Ideological Infection

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Abstract

Many policy problems are inherently dynamic. Outcomes worsen over time if policy is not adapted to changing circumstances. However, even if everyone agrees on how to address the problem, policy negotiations do not occur in a vacuum. Consequently, disagreements on more contentious ideological issues can spillover, distorting preferences and outcomes on the common-values policy problem. In this paper, we develop a dynamic bargaining model to study when and how this *ideological infection* emerges. We find that dynamic policy problems are vulnerable to ideological infection precisely because the costs of inaction on these issues increase over time. Furthermore, we show that inefficient policies on the common-values dimension are inevitable when players anticipate conflict to rapidly intensify on the ideological dimension.

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1 Introduction

Many policy problems are inherently dynamic. A crisis worsens over time if it is unaddressed, thus requiring increasingly bolder interventions; infrastructure deteriorates, necessitating greater investment tomorrow; and anticipated innovations alter the optimal level of investment in green technologies in the future. Given the growing price of inaction on these issues, we may expect little underlying disagreement among policymakers, at least on the optimal direction to move policy. However, solutions to these problems are rarely negotiated in a vacuum. Policy proposals to address such problems are often negotiated, either explicitly or implicitly, in conjunction with other issues that are more contentious.[1] In turn, this bundling of issues may alter the bargaining parties' strategic incentives to reach an agreement on these dynamic policy problems.

In this paper, we show that, in these environments, conflict on ideological issues can spillover and infect bargaining over an evolving common-values policy problem. For example, due to the joint interests of the United States and China in addressing climate change, China's Foreign Minister Wang Yi described the issue as an "oasis," but went on to state that "surrounding the oasis is a desert, and the oasis could be desertified very soon. China-U.S. climate co-operation cannot be separated from the wider environment of China-U.S. relations."[2] Ee develop a model to study when and how such *ideological infection* emerges. We find that the types of policy problems described above are vulnerable to being infected by other more contentious issues precisely because of their dynamic nature, whereby the costs of inaction compound over time. Furthermore, inefficient common-values policies become inevitable when players also anticipate rapidly intensifying disagreements on the dimension of conflict. Interestingly, the players delay coming to an efficient agreement today despite knowing their opponent will become more entrenched tomorrow, and thus less willing to yield ideological concessions.

In our model, two players repeatedly bargain over both an ideological issue and a common-values issue. The proposer each period offers a policy on each dimension, and the veto player accepts or rejects the entire bundle of policies. On each issue, the policy that is implemented today becomes the status quo tomorrow. To model the dynamic nature of policy issues, we allow the players' preferences on both dimensions to evolve over time. On the ideological dimension players disagree on the optimal policy, and this disagreement may grow over time. This can capture situations in which political parties expect increasing polarization among

[1] Bundling unrelated issues together in larger omnibus bills is commonly used in legislative bargaining (Krutz, 2001; Clinton and Lapinski, 2006; Hazama and Iba, 2017; Meßerschmidt, 2021). Similarly, issue linkage is an important tactic in international negotiations (Tollison and Willett, 1979; Keohane, 1984).

[2] https://www.reuters.com/world/asia-pacific/china-holds-virtual-climate-meeting-with-us-describes-environment-policy-oasis-2021-09-02/

their respective constituents or members, and thus anticipate they will have to appeal to more extreme preferences in the future to maintain the support of their respective bases (Peltzman, 1984). Alternatively it may capture changes in the environment that intensify preferences over an issue, e.g., two countries negotiating over a territory that they anticipate will be more valuable in the future. In contrast, the players' preferences on the common-values dimension are fully aligned and thus change in the same way over time, reflecting an increasing need for investment, shifts in the optimal policy due to technological change, or the worsening of a crisis.

As highlighted above, the ability to link multiple issues together can alter strategic incentives when bargaining. In a static world, this form of issue linkage generates no detrimental spillovers from the ideological issue to the common-values one. The efficient policy on the dimension of agreement also maximizes the ideological concessions the proposer can obtain. As such, in equilibrium, there is no ideological infection of preferences over the common-values dimension, and hence no impediment to the players adopting the commonly beneficial policy. In a dynamic setting, however, this is not always true. The policy implemented on the common-values dimension today influences the ideological concessions the proposer can extract from the veto player tomorrow. In turn, this generates the potential for players to prefer inefficient policies.

We find that a necessary condition for the players' preferences on the common-values dimension to become infected is that the marginal cost of inefficiency on this dimension increases over time. This creates *compounding costs* on the common-values issue, so that any residual inefficiency the players inherit from the past becomes more and more detrimental over time. This condition captures the way in which many policy problems evolve. Underinvestment today becomes even more costly as infrastructure continues deteriorating, or a pandemic that is left unaddressed spreads more and more rapidly.

Increasing marginal costs of inefficiency ensures that, for one or both players, today's price of distorting the common-values policy away from the optimum is smaller than the ideological concessions this residual inefficiency will buy in the future. Absent this compounding, neither player is incentivized to pursue an inefficient common-values policy today, since any future gain on the ideological dimension is completely offset by the cost imposed today by not moving to the optimal common-values policy. Thus, the equilibrium common-values policy is always efficient when there are not increasing marginal costs of inefficiency on the common-values dimension.

Next, we show that, in the presence of compounding costs of inefficiency, rapidly intensifying disagreement on the dimension of conflict is sufficient for ideological infection to emerge. To see why, suppose that the conflict on the ideological dimension increases slowly.

In this case, the evolution on the common-values dimension provides the proposer enough leverage to pass his ideal point tomorrow, even if the policy is efficient today. Thus, the proposer has no incentive to pursue an inefficient policy today. Under certain values of the status quo, this is enough to ensure an efficient policy in equilibrium, even with compounding costs on the common-values dimension.

Conversely, when conflict on the ideological dimension intensifies rapidly, ideological infection of the common-values dimension becomes unavoidable. In this scenario, the proposer prefers to *undershoot* the optimal common-values policy today, e.g., underinvest in infrastructure or only partially address a crisis, to strengthen his bargaining position on the ideological dimension tomorrow. Instead, the veto player prefers to *overshoot* the optimal policy today to constrain the proposer tomorrow, e.g., investing to not only fix but also prevent future infrastructure decay or further crises. As a consequence, the equilibrium common-values policy is always inefficient.

Finally, we characterize the form this inefficiency takes in equilibrium and the location of the ideological policy. Depending on the environment, inefficiency may manifest as proposer-induced undershooting or veto-induced overshooting. Importantly, we find that, under some conditions, the proposer undershoots the optimal common-values policy *and* chooses an ideological policy to the left of both its first and second-period ideal points.

This form of undershooting on both the common-values and ideological dimension highlights that the reason inefficient policies are implemented is *not* because the proposer does not 'need' leverage today. In our model, inefficiency occurs even when undershooting on the dimension of agreement fails to give the proposer enough leverage to obtain his first-period ideal policy on the ideological dimension. In this case, by moving closer to the efficient policy the proposer could reduce the costs of inefficiency on the common-values dimension *and* move ideological policy in his preferred direction. However, increasing marginal costs of inefficiency allow the proposer to better leverage the common-values dimension for ideological concessions in the future. As a result, the proposer sometimes deliberately incurs immediate costs on the common-values dimension *and* leaves gains on the table on the ideological dimension.

1.1 Applications

China, the United States, and Climate Change. The US-China climate cooperation mentioned above provides an example of ideological infection under evolving policy problems. The warning of China's Foreign Minister underscores that climate cooperation cannot be entirely isolated from other aspects of the bilateral relationship, and suggests that a cooperative approach to climate change might be difficult to sustain if tensions in other areas continue to escalate. Indeed, climate change and the issue of Taiwan are both rapidly evolving. After a period of fragile reconciliation beginning in the late 1980s, tensions between China and Taiwan started intensifying with the election of Tsai Ing-wen, from the traditionally pro-independence Democratic Progressive Party, as Taiwan's first female president in 2016. In this same period, the US has increased its economic relations with Taiwan. These developments have made the issue more salient for both of the bargaining parties. On the other hand, climate-induced disasters have grown increasingly severe and frequent,[3] Thus, although both countries agree on the urgent need to address an increasingly costly climate crisis, their cooperation on climate policy is vulnerable because it can be used as leverage to gain concessions on the issue of Taiwan's sovereignty.[4]

Congress, Conflict Expansion, and Ideological Polarization. Two important observations often characterize accounts of American politics. First, political parties are growing increasingly polarized. Looking at the United States Congress, measures of polarization were quite low until the mid 1970s, but have seen a steep increase since that time (Barber et al., 2015). Second, while political conflict between the parties has remained organized along classic dimensions of polarization, other issues 'have been absorbed into it' (Barber et al.2015, p. 23). Indeed, Lee (2005) describes how partisan divisions now extend to issues such as good government, disaster relief, and transportation programs, areas where we would expect the preferences of 'both parties and all voters [to be] located at a single point' (Stokes 1963, p. 372).[5] As such, political parties appear to be polarized on virtually all policy dimensions, including those with little or no ideological connotation. Consequently, in recent decades we have witnessed a stark decrease in the ability of Congress to legislative efficiently, even on common-values issues (Layman et al., 2006). This case highlights how increasing polarization can generate ideological infection of common-values policy problems, with the consequence that 'public policy does not adjust to changing economic and demographic circumstances' (Barber et al. 2015, p. 41). In the most severe form of this inefficiency, Congress appears unwilling to address even the most pressing issues facing the country, instead choosing to 'kick the can down the road (...) and govern by (artificial) crises' (Barber et al. 2015, p. 41).

[3] https://www.pbs.org/newshour/science/scientists-confirm-global-floods-and-droughts-worsened-by-climate-change.

[4] https://time.com/6295941/us-china-climate-cooperation-challenge/.

[5] The insights of our model would still apply if, for example, there is some disagreement about the optimal degree of disaster relief, but a continuing disaster moves preferences in the same direction.

