}(x) + u'_{R2}(x) = 0$. Thus, we must have $x = \frac{\hat{x}_{R1}+\hat{x}_{R2}}{2}$. Taking derivatives of the objective function and setting $(x, y) = (\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2}, \hat{y})$ yields

$$\frac{\partial^2}{\partial x^2} = -4$$
$$\frac{\partial^2}{\partial y^2} = -2\Big(\theta + \frac{u'_{R2}(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2})}{u'_{L2}(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2})}\Big)$$
$$\frac{\partial^2}{\partial x \partial y} = 0.$$

Thus, a necessary condition for $(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2}, \hat{y})$ to be a maximizer is that $\theta + \frac{u'_{R2}(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2})}{u'_{L2}(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2})} > 0$. Hence, if $\theta < -\frac{u'_{R2}(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2})}{u'_{L2}(\frac{\hat{x}_{R1}+\hat{x}_{R2}}{2})}$ then $y = \hat{y}$ is not optimal.

Risk Aversion

The proofs required to show we get inefficiency in the bad status quo case only used that u_{it} and v_t are single-peaked and everywhere differentiable. Additionally, $u_{L2}(\hat{x}_2^R) \geq u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$ is still clearly sufficient for efficiency. Now instead consider $v_t(y) = -|y - \hat{y}_t|$ and assume $u_{L2}(\hat{x}_2^R) \leq u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$. We show that the optimal first period policy sets $y_1 = \hat{y}_1$.

In the second period, $\overline{x}(x, y)$ solves $u_{L2}(z) = u_{L2}(x) - |y - \hat{y}_2|$. Since $y < \hat{y}_2$ we can write this as $u_{L2}(z) = u_{L2}(x) - (\hat{y}_2 - y)$ and $\overline{x}$ is still differentiable in y.

Clearly R never overshoots when unconstrained in the first period, thus, consider $y \leq \hat{y}_1$. As before, R will not choose $\overline{x}(x,y) > \hat{x}_2^R$. Thus, we can write its problem as

$$\max_{x,y} \; u_{R1}(x) - (\hat{y}_1 - y) + u_{R2}(\overline{x}(x,y))$$
$$\text{s.t. } \hat{y}_1 \geq y$$
$$u_{L2}(x) + v_2(y) \geq u_{L2}(\hat{x}_2^R).$$

This yields KKT conditions

$$u'_{R1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{R2}(\overline{x}(x,y)) + \lambda_2 u'_{L2}(x) = 0 \tag{29}$$

$$1 + \frac{\partial \overline{x}}{\partial y} u'_{R2}(\overline{x}(x,y)) - \lambda_1 + \lambda_2 = 0 \tag{30}$$

$$\lambda_1(\hat{y}_1 - y) = 0 \tag{31}$$

$$\lambda_2 \Big(u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) \Big) = 0 \tag{32}$$

If $\lambda_1 > 0$ then $y = \hat{y}_1$ and we are done. Assume $\lambda_1 = 0$. If $\lambda_2 = 0$ then the first and second conditions become $u'_{R1}(x) + \frac{u'_{R2}(\overline{x}(x,y))}{u'_{L2}(x,y)} u'_{L2}(x) = 0$ and $1 + \frac{u'_{R2}(\overline{x}(x,y))}{u'_{L2}(\overline{x}(x,y))} = 0$, respectively. Combining conditions, this implies that $u'_{R1}(x) = u'_{L2}(x)$, which does not hold for any x. Next, assume $\lambda_2 > 0$. In this case, $\overline{x}(x,y) = \hat{x}_2^R$. The second condition then becomes $1 + \lambda_2 = 0$, which never holds.

General Movements

Lemmas A.1 and A.3 hold unchanged. To complete argument consider Lemma A.5. We have same KKT conditions (22) - (25). The argument for x, y such that $u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) > 0$ is unchanged. Next, consider $u_{L2}(x) + v_2(y) - u_{L2}(\hat{x}_2^R) > 0$. Assume $y \in [\hat{y}_1, \hat{y}_2)$ If $\hat{x}_2^L < \hat{x}_1^R$ the argument is the same as before. If $\hat{x}_2^L > \hat{x}_1^R$ then for (22) to hold we must have $u'_{R1}(x) + \lambda_2 u'_{L2}(x) = 0$, which implies that $u'_{R1}(x) < 0$ and $u'_{L2}(x) > 0$, thus, $x \in (\hat{x}_1^R, \hat{x}_2^L)$. Therefore $u_{L2}(x) + v_2(y) > u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$. But by assumption $u_{L2}(x) + v_2(y) = u_{L2}(\hat{x}_2^R) < u_{L2}(\hat{x}_1^R) + v_2(\hat{y}_1)$, a contradiction.