The Shutdown, Budget and Debt Ceiling Negotiations. After Democrats and Republicans failed to reach a timely agreement on a federal budget bill in the fall of 2013, the US government was forced to suspend most of its routine operations from October 1 to October 17. The consequences included, but were not limited to, approximately 800,000 federal employees being indefinitely furloughed, and another 1.3 million required to report to work without known payment dates.

Despite these negative consequences, the shutdown was the result of a deliberate bargaining strategy. Several factions within the Republican party aimed to create a funding gap and use the threat of a shut down to increase their bargaining leverage and secure policy concessions on other issues, in particular, defunding the Affordable Care Act. Indeed, several Republicans openly welcomed the shutdown. Minnesota Representative Michele Bachmann, for example, stated she was 'very excited' about 'getting exactly what we wanted'.[6]

For their part, Democrats also recognized the strategic implications of bundling a resolution to the shutdown with more ideological issues. Hours before the start of the shutdown, Republicans in the House attempted to start budget conference-committee negotiations, but Democratic Senate Majority Leader Harry Reid declared: 'We will not go to conference with a gun to our head'.[7] In the days that followed, the House of Representatives proposed several piecemeal bills to fund specific critical agencies and the city of Washington, D.C.. However, the Democratic Senate leadership and President Obama refused to approve these bills, likely recognizing that this would weaken their position in future negotiations. Indeed, President Obama sought to further expand the scope of negotiations by explicitly linking the government shutdown to the impending debt ceiling crisis, stating that he would not reopen budget negotiations until Republicans agreed to passage of a bill raising the debt limit.[8]

In each of these cases, our model offers a possible explanation for why a dynamic policy problem, over which the bargaining players have roughly similar preferences, becomes entangled in ideological conflict, leading to infected preferences on the common-values issue and inefficiencies in policy implementation.

1.2 Contribution to the Literature

Our research contributes to the extensive political economy literature that explores bargaining with an endogenous status quo.[9] However, the mechanism that generates inefficiency in our setting differs from those in the literature. Previous papers have found inefficiencies that

[6] http://www.washingtonpost.com/politics/on-cusp-of-shutdown-house-conservatives-excited-say-they-are-doing-the-right-thing/2013/09/28/2a5ab618-285e-11e3-97e6-2e7cad1b77e-story.html, archived at http://perma.cc/69K5-27BG.

[7] https://www.nbcnews.com/video/reid-we-will-not-go-to-conference-with-a-gun-to-our-head-51143747771.

[8] https://www.bbc.com/news/world-us-canada-24375591.

[9] See Eraslan et al. (2022) for a review of this literature.

stem from motives such as insurance against turnover (Buisseret and Bernhardt, 2017) or the possibility of developing future conflict on the issue (Riboni and Ruge-Murcia, 2008; Zápal, 2011; Dziuda and Loeper, 2016; Austen-Smith et al., 2019). In contrast, in our model, distortions are due to the multidimensionality of the policy space combined with the evolution of preferences over time.[10] We demonstrate that in a multidimensional world preferences over a common-values issue can become distorted even when the proposer is guaranteed to remain in power indefinitely, and the players are certain they will never disagree on this dimension.[11] As such, the potential for inefficiency to emerge may be even more severe than previously shown.

Closer to our paper, Callander and Martin (2017) studies an endogenous status quo bargaining model where policies have an ideological and a (common-values) quality component. Quality decays over time but can be restored. However, in Callander and Martin (2017) there is never inefficiency on the equilibrium path of play, i.e., the proposer never underinvests in quality for future leverage. Our analysis highlights two important features of the environment that drive this difference in outcomes. First, we find that the proposer develops a preference for inefficiency only when players anticipate intensifying disagreements on the dimension of conflict in the future. Polarization on the ideological dimension does not change over time in Callander and Martin (2017), and therefore proposer-induced inefficiency cannot arise. Second, in Callander and Martin (2017) parties cannot overinvest in quality today to avoid future decay. Thus, the veto-player induced overshooting inefficiency that emerges in our setting cannot occur in their model.

Two previous papers have analyzed how dynamic incentives can lead to inefficient agreements and delay when players bargain over multiple issues (Acharya and Ortner, 2013; Lee, 2020).[12] The emergence of inefficiency in both of these papers depends on two assumptions. First, not all issues (or goods) are immediately available for the players to bargain over. Second, the two players place different weights on each issue.[13] Under these assumptions,

[10]Previous papers in this literature that incorporate multiple policy dimensions have focused on issues of existence (Duggan and Kalandrakis, 2012) and indeterminacy (Anesi and Duggan, 2018). Chen and Eraslan (2017) also analyzes dynamic bargaining with multiple policy dimensions, but assumes parties can only address one issue at a time. Instead, we specifically focus on the effects of bundling different dimensions.

[11]This distinguishes our work from papers that find strategic polarization on a single dimension that can exhibit conflict (Dziuda and Loeper, 2018). Penn (2009) allows for multiple dimensions and characterizes how continuing policies distort preferences. However, proposals are exogenous in her model, which focuses on voting behavior. In contrast endogenous proposals are a crucial determinant of preferences our model.

[12]Other works study the effects of bundling different dimensions, but consider a one-shot interaction or assume bargaining concludes once an agreement is reached (e.g., Fershtman, 1990; Jackson and Moselle, 2002; Chen and Eraslan, 2013; Câmara and Eguia, 2017). Consequently, the inefficiencies we find due to evolving preferences and an endogenous status quo do not arise in these models.

[13]In Lee (2020), on each issue both players prefer the alternative policy to the status quo, but the payoff they obtain from the alternative is different.

agreements may be delayed because one player values today's issues less and waits to bundle with his preferred issues when it becomes available in the future. In our model, neither of these features are present, rather we assume preferences can evolve over time and inefficiency in multi-issue bargaining arises due to increasing marginal costs on the common-values dimension.[14]

This difference in mechanisms is emphasized by our finding that the proposer sometimes pursues an inefficient common-values policy even when doing does not allow him to pull the ideological policy at least to his first-period ideal point. That is, the proposer undershoots on the common-values despite "needing" more leverage today on the ideological dimension. Instead, in Acharya and Ortner (2013) and Lee (2020) players may not "need" leverage today if not all issues are available to bargain over. Under this condition, asymmetry in the players' evaluation of the different issues then allows for the costs of inefficiency to be lower than the future gains, leading to delay. In the Appendix, we formalize the difference between these papers and our mechanism. We shut down compounding costs on the common-values issue but allow the players to weight the dimensions differently. We show that, while the policy outcome can be inefficient, the proposer only ever maintains leverage on the common-values dimension if he also pulls the ideological policy at least as far as his first-period ideal point. That is, in this setting, which resembles Acharya and Ortner (2013) and Lee (2020), the proposer only keeps leverage for tomorrow when he does not need more leverage today. Thus, our paper complements these works by uncovering a novel mechanism by which dynamic bargaining over multiple issues can cause inefficiency, one which applies to evolving policy problems.

Finally, our work is also related to studies that analyze multidimensional bargaining where players can make transfers to each other (e.g., Austen-Smith and Banks, 1988; Diermeier and Merlo, 2000). Similar to a transfer, our proposer can use the common-values dimension to obtain favorable policy on the partisan dimension. However, the common-values dimension differs from a transfer because both players benefit from moving policy to the common-values ideal point, therefore inefficiency is costly for the proposer as well. Thus, leaving leverage for the future on the common-values dimension is always Pareto inefficient. Other studies analyze how bargaining players divide a budget between private benefits and public goods (e.g., Battaglini and Coate, 2007; Volden and Wiseman, 2007). In such settings, a proposer may have incentives to underprovide on the public good dimension to extract the maximum private transfers for itself. In this context, Bowen et al. (2014) show that an

[14] A further difference is that we consider a setting where players can pass policy on the same issue multiple times, while agreement in Acharya and Ortner (2013) and Lee (2020) settles (at least partially) the issue. Additionally, there is no scope for veto-player induced overshooting in either paper. On the other hand, our model abstracts from elections, whereas Lee (2020) explicitly incorporates voters.

endogenous status quo, i.e., mandatory public good spending, can actually help *correct* the inefficiencies that would emerge in a static setting. This contrasts with our setup, where the endogenous status quo is precisely what generates inefficiency.

2 The Model

Players and policies. There are two players, a proposer (P) and veto (V), who interact over two periods, $t \in \{1, 2\}$. The policy space is composed of an ideological dimension $X = \mathbb{R}$ and a common-values dimension $Y = \mathbb{R}$. In every period t, the players bargain to determine a policy outcome $(x_t, y_t) \in X \times Y = \mathbb{R}^2$.

Preferences. The stage utility to player $i \in \{P, V\}$ in period t from a policy outcome (x, y) is $u_{it}(x) + v_t(y)$, where we define $u_{it}(x) = u(x - \hat{x}_t^i)$ and $v_t(y) = v(y - \hat{y}_t)$. We assume v is single-peaked at $\hat{y}_t$, u is strictly concave and single-peaked at $\hat{x}_t^i$, and all functions are twice-differentiable. Thus, player i's statically optimal policy in period t is given by its ideal point $(\hat{x}_t^i, \hat{y}_t) \in \mathbb{R}^2$. Additionally, let $u_{it}(\hat{x}_t^i) = v_t(\hat{y}_t) = 0$. In each period, player P's preferred ideological policy is to the right of player V, $\hat{x}_t^V < \hat{x}_t^P$, and conflict on the ideological dimension (weakly) increases over time, $\hat{x}_2^V \leq \hat{x}_1^V < \hat{x}_1^P \leq \hat{x}_2^P$. To reduce the number of cases, on the common-values issue we assume the shared ideal policy (weakly) increases over time, $\hat{y}_1 \leq \hat{y}_2$.

The sequence of ideal policies is common knowledge. Thus, the evolution of preferences is deterministic and the parties in our model face no uncertainty.

Player i's payoff in the dynamic game is given by:

$$\sum_{t \in \{1,2\}} u_{it}(x_t) + v_t(y_t),$$

where for simplicity we assume no discounting.

Political environment. At the start of each period $t \in \{1, 2\}$ player P makes a proposal $(x_t, y_t) \in \mathbb{R}^2$, which consists of a policy on the ideological issue, $x_t \in X$, and a policy on the common-values issue, $y_t \in Y$. Next, player V decides whether to accept or reject the proposal. If the proposal is accepted then the policy outcome in period t is (x_t, y_t). If the proposal is rejected then the policy outcome in period t is $(x_t^q, y_t^q) \in \mathbb{R}^2$, where (x_t^q, y_t^q) is the status quo in period t. Thus, proposals on the two dimensions are bundled together.

The policy outcome in the current period becomes the status quo in the subsequent period. Thus, if (x_1, y_1) is the policy outcome in period 1 then the status quo in period 2 is

$(x_2^q, y_2^q) = (x_1, y_1)$. The status quo at the beginning of the game is exogenously set at (x_1^q, y_1^q).

Discussion of the model. In our baseline model there is no turnover in proposers, no uncertainty about changes in ideal points, and no asymmetry across players' utility functions (besides ideal points). These assumptions allow us to isolate the mechanism that drives our results, while shutting down features that have previously been shown to cause inefficiency. Additionally, we consider a two-period model in order to obtain sharper results. However, none of these assumptions undermine the core mechanism of our model. We later consider extensions relaxing each of these assumptions.

In order to more clearly illustrate our results, we assume the players share exactly the same ideal policy on the Y dimension. However, our intuitions apply broadly to cases in which players face some disagreement on this dimension, but the status quo is outside of the gridlock interval and the players' ideal points shift in the same direction over time. This would guarantee that, on the Y dimension, players always agree on the optimal direction of policy change. In contrast, the assumption that $\hat{x}_2^V \leq \hat{x}_1^V < \hat{x}_1^P \leq \hat{x}_2^P$ ensures that the X dimension *always* features conflict and thus distinguishes it from a common-values dimension.[15]. Absent this assumption, a policy that is in the gridlock interval on X in the first period can become unstuck, even without the Y dimension, if the ideal points of both players move closer together or shift in the same direction in the second period. Indeed, our analysis highlights that in a dynamic setting distinguishing policy issues that feature conflict from those with common values depends both on the current location of ideal points and how these ideal points change over time.

The key feature of our model is that ideal points are indexed by the time period t, hence, the players' preferences may change over time. Figure 1 depicts an example of this evolution. To clearly illuminate our mechanism we limit the number of degrees of freedom by fixing the shapes of v and u across periods. However, under relatively mild assumptions, which ensure that players still become more entrenched on the X dimension between periods, our results are robust even if we allow these shapes to change as well. Thus, the insights of our model can apply to a broad number of ways in which preferences may change. We discuss the effects of letting v and u change over time in Section 4.

A definition of efficiency. We conclude this section by establishing terminology for a policy to be efficient.

[15] Likewise, assuming u is concave ensures that the players become more resistant to changes on the X dimension when their ideal points move apart, capturing the idea of increasing conflict. We further discuss this point in Section 4.1

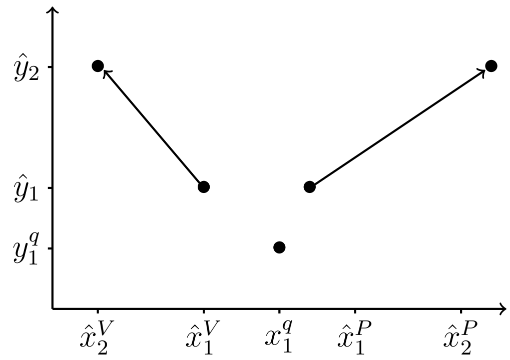

Figure 1: Example of increasing polarization in the evolution of preferences.

Definition 1. *A policy outcome (x_t, y_t) in period t is efficient if it sets the common-values policy at $y_t = \hat{y}_t$. Otherwise, a policy (x_t, y_t) is inefficient.*

According to our definition, an inefficient outcome is always Pareto inefficient as well. Specifically, Pareto efficiency requires $(x_t, y_t) \in [\hat{x}_t^V, \hat{x}_t^P] \times \{\hat{y}_t\}$, i.e., efficiency on Y *and* policy in the gridlock interval on X. Given our focus on understanding when players cannot agree on the common-values issue, our definition sidesteps that even with efficiency on the common-values dimension the outcome may be Pareto inefficient if the ideological policy x_t is not in the interval $[\hat{x}_t^V, \hat{x}_t^P]$.

3 Analysis

Moving to the analysis, our solution concept is subgame perfect equilibrium and we proceed by backwards induction. In the second period, players only consider their static payoffs. Thus, V accepts any policy (x, y) such that:

$$u_{V2}(x) + v_2(y) \geq u_{V2}(x_2^q) + v_2(y_2^q). \tag{1}$$

V is only willing to accept an ideological policy that moves farther away from its ideal point on X if the proposal improves on the common-values status quo. Consequently, if the inherited status quo is inefficient, $y_2^q \neq \hat{y}_2$, then P can extract concessions on the conflict dimension by proposing a bundle that moves the common-values policy closer to $\hat{y}_2$.

In equilibrium, P chooses its proposal to maximize $u_{P2}(x) + v_2(y)$ subject to (1). Let $\overline{x}(x_2^q, y_2^q)$ be the upper solution to:

$$u_{V2}(x) + v_2(\hat{y}_2) = u_{V2}(x_2^q) + v_2(y_2^q). \tag{2}$$

Lemma 1 characterizes P's optimal second-period proposal.

Lemma 1. *In the second period P proposes $y_2^* = \hat{y}_2$ and $x_2^*(x^q, y^q) = \min\left\{\hat{x}_2^P, \overline{x}(x_2^q, y_2^q)\right\}$.*

Proposing $y = \hat{y}_2$ maximizes V's utility from the offer on the common-values dimension, and thus maximizes V's willingness to accept a worse payoff on the ideological dimension. As such, the efficient policy $y = \hat{y}_2$ both maximizes P's payoff on the common-values dimension *and* the extent to which P can move the outcome towards its ideal policy $\hat{x}_2^P$. Therefore, the equilibrium policy outcome is always efficient, emphasizing that the ability to bundle dimensions does not lead to inefficiency absent dynamic motives.

Turning to the first period, let $U_q = u_{V1}(x_1^q) + v_1(y_1^q) + u\big(x_2^*(x_1^q, y_1^q)\big)$ denote V's dynamic equilibrium payoff from keeping the status quo. Thus, in the first period V accepts a proposal (x, y) if:

$$u_{V1}(x) + v_1(y) + u_{V2}\big(x_2^*(x, y)\big) \geq U_q,$$

and rejects otherwise. Facing this constraint from V, in the first period player P chooses $(x, y) \in \mathbb{R}^2$ to solve the following maximization problem:

$$\begin{aligned} \max_{x,y} \;\; & u_{P1}(x) + v_1(y) + u_{P2}\big(x_2^*(x, y)\big) \\ \text{s.t. } \;\; & u_{V1}(x) + v_1(y) + u_{V2}\big(x_2^*(x, y)\big) \geq U_q \end{aligned} \tag{3}$$

In equilibrium, the first-period policy thus balances P's static preferences against its dynamic incentives, while still appeasing V. As such, the policy depends crucially on the players' dynamic preferences over policies. In particular, players' optimal policies induced by these strategic considerations are central to our concept of ideological infection. It is therefore useful to introduce the following definition of a player's dynamic ideal point.

Definition 2. *Player i's dynamic ideal point $(\hat{x}_d^i, \hat{y}_d^i)$ solves*

$$\max_{x,y} u_{i1}(x) + v_1(y) + u_{i2}\big(x_2^*(x, y)\big).$$

Thus player i's dynamic ideal point is the policy that i would choose to implement today, anticipating bargaining tomorrow.

Lemma 2 provides an initial characterization of these dynamic ideal points.

Lemma 2. *On the Y dimension $\hat{y}_d^P \leq \hat{y}_1 \leq \hat{y}_d^V$. On the X dimension $\hat{x}_2^V \leq \hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{x}_1^P \leq \hat{x}_d^P \leq \hat{x}_2^P$.*

Although both players prefer the efficient common-values policy today, their incentives to influence future policy outcomes can lead to divergent dynamic preferences. From equation (1), we see that moving y_2^q further from $\hat{y}_2$ increases $\overline{x}(x_2^q, y_2^q)$, i.e., it increases the proposer's second-period leverage. As such, the proposer's second-period equilibrium payoff increases when the inherited common-values policy is further from $\hat{y}_2$, as shown in Figure 2. Conversely, the veto player's second-period payoff increases when y_1 moves closer to $\hat{y}_2$, as this limits the proposer's leverage in the second period. Consequently, P prefers a policy that weakly *undershoots* the efficient common-values policy, $\hat{y}_d^P \leq \hat{y}_1 < \hat{y}_2$, while V prefers a policy that weakly *overshoots* it, $\hat{y}_1 \leq \hat{y}_d^V < \hat{y}_2$. As we will show, under some conditions these inequalities hold strictly, leading to ideological infection.

Notice that each player's dynamic preferences on the conflict dimension can also be distorted from its static ideal policy. For player i a policy closer to $\hat{x}_2^i$ improves its equilibrium policy payoff tomorrow and worsens the other player's. As such, each player's dynamic ideal point lies in between the first and second-period optimum.

It is important to note that this distortion of preferences on X would emerge even in a model without the Y dimension. Absent the Y dimension, policy is always stuck (in the gridlock interval) and P's dynamic ideal point is again in $(\hat{x}_1^P, \hat{x}_2^P)$, as this balances its payoff from today versus tomorrow. In contrast, in a model with only the Y dimension there is no difficulty in agreeing to the efficient policy today and tomorrow. Consequently, any preference divergence on the common-values dimension is due solely to the existence of multiple dimensions.

We now formally introduce the concept of ideological infection.

Definition 3. *If $\hat{y}_d^P = \hat{y}_d^V = \hat{y}_1$ there is no ideological infection. Otherwise, if $\hat{y}_d^P \neq \hat{y}_d^V$ then there is ideological infection. In particular, if $\hat{y}_d^i \neq \hat{y}_1$ then i's preferences are infected.*

Ideological infection emerges when one (or both) players prefer an inefficient common-value policy in the first period. In the subsequent sections, we unpack the conditions under which ideological infection occurs, when infection leads to inefficient policy outcomes, and the form of inefficiency that emerges.

3.1 The Role of Compounding Costs of Inefficiency

We first provide a necessary condition for ideological infection to emerge. Although players have dynamic incentives to distort policy in the first period, any inefficiency exploited to gain an advantage tomorrow imposes a cost on both players today. Thus, the players' dynamic incentives do not necessarily lead to infection of preferences on the common-values dimension. We show that the key feature is whether the gains from moving policy to be more efficient are

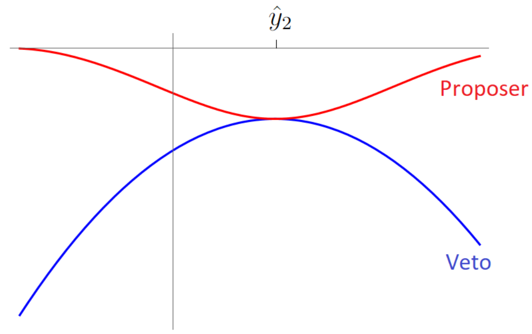

Figure 2: Second-period equilibrium payoffs as a function of y_2^q, if players have quadratic utility on both dimensions.

relatively greater in the second period or the first. Specifically, whether ideological infection can occur in equilibrium depends on the marginal cost of an inefficient policy in the first period versus the second period. We describe this condition with the following definition:

Definition 4. *The costs of inefficiency are compounding over time if the following condition holds:*

$$v_2'(y) > v_1'(y) \text{ for } y \leq \hat{y}_1. \tag{4}$$

Otherwise, if $v_2'(y) \leq v_1'(y)$ for $y \leq \hat{y}_1$ then the costs of inefficiency are not compounding over time.

Notice that if the costs of inefficiency do not compound over time then it must be the case that $\hat{y}_1 = \hat{y}_2$, as otherwise $v_2'(\hat{y}_1) > 0 = v_1'(\hat{y}_1)$. Instead, if (4) does hold then we must have $\hat{y}_1 < \hat{y}_2$. Thus, in our baseline model, the costs of inefficiency compounding is equivalent to movement in the common-values ideal point. We adopt the definition of compounding in terms of condition (4), rather than the change in $\hat{y}_t$, because we later allow the shape of v to change over time. Under this generalization, the two concepts are no longer equivalent and it becomes clear that increasing marginal costs of inefficiency is the crucial necessary condition for ideological infection.

To see the importance of compounding costs for infection, first consider P's preferences. To clarify the discussion, assume $x_2^*(x, y) = \overline{x}(x, y)$, thus, P's dynamic ideal point solves:

$$u_{P1}'(x) + \frac{u_{P2}'(\overline{x}(x, y))}{u_{V2}'(\overline{x}(x, y))} u_{V2}'(x) = 0 \tag{5}$$

$$v_1'(y) + \frac{u_{P2}'(\overline{x}(x, y))}{u_{V2}'(\overline{x}(x, y))} v_2'(y) = 0. \tag{6}$$

Equations (5) and (6) together yield that at P's dynamic ideal point:

$$\frac{u'_{P1}(x)}{u'_{V2}(x)} = \frac{v'_1(y)}{v'_2(y)} = -\frac{u'_{P2}(\overline{x}(x,y))}{u'_{V2}(\overline{x}(x,y))} > 0. \tag{7}$$

Condition 7 captures that P faces a tradeoff between maximizing its first-period payoff versus setting itself up for the future by minimizing V's utility from the status quo in the second period. Indeed, we can rewrite the first equality of (7) as $\frac{u'_{P1}(x)}{v'_1(y)} = \frac{u'_{V2}(x)}{v'_2(y)}$. That is, at (x^d_P, x^d_V), P's first-period marginal rate of substitution between dimension X and dimension Y is equal to L's second-period marginal rate of substitution between X and Y. For $x < \hat{x}^P_1$, increasing x increases P's payoff today and tomorrow, but for $x > \hat{x}^P_1$ further increasing x decreases its first-period payoff. On the common-values dimension, increasing y towards $\hat{y}_1$ improves P's first-period utility on Y *and* makes V more willing to shift policy to the right on X, which improves P's first period ideological payoff. However, it comes at the cost of increasing $v_2(y)$, which decreases P's leverage in the second period. The optimal policy bundle for P balances these considerations.

Consequently, whether P's preferences on Y are distorted depends on if improving the policy on Y makes V more willing to concede policy on the X dimension today or tomorrow. Under $v'_2(y) \leq v'_1(y)$ for $y < \hat{y}_1$, the increase in V's payoff from moving y towards $\hat{y}_1$ is weakly greater in the first period than in the second. This induces P to use any available leverage today rather than save it for tomorrow. The opposite holds if $v'_2(y) > v'_1(y)$ for $y < \hat{y}_1$, i.e., costs of inefficiency compound over time. In this case, the proposer can obtain greater concessions from the veto player by reducing inefficiency on the common value dimension in the second period, which generates incentives to pursue inefficient policies that undershoot $\hat{y}_1$ in the first.

Figure 3 further illustrates the implications of compounding costs for ideological infection, considering an example where v is concave. Recall that an inefficient policy that undershoots $\hat{y}_1$ imposes costs on both parties in the first period and increases the cost to the veto player for maintaining the status quo in the second period. In turn, this allows P to pull x_2 closer to $\hat{x}^P_2$. Under concavity, the condition that $v'_2(y) > v'_1(y)$ for $y < \hat{y}_1$ implies that:

$$|v_2(y) - v_2(\hat{y}_1)| > |v_1(y) - v_1(\hat{y}_1)| \tag{8}$$

for any $y < \hat{y}_1$. In turn, inequality (8) indicates that the increased second-period cost on the veto player is greater than the cost that *both* parties pay for inefficiency in the first period (as shown in Figure 3). This wedge in the cost of residual inefficiency today and tomorrow creates the possibility for P to benefit from a policy that undershoots the static optimum

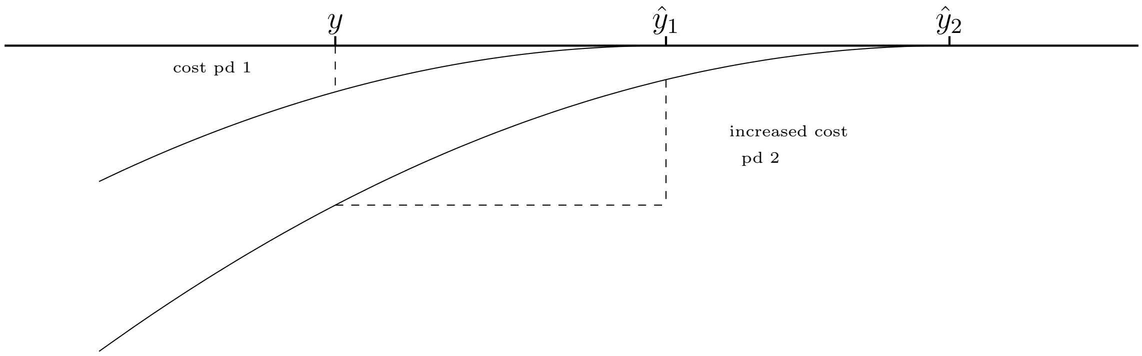

Figure 3: Common-values utility in first and second period.

on Y. If instead condition $v_2'(y) \leq v_1'(y)$ for $y < \hat{y}_1$, then tomorrow's ideological gains are always lower than today's cost of inefficiency, and no ideological infection emerges.

Finally, a similar logic explains V's dynamic preferences. Again, assume $x_2^*(x, y) = \overline{x}(x, y)$, i.e., it is not optimal to choose (x, y) such that P gets its second-period ideal point. V's dynamic ideal point then solves $\left(x_d^V, y_d^V\right)$ solves:

$$u_{V1}'(x) + u_{V2}'(x) = 0 \tag{9}$$

$$v_1'(y) + v_2'(y) = 0. \tag{10}$$

Here, V's second-period equilibrium payoff is equivalent to its first-period payoff because P holds the proposal power. In turn, V's dynamic ideal point acts as if policy will be stuck at the first-period policy outcome. Accordingly, V wants to exactly balance its marginal utility on the common-values dimension from today and tomorrow (and same for the ideological dimension). If costs of inefficiency do not compound over time, then we have that $v_2'(y) \leq v_1'(y)$, and thus $\hat{y}_1 = \hat{y}_2$. When $\hat{y}_1 = \hat{y}_2$, the efficient first-period policy is also the policy that minimizes the proposer's leverage in the second period. Consequently, the efficient policy is dynamically optimal for the veto player. As such, $\frac{v_2'(y)}{v_1'(y)} \leq 1$ immediately removes any distortion of V's preferences. If instead $\frac{v_2'(y)}{v_1'(y)} > 1$ then the wedge in the cost of residual inefficiency opens the possibility that V benefits from overshooting $\hat{y}_1$, and therefore V's preferences may be infected.

Building on this discussion, Proposition 1 shows that the absence of compounding eliminates ideological infection.

Proposition 1. *If $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$ then there is no ideological infection, $\hat{y}_d^P = \hat{y}_d^V = \hat{y}_1$. Furthermore, the policy outcome is efficient, $y_1^* = \hat{y}_1$.*

Absent ideological infection neither player has a preference for inefficiency, and the players have no difficulty coming to an efficient agreement. Thus, on issues for which the cost of inefficiency does not compound over time, efficiency prevails.

3.2 The Role of Increasing Ideological Conflict

Our previous results establish that compounding in the costs of inefficiency is a necessary condition for ideological infection. We now complete the analysis by showing that whether infection actually emerges in equilibrium also depends on the evolution of preferences on the X dimension. We find that, with increasing marginal costs of inefficiency, a sufficient condition for infection is rapidly increasing conflict on the ideological dimension. Throughout this section we maintain the assumption that costs of inefficiency are compounding over time.

Assumption 1. *The costs of inefficiency are compounding over time:*

$$v_2'(y) > v_1'(y) \text{ for all } y \leq \hat{y}_1. \tag{11}$$

To characterize the conditions under which each player's preferences are infected we first define two cut-points:

$$\overline{u}_P \equiv u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1), \tag{12}$$

$$\overline{u}_V \equiv u_{V1}(\hat{x}_\alpha^V) + v_1(\hat{y}_\alpha^V) + u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V), \tag{13}$$

where $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ is defined as the solution to the equations (9) and (10). Thus, $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ characterizes V's dynamic ideal point in the case where P does not obtain $\hat{x}_2^P$ in the second period. Proposition 2 now characterizes when each player's preferences are infected.

Proposition 2.

1. *V's preferences are infected if and only if $u_{V2}(\hat{x}_2^P) < \overline{u}_V$; and*

2. *P's preferences are infected if and only if $u_{V2}(\hat{x}_2^P) < \overline{u}_P$.*

Furthermore, $\overline{u}_P < \overline{u}_V$.

Whether compounding costs of inefficiency generate infection depends on the anticipated degree of conflict in the future, characterized by $u_{V2}(\hat{x}_2^P)$. To see why, consider the condition for P's preferences to *not* be infected:

$$u_{V2}(\hat{x}_2^P) + v_2(\hat{y}_2) \geq \overline{u}_P = u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1) \tag{14}$$

In this case, the increase in conflict on the ideological dimension is low relative to the change on the common-values dimension. Specifically, if condition (14) is satisfied then P has enough leverage in the second period to pass its ideal point, even if the status quo is at the first-period efficient policy $\hat{y}_1$. As such, P has no incentive to undershoot on the common-values dimension, or implement an extreme policy on the ideological dimension. In contrast, if the players anticipate significant conflict in the second period, $u_{V2}(\hat{x}_2^P) < \overline{u}_P$, then P does not have enough leverage to get its preferred policy when the inherited status quo is $(\hat{x}_1^P, \hat{y}_1)$. As a consequence, P's dynamically optimal policy undershoots the efficient $\hat{y}_1$.

Notice that we can rewrite condition (14) as $u_{V2}(\hat{x}_1^P) - u_{V2}(\hat{x}_2^P) \leq v_2(\hat{y}_2) - v_2(\hat{y}_1)$. Thus, under this condition, the conflict on the ideological dimension increases slowly relative to the evolution of the common-values issue. Furthermore, if the players' ideal points on X are the same in period 2 and in period 1 then $u_{V2}(\hat{x}_1^P) = u_{V2}(\hat{x}_2^P)$ and condition (14) always holds. Consequently, infection of the proposer's preferences requires conflict on the ideological dimension to increase over time. Furthermore, this increase needs to be sufficiently rapid relative to the evolution of the common-values dimension.

A similar calculation determines whether the veto's preferences are infected. If there is sufficiently little disagreement in the second period, then the proposer can obtain $\hat{x}_2^P$ even if the first-period policy overshoots $\hat{y}_1$. In turn, inefficiency does not constrain P in the second period, and V's optimal first-period policy is $(\hat{x}_1^V, \hat{y}_1)$. Additionally, because P holds the bargaining power, the anticipated amount of conflict needed to induce infection in V's preferences is lower than the amount needed to infect V, $\overline{u}_V < \overline{u}_P$. Specifically, it is easier for P to move policy to $(\hat{x}_2^P, \hat{y}_1)$ from $(\hat{x}_1^P, \hat{y}_1)$ than from $(\hat{x}_1^V, \hat{y}_1)$.

We note that, if $\hat{y}_2 - \hat{y}_1$ is not too large, then $u_{V2}(\hat{x}_1^P) < \overline{u}_V$. Therefore, infection of V's preferences can emerge even if there is no change on the ideological dimension across periods. However, if $\hat{y}_2 - \hat{y}_1$ is sufficiently large, then infection of the veto player's preferences also requires rapidly increasing disagreement on the dimension of conflict.

The above discussion highlights that a significant increase in the intensity of the ideological conflict from the first to the second period is necessary and sufficient to ensure that *both* parties' preferences are infected (illustrated in Figure 4). As Proposition 3 now establishes, this makes an inefficient equilibrium policy inevitable.

Proposition 3.

1. *Assume V's preferences are infected but P's preferences are not. If $U_q \leq u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ then the equilibrium policy is efficient. Otherwise, the equilibrium policy is inefficient for almost all $(U_q, \hat{x}_1^V)$.*

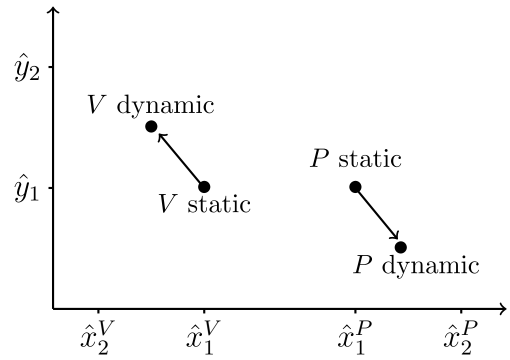

Figure 4: Dynamic ideal points under rapidly increasing polarization.

2. *If both players' preferences are infected then the equilibrium policy is inefficient for almost all* $(U_q, \hat{x}_1^V)$*.*

If only the veto player's preferences are infected, i.e., $\overline{u}_P < u_{V2}(\hat{x}_2^P) < \overline{u}_V$, then the equilibrium policy on the common-values dimension may still be efficient. When the initial status quo U_q is bad for the veto player, the proposer can pass its optimal bundle. In particular, following our previous discussion, P can obtain its statically optimal bundle in both periods and the equilibrium is efficient. Otherwise, if U_q is high, then the proposer is constrained in the first period. Thus, to appease the veto player, P proposes an inefficient policy, even though P's own preferences are not infected.

Instead, if the preferences of both players are infected, which occurs when $u_{V2}(\hat{x}_2^P) < \overline{u}_P$, then inefficiency is inevitable. The efficient policy always leaves P with too little or too much leverage in the future. Thus, by Proposition 2, we should expect inefficiency to be most pervasive when players anticipate the conflict to intensify rapidly.

Having established the conditions for the emergence of inefficiency, we conclude this section by analyzing the nature this inefficiency takes in equilibrium and how it influences the policy on the ideological dimension. In particular, the value of U_q is crucial in determining the policy outcome. Suppose that $u_{V2}(\hat{x}_2^P) < \overline{u}_P$, so the policy is (almost) always inefficient.

When U_q is very low, the status quo is highly favorable to the proposer. Consequently, P can pass a policy close to its unconstrained optimum; hence, $y_1^* < \hat{y}_1$ and $x_1^* > \hat{x}_1$, aligning with the proposer's dynamic preferences. A similar symmetric logic holds when U_q is very high. The status quo is very favorable for the veto player, and therefore P has little leverage to pull the conflict-dimension policy close to its first-period ideal. Thus, P needs to propose a policy close to V's dynamic ideal point, binding himself in the future to obtain larger concessions today. The equilibrium in this case is characterized by a veto-player-induced inefficiency. The common-values policy overshoots the first-period ideal, $y_1^* \in (\hat{y}_1, \hat{y}_d^V)$, and the conflict policy remains below P's ideal point, $x_1^* < \hat{x}_1^P$.

More interesting is the case where neither player is initially strongly advantaged by the status quo, i.e., when U_q is intermediate. Intuition may suggest that undershooting on the common-value dimension should only emerge when the proposer does not need leverage in the first period. That is, if P can pass $\hat{x}_1^P$ without moving the Y dimension status quo all the way to the efficient $\hat{y}_1$, and thus chooses to maintain some inefficiency to increase leverage for the future. Our next result shows that this is not always the case in our setting:

Proposition 4. *Assume $u_{V2}(\hat{x}_2^P) < \overline{u}_P$. There exists an open interval $(\underline{U}_q, \overline{U}_q)$, such that, if $U_q \in (\underline{U}_q, \overline{U}_q)$ then $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$.*

When V's payoff from the status quo is intermediate the proposer undershoots on the common-values dimension, $y_1^* < \hat{y}_1$, *and* proposes an ideological policy to the left of both its first- and second-period ideal points, $x_1^* < \hat{x}_1^P$. In this case, the proposer *could* move policy closer to $\hat{x}_1^P$ — potentially even obtaining its first-period ideologically preferred policy. However, doing so requires satisfying the veto player by moving the common-values policy closer to $\hat{y}_2$, reducing P's future leverage. In equilibrium, undershooting occurs despite the proposer needing more leverage today. The reason P is willing to forgo gains today is precisely because there are compounding costs of inefficiency on the Y dimension: the proposer can buy *even more* concessions tomorrow than it can today by implementing the efficient $\hat{y}_1$ and moving x further right. Thus, P chooses to incur immediate costs on the common-values dimension *and* forgoes gains on the ideological dimension.

Figure 5 provides an illustration of the first-period equilibrium under the assumption that the players' have quadratic-loss preferences on each dimension in each period.[16]

4 Extensions

In this section we extend the model in several directions. First, we provide a more general condition that captures the concept of increasing conflict on the ideological dimension and allows for the shape of the players' utility functions to change over time. Next, we examine the conditions for ideological infection of the players' preferences to emerge if there is turnover in the proposer, or if the change in ideal points between periods is stochastic. Finally, we study bargaining over a longer time horizon. This final analysis further emphasizes how the speed at which conflict increases on X relative to the compounding costs of inefficiency on Y matters for the persistence of inefficient policies.

[16]Under quadratic utility whether the equilibrium policy undershoots or overshoots $\hat{y}_1$ is fully determined by a unique cutoff in U_q. However, providing a full characterization under more general functional forms is challenging, as the equilibrium policy may cross $\hat{y}_1$ multiple times and thus not be monotonic in U_q.

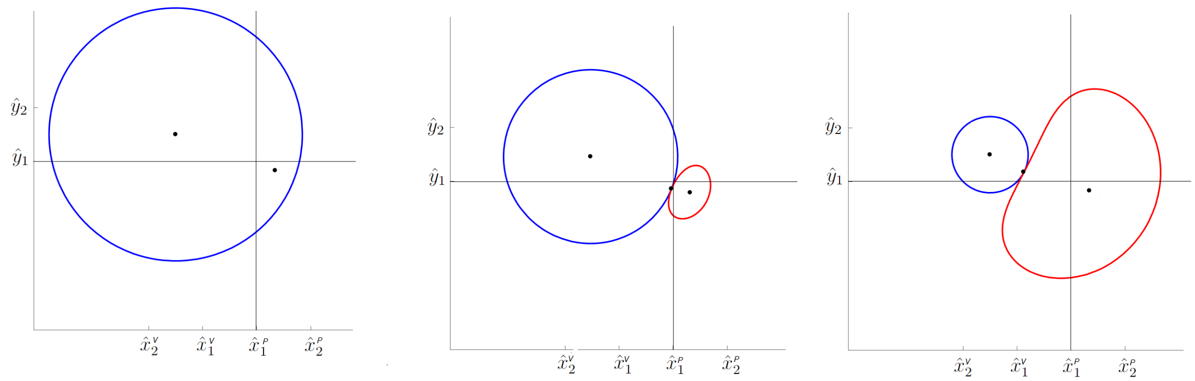

Figure 5: Players' dynamic ideal points and indifference curves with quadratic utility (blue for the veto player, red for the proposer). The left-most panel considers $U_q < \underline{U}$. In the middle panel we have $U_q \in (\underline{U}, \overline{U})$. In the right-most panel we set $U_q > \overline{U}$. Generated from a numerical example where players have quadratic utility over both dimensions.

4.1 Changing Shapes of the Utility Functions

In our baseline model, we allow the players' ideal points to change over time but assume the shape of their utility function is fixed. Here, we relax this assumption and allow both the location of the ideal policies and the shape of the utility functions to evolve. We thus index the functions u and v by time so that $u_{it}(x) = u_t(x - \hat{x}_t^i)$ and $v_t(y) = v_t(y - \hat{y}_t)$, but maintain all the other assumptions.

We begin by providing a condition on how the utility functions over the X dimension change over time that ensures our earlier results carry through to this richer setting.

Proposition 5. *Assume the following conditions on u_1 and u_2:*

$$u'_{V2}(x) \leq u'_{V1}(x) \text{ and } u'_{P1}(x) \leq u'_{P2}(x) \text{ for all } x \in [\hat{x}_2^V, \hat{x}_2^P]. \tag{15}$$

Then the previous results hold exactly as stated.

Assumption (15) states that for each player the marginal cost of moving policy away from its ideal point on X becomes greater over time. This assumption is consistent with our consideration of X as a dimension of disagreement on which conflict is increasing is over time. Thus, under this definition of increasing ideological conflict the central result from our baseline model continues to hold. Even though both players expect their opponent to become more entrenched on the ideological dimension in the future, and thus less willing to compromise, compounding costs on the common-values dimension generate incentives to delay coming to an efficient agreement.

If the above condition fails, then the utility function over the X dimension can become "flatter" over time despite the ideal points pulling further apart. In this case, our model delivers a less surprising result. If players become less sensitive over time to changes on the conflict dimension, and thus more willing to compromise tomorrow, this naturally creates incentives to delay agreeing to an efficient policy today, even absent compounding costs of inefficiency. That is, even if preferences on the Y dimension do not change, the proposer may want to undershoot the efficient policy to preserve leverage because the veto player becomes close to indifferent over policies on the X dimension in the second period. As such, any amount of inefficiency is more valuable and P's preferences are infected. Here, the key condition for P's preferences to not be infected is that, for all $x > \hat{x}_1^P$ and $y \leq \hat{y}_2$:

$$\frac{u'_{V2}(x)}{u'_{P1}(x)} \geq \frac{v'_2(y)}{v'_1(y)}. \tag{16}$$

Under the increasing conflict condition, $\frac{u'_{V2}(x)}{u'_{P1}(x)} > 1$ for all $x > \hat{x}_1^P$, thus, $v'_2(y) \leq v'_1(y)$ is sufficient for (16) to hold. If the increasing conflict condition fails, then it is possible that $\frac{u'_{V2}(x)}{u'_{P1}(x)} < 1$ for $x > \hat{x}_1^P$. Consequently, (16) can fail and infection of the proposer's preferences can occur even if, for example, $v_1(y) = v_2(y)$. However, Condition (16) still maintains a similar flavor as the original result, whereby infection of the proposer's preferences is avoided as long as the marginal cost of inefficiency is relatively smaller tomorrow than today. We also note that the same logic and discussion applies if we relax concavity of u and only require u to be quasi-concave over X: pulling apart the players' ideal points can make them less sensitive to changes over the relevant policy region (even if the shape of u does not change in this case). Overall, given our substantive interest in situations where the actors are becoming more antagonistic and policy problems worsen over time, we have focused our analysis on the case where the u_ts are concave and condition (15) holds.

Finally, now that we allow the shape of the utility functions to change over time we point out that Condition (4) does not consider the case where $\hat{y}_1 = \hat{y}_2$ but $v'_2(y) > v'_1(y)$ for $y < \hat{y}_1$. This streamlines the presentation of our results, but it is not crucial for infection of P's preferences.[17] That is, a similar logic of compounding can lead to infection of P's preferences if the ideal point on Y remains the same but the marginal cost of inaction increases over time (except at $\hat{y}_1 = \hat{y}_2$), e.g., $v_2(y) = \theta v_1(y)$ with $\theta > 1$. In this case, V's preferences are not infected, as it is not possible for y to overshoot $\hat{y}_1$. However, P is still incentivized to undershoot $\hat{y}_1$ because the compounding makes V more willing to yield concessions in period 2.[18]

[17] Furthermore, we note that Condition (4) allows the change in $\hat{y}_1$ to be arbitrarily small.

[18] For example, consider the following numerical specification: $u_t(x - \hat{x}_t^i) = -(x - \hat{x}_t^i)^2$, $\hat{x}_1^P = -\hat{x}_1^V = 1$,

4.2 Turnover

Up to this point, we have assumed that player P is always the proposer, emphasizing that inefficiency and ideological infection in our setting do not stem from fear of the other player taking power. We now turn our attention to the implications of turnover for our mechanism. Specifically, we assume P is the proposer in period 1 and remains so with probability $\rho \in (0, 1)$ in period 2, while V becomes the proposer with probability $1 - \rho$.

Now, the second-period outcome depends both on the first-period policy *and* the identity of the player selected to be the proposer. If P remains the proposer, the equilibrium outcome is as characterized in the baseline model. Letting $\overline{x}_V(x_2^q, y_2^q) \equiv \overline{x}(x_2^q, y_2^q)$, the second-period outcome is then $x_P^*(x, y) = \min\{\hat{x}_2^P, \overline{x}_V(x, y)\}$. Suppose instead that P becomes the veto player in the second period. Then, the relevant threshold characterizing the set of acceptable policies is $\underline{x}_P(x_2^q, y_2^q)$, which is defined as the lower solution to:

$$u_{P2}(x) = u_{P2}(x_2^q) + v_2(y_2^q).$$

Therefore, the second-period outcome if V becomes the proposer is $x_V^*(x, y) = \max\{\hat{x}_2^V, \underline{x}_P(x_2^q, y_2^q)\}$.

P's problem in the first period can now be written as:

$$\max_{x,y} \; u_{P1}(x) + v_1(y) + \rho u_{P2}\big(x_P^*(x, y)\big) + (1 - \rho)u_{P2}\big(x_V^*(x, y)\big) \tag{17}$$
$$\text{s.t. } u_{V1}(x) + v_1(y) + \rho u_{V2}\big(x_P^*(x, y)\big) + (1 - \rho)u_{V2}\big(x_V^*(x, y)\big) \geq U_q$$

We demonstrate that, similar to the baseline model, both players' preferences are always infected when ideological conflict intensifies rapidly.

Proposition 6. *If $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$, then both players' preferences are infected for almost all values of $\rho \in (0, 1)$.*

To understand these conditions, we focus first on P's preferences. Suppose $\rho = 1$, so that P is certain to remain in power in the second period. The analysis of the baseline model highlights that P's preferences are infected if and only if, by setting $y_1 = \hat{y}_1$ (and $x_1 = \hat{x}_1^P$), P does not maintain enough leverage to obtain its static optimum in the second period. This condition ensures that any marginal movement away from $\hat{y}_1$ impacts the outcome of the second-period bargaining, generating the distortion. Symmetrically, when $\rho = 0$ and P

$\hat{x}_2^P = -\hat{x}_2^V = 7$, $v_1(y) = -\frac{1}{4}(y-1)^2$, and $v_2(y) = -(y-1)^2$. If the efficient policy is implemented in the first period, $y = \hat{y} = 1$, then the optimal x for P is the midpoint $\frac{\hat{x}_1^P + \hat{x}_2^P}{2} = 4$. P's dynamic payoff in this case is $-(4-1)^2 - (4-7)^2 = -18$. Instead, consider the policy $x = 3.5$ and $y = -2.4$. In the second period, the veto player is willing to accept $\overline{x}(3.5, -2.4) \approx 4.03$. Then P's dynamic payoff from the inefficient policy $(3.5, -2.4)$ is $-(3.5-1)^2 - \frac{1}{4}(-2.4-1)^2 - (4.03-7)^2 \approx -17.93 > -18$.

is certain to lose power in the second period, infection of his preferences emerges whenever implementing $y_1 = \hat{y}_1$ (and $x_1 = \hat{x}_1^P$) in period 1 implies V does not have enough leverage to pass his optimum in the second period. When ρ is between 0 and 1, marginal movements away from $\hat{y}_1$ influence the second-period outcome as long as *at least one* of these conditions is satisfied. As a consequence, either one of these conditions is sufficient for P's preferences to be infected. A similar logic applies to player V. Eliminating the least binding conditions, we obtain that both players' preferences are infected if $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$, as stated in Proposition 6.

Notice that the above discussion has an important implication: in our setting, turnover can *generate* ideological infection. When $u_{V2}(\hat{x}_2^P) - u_{V2}(\hat{x}_1^P) > u_{P2}(\hat{x}_2^V) - u_{P2}(\hat{x}_1^V)$, the condition for V to be constrained as a second-period proposer after passing his first-period optimum is less binding than the analogous condition for P. When P is sure to remain in power in the second period, this is irrelevant. However, as described above, when $\rho < 1$ P worries about reducing V's leverage should he be selected as the second-period proposer, and $u_{P2}(\hat{x}_2^V) - u_{P2}(\hat{x}_1^V) < v_2(\hat{y}_1)$ is enough to ensure P's preferences are infected.

However, our final result shows that turnover can mitigate inefficiency on the intensive margin, by reducing the *degree* to which players' preferences are infected.

Proposition 7. *Suppose each player i's dynamic ideal point is such that* $\overline{x}_V(\hat{x}_d^i, \hat{y}_d^i) < \hat{x}_2^P$ *and* $\underline{x}_P(\hat{x}_d^i, \hat{y}_d^i) > \hat{x}_2^V$*. Then,* $\hat{y}_d^P$ *is decreasing in* ρ *and* $\hat{y}_d^V$ *is increasing in* ρ*.*

Consider the incentives of the proposer. If ρ is high then P is confident of remaining the proposer tomorrow and therefore wants to undershoot in the first period. However, as ρ decreases, P becomes increasing likely to lose power, and moves y towards $\hat{y}_1$ to offset the downside of keeping leverage in case V becomes the proposer. Eventually, the probability of remaining proposer is sufficiently low that P begins to overshoot as insurance against V becoming the proposer tomorrow. Thus, there is a unique value of ρ for which the proposer's incentives to under and overshoot exactly compensate each other, eliminating infection. For all other values, infection persists. This implies that, for values of ρ below this cutoff, increasing turnover reduces the degree to which P's preferences are distorted. This finding suggests that during times of rapidly intensifying ideological conflict, electoral uncertainty over who will hold power tomorrow can partially mitigate ideological infection and its consequences, but cannot completely eliminate such distortions.

4.3 Stochastic Preferences

In our analysis thus far, we have assumed the players can perfectly anticipate how the optimal common-value policy will evolve over time. This assumption is useful isolate the mechanism behind our results, but it is an obvious simplification. In this section, we discuss the effects of relaxing this assumption and consider a version of the model where the second-period optimal common-value policy is ex-ante unknown. Suppose that $\hat{y}_2 = \hat{y}_1 + \epsilon$, where ϵ is drawn from a continuous distribution G with density g and full support over the real line.

For ease of exposition, we focus on infection of P's preferences. Now, in the second period the set of policies V is willing to accept depends on the realization of the shock on the common-values dimension. Specifically, $\overline{x}(x_2^q, y_2^q; \epsilon)$ solves:

$$u_{V2}(x) = u_{V2}(x_2^q) + v(y_2^q - \hat{y}_1 - \epsilon).$$

Let $\underline{\epsilon}(x_2^q, y_2^q)$ and $\overline{\epsilon}(x_2^q, y_2^q)$ be the lower and upper solutions, respectively, to $\overline{x}(x_2^q, y_2^q; \epsilon) = \hat{x}_2^P$. Extreme shocks shift $\hat{y}_2$ far from y_2^q, which gives P enough leverage to obtain its ideal point in the second period. Therefore, P's dynamic ideal point $(\hat{x}_d^P, \hat{y}_d^P)$ solves

$$\max_{x,y} u_{P1}(x) + v_1(y) + \int_{\underline{\epsilon}(x,y)}^{\overline{\epsilon}(x,y)} u_{P2}\big(\overline{x}(x,y;\epsilon)\big) g(\epsilon) d\epsilon.$$

Notice, if the support of G was such that $\epsilon \in \big(\underline{\epsilon}(\hat{x}_1^P, \hat{y}_1), \overline{\epsilon}(\hat{x}_1^P, \hat{y}_1)\big)$ with probability 0 then P's preferences are not infected. Similar to the case of $u_{V2}(\hat{x}_2^P) + v_2(\hat{y}_2) \geq u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ in Proposition 2, P anticipates having sufficient leverage tomorrow to obtain $\hat{x}_2^P$, even if the efficient policy is implemented today. When instead G has full support on $\mathbb{R}$ then $(\hat{x}_d^P, \hat{y}_d^P)$ must solve:

$$u'_{P1}(x) + \int_{\underline{\epsilon}(x,y)}^{\overline{\epsilon}(x,y)} \frac{u'_{P2}\big(\overline{x}(x,y;\epsilon)\big)}{u'_{V2}\big(\overline{x}(x,y;\epsilon)\big)} u'_{V1}(x) g(\epsilon) d\epsilon = 0 \tag{18}$$

$$v'_1(y) + \int_{\underline{\epsilon}(x,y)}^{\overline{\epsilon}(x,y)} \frac{u'_{P2}\big(\overline{x}(x,y;\epsilon)\big)}{u'_{V2}\big(\overline{x}(x,y;\epsilon)\big)} v'_2(y;\epsilon) g(\epsilon) d\epsilon = 0. \tag{19}$$

The first order conditions highlight that the results of this enriched model align with our baseline findings. First, suppose that the shock ϵ has a zero mean, and both the distribution G and the v function are symmetric. Substantively, this implies that players' expect there will be no change on the Y dimension. Condition 19 shows that, in this case, infection of the proposer's preferences is avoided. Under the assumed symmetry conditions, $\underline{\epsilon}(x, \hat{y}_1)$ and $\overline{\epsilon}(x, \hat{y}_1)$ are centered around 0. In turn, this implies that $\int_{\underline{\epsilon}(x,\hat{y}_1)}^{\overline{\epsilon}(x,\hat{y}_1)} \frac{u'_{P2}\big(\overline{x}(x,\hat{y}_1;\epsilon)\big)}{u'_{V2}\big(\overline{x}(x,\hat{y}_1;\epsilon)\big)} v'_2(\hat{y}_1;\epsilon) g(\epsilon) d\epsilon = 0$ under a symmetric v. In other words, even though change on the common-values dimension *may* occur, the symmetry conditions imply that the possible dynamic downsides for the proposer of a policy undershooting the static optimum exactly compensate the upsides. *In expectation*, neither player benefits from an inefficient policy. As such, $\hat{y}_1$ is the unique solution to the proposer's maximization problem, and ideological infection is avoided.

Conversely, a positive mean shock, and thus the mere expectation of a worsening of the status quo on the common-values dimension, is enough to guarantee infection of the proposer's preferences. In this case, the full support of G implies that, with *some* probability, the proposer will dynamically benefit from undershooting on the common-values dimension. This is enough to distort his preferences.

While the above discussion only considers uncertainty over the common-values dimension, similar intuitions apply to the conflict dimension. Suppose the players anticipate that ideological polarization will (weakly) increase in the second period, but do not know by how much. An intuitive condition is sufficient to generate ideological infection: that there is a strictly positive probability that the realization of the second-period bliss points satisfies the conditions identified in Proposition 2.

In this case, although the parties cannot perfectly predict the future location of their respective optimal policies, it is possible they will become so polarized that inefficiency in the first period could yield gains for the proposer in the second.[19] This generates incentives for the proposer to distort policy in the first period, resulting in $y_1^* \neq \hat{y}_1$. Thus, considering uncertainty over either, or both, dimensions does not alter our main insights from the baseline model.

4.4 Long-run Outcomes

We now extend our analysis to study when bargaining over a dynamic policy problem and a issue with increasing conflict leads to inefficiency over the long run. Bargaining proceeds as before, but unfolds over a finite number of periods, $t = 1, 2, ..., T$, where throughout we assume T is large.

Let $u(x - \hat{x}_t^i) = -(x - \hat{x}_t^i)^2$ and $v(y - \hat{y}_t) = -(y - \hat{y}_t)^2$. Additionally, we specify the evolution of ideal points as follows: $\hat{y}_t = \gamma t$ and $\hat{x}_t^P = -\hat{x}_t^V = t^\eta$, with $\gamma > 0$ and $\eta > 0$. Thus, the common-values ideal point increases linearly in time, while the evolution of ideal points on the conflict dimension may be concave or convex, and the evolution of these processes are governed by γ and η.

We study subgame perfect equilibria and, given the finite horizon, analyze the model via backwards induction. Notice that at period t any history leading to the same status quo (x_t^q, y_t^q) yields the same continuation game. As such, we focus on strategy profiles where policy proposals only depend on the time period (which also captures the players' ideal

[19] Recall that condition for infection identified in Proposition 2 indicates that if R implements his static optimum in the first period, $(\hat{x}_1^P, \hat{y}_1)$, he will not have enough leverage to get his static optimum in the second.

points) and the inherited status quo, and acceptance decisions only depend on these factors plus the proposed policy. We write player i's continuation payoff from strategy starting at time t as $w_t^i(x_t^q, y_t^q)$, suppressing dependence on the strategy profile. With this notation in hand, a strategy profile constitutes an equilibrium if for all status quo policies (x^q, y^q) and all t, the following conditions hold:

- for any proposal (x, y), V accepts if and only if

$$u_{Vt}(x) + v_t(y) + w_{t+1}^V\big(x, y\big) \geq u_{Vt}(x_t^q) + v_t(y_t^q) + w_{t+1}^V\big(x_t^q, y_t^q\big).$$

- P's proposal, $x_t^*(x^q, y^q)$, solves

$$\max_{(x,y)} \; u_{Pt}(x) + v_t(y) + w_{t+1}^P\big(x, y\big)$$
$$\text{s.t. } u_{Vt}(x) + v_t(y) + w_{t+1}^V\big(x, y\big) \geq u_{Vt}(x_t^q) + v_t(y_t^q) + w_{t+1}^V\big(x_t^q, y_t^q\big).$$

Of course, in the last period of the game the players always agree on the efficient policy, $y_T^* = \hat{y}_T$, and the ideological policy $x_T^* = \min\left\{\hat{x}_T^P, \overline{x}(x_T^q, y_T^q)\right\}$, where $\overline{x}$ is defined as in the two-period model.

In this setting, the case of $\gamma = 0$ corresponds to no movement in the ideal common-values policy, i.e., condition (4) fails in each period. For the same logic as in the baseline model, the absence of compounding costs of inefficiency implies there is no ideological infection. The following propositions consider the case where $\gamma > 0$, i.e. the optimal common-values policy changes over time. As we saw in the baseline model, compounding of the costs of inefficiency are necessary but not sufficient for ideological infection. Specifically, from Proposition 2, P's preferences are not infected when the change in preferences on X is sufficiently slow relative to the change in $\hat{y}$, such that P can obtain its ideal point in both periods. Similarly, the evolution of preferences on the conflict dimension plays a crucial role for whether inefficiency can be sustained in the long-run.

Proposition 8. *If $\eta < \frac{1}{2}$ then there exists $\hat{t} < T$ such that the equilibrium policy outcome is $x_t^* = \hat{x}_t^P$ and $y_t^* = \hat{y}_t$ in every period $t \geq \hat{t}$. Furthermore, for γ sufficiently large $\hat{t} = 1$.*

When $\eta < \frac{1}{2}$, in which case the evolution of preferences on the X dimension is concave, the ideological conflict is increasing but eventually this increase is very small. Thus, in the long run, the change in conflict on X is slow relative to changes on the common-values dimension, where $\hat{y}_t$ is increasing linearly. Proposition 8 then confirms our insights from the baseline model. The players eventually reach a period where P is able to pass its static ideal policy $(\hat{x}_t^P, \hat{y}_t)$, and from there P has enough leverage to implement his ideal point on the conflict dimension in every subsequent period. Thus, the parties always reach the efficient common-values policy before the end date T. Furthermore, when the evolution of the common-values dimension is sufficiently rapid the parties reach efficiency immediately.

Next, we consider the case of rapidly growing polarization. Similar to the baseline, infection in any non-final period may be inevitable under rapid polarization.

Proposition 9. *If $\eta > \frac{1}{2}$ then $y_t^* \neq \hat{y}_1$ in every period.*

When $\eta > \frac{1}{2}$, so that the evolution of the conflict dimension is convex, the game eventually reaches a state where implementing an efficient policy leaves the proposer with insufficient leverage to obtain its optimal bundle in period $T - 1$. Anticipation of the eventual need for leverage rolls back to the previous periods, and leads to policy outcomes always being inefficient.

5 Conclusion

Addressing policy problems often requires the agreement of multiple parties. However, bargaining parties regularly have trouble reaching an efficient solution, even on issues where they all agree that the situation will grow increasingly worse if there is a lack of action. Our model shows that agreements on these common-values policy problems are vulnerable to being distorted by disagreements on other issues precisely because they worsen over time. Furthermore, if parties anticipate being more entrenched on the conflict dimension in the future, then ideological infection of preferences over the common-values issue is inevitable.

Our analysis provides insight into a number of contexts where parties have failed to adapt policy to deteriorating circumstances. Climate change negotiations between China and the United States have been hampered by disagreements over Taiwan. In the United States, political parties now appear polarized on nearly every issue, including those with little ideological content. Additionally, despite the severe costs, Republicans and Democrats have not always managed to avoid a government shutdown due to strategic incentives to gain an advantage on ideological issues. Our paper uncovers the conditions under which these issues of joint interest become infected by issues of conflict.

A Proofs of Main Results

As discussed in Section 4.1, we can generalize the baseline model to allow the shapes of the utility functions to change between periods. In proving the results, we now index u and v by the time period: $u_{it}(x) = u_t(x - \hat{x}_t^i)$ and $v_t(y) = v_t(y - \hat{y}_t)$ and impose the following assumption throughout:

Assumption 2.

$$u'_{V2}(x) \leq u'_{V1}(x) \text{ and } u'_{P1}(x) \leq u'_{P2}(x) \text{ for all } x \in [\hat{x}_2^V, \hat{x}_2^P]. \tag{20}$$

Lemma A.1. *Any (x, y) such that $\overline{x}(x, y) > \hat{x}_2^P$ with $y \neq \hat{y}_1$ or $x > \hat{x}_1^P$ does not solve (3).*

Proof. For a contradiction, assume there exists (x, y) such that $\overline{x}(x, y) > \hat{x}_2^P$ is optimal and $y \neq \hat{y}_1$. Since $\overline{x}(x, y)$ is continuous in y there exists y' closer to $\hat{y}_1$ such that $\overline{x}(x, y') > \hat{x}_2^P$. Furthermore, because $u_{V1}(x)+v_1(y')+u_{V2}(\hat{x}_2^P) > u_{V1}(x)+v_1(y)+u_{V2}(\hat{x}_2^P)$ the policy (x, y') must also satisfy V's acceptance constraint. Evaluating the objective function at (x, y') and (x, y) immediately yields $u_{P1}(x)+v_1(y')+u_{P2}(\hat{x}_2^P) > u_{P1}(x)+v_1(y)+u_{P2}(\hat{x}_2^P)$, contradicting that (x, y) solves problem (3). A similar argument shows that if $\overline{x}(x, y) > \hat{x}_2^P$ and $x > \hat{x}_1^P$ then there exists some profitable deviation $x' \in (\hat{x}_1^P, x)$, which improves the first-period payoffs of P and V without changing second-period payoffs. □

Lemma A.1 establishes an initial characterization of the optimal proposal when $\overline{x}(x, y) > \hat{x}_2^P$. If instead (x, y) is such that $\overline{x}(x, y) \leq \hat{x}_2^P$, then we can write the proposer's problem (3) as:

$$\begin{aligned}
\max_{x,y} \quad & u_{P1}(x) + v_1(y) + u_{P2}\big(\overline{x}(x, y)\big) \\
\text{s.t.} \quad & u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y) \geq U_q \\
& u_{V2}(x) + v_2(y) \geq u_{V2}(\hat{x}_2^P)
\end{aligned}$$

System (21) yields the KKT conditions for this problem:[20]

[20] It is straightforward to show that if the constraint qualification fails at some point (x, y) then it must be that $x \in [\hat{x}_2^V, \hat{x}_\alpha^V)$ and $y \in (\hat{y}_\alpha^V, \hat{y}_2]$. However, clearly this cannot be optimal as a deviation to $x = \hat{x}_d^V$ and $y = \hat{y}_d^V$ is always accepted by V and improves P's dynamic payoff. Thus, the KKT conditions will hold at any solution.

$$u'_{P1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{P2}\big(\overline{x}(x,y)\big) + \lambda_1 \Big[u'_{V1}(x) + u'_{V2}(x)\Big] + \lambda_2 u'_{V2}(x) = 0 \tag{21a}$$

$$v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{P2}\big(\overline{x}(x,y)\big) + \lambda_1 \Big[v'_1(y) + v'_2(y)\Big] + \lambda_2 v'_2(y) = 0 \tag{21b}$$

$$\lambda_1 \Big[u_{V1}(x) + v_1(y) + u_{V2}\big(\overline{x}(x,y)\big) - U_q\Big] = 0 \tag{21c}$$

$$\lambda_2 \Big[u_{V2}(x) + v_2(y) - u_{V2}(\hat{x}_2^P)\Big] = 0 \tag{21d}$$

$$\lambda_1, \lambda_2 \geq 0 \tag{21e}$$

Lemma A.2. *Any (x,y) such that $\overline{x}(x,y) = \hat{x}_2^P$ never solves (3).*

Proof. Suppose there exists (x,y) that solves (3) such that $\overline{x}(x,y) = \hat{x}_2^P$. Thus, (x,y) must solve system (21). In particular, consider condition (21b). Letting $\overline{x}(x,y) = \hat{x}_2^P$, then $u'_{P2}\big(\overline{x}(x,y)\big) = 0$ and this condition becomes:

$$v'_1(y) + \lambda_1 \Big[v'_1(y) + v'_2(y)\Big] + \lambda_2 v'_2(y) = 0. \tag{22}$$

First, notice that if such an (x,y) is optimal then we must have $y \leq \hat{y}_1$. If $y \in (\hat{y}_1, \hat{y}_2]$ then P could deviate to $y = \hat{y}_1$, which would maintain $x_2^*(x,y) = \hat{x}_2^P$ and improve the first-period payoff of both players, contradicting that (x,y) solves problem (3).

Second, if $y < \hat{y}_1$ then the LHS of (22) is strictly positive by $\hat{y}_1 \leq \hat{y}_2$, contradicting that (22) holds. Therefore, if (x,y) is such that $\overline{x}(x,y) = \hat{x}_2^P$ and solves problem (3) then we must have $y = \hat{y}_1$.

To finish the proof we now show that $y = \hat{y}_1$ also leads to a contradiction. If $y = \hat{y}_1$ then (22) reduces to:

$$\big(\lambda_1 + \lambda_2\big) v'_2(\hat{y}_1) = 0. \tag{23}$$

We consider two cases depending on $v'_2(\hat{y}_1)$. First, if $v'_2(\hat{y}_1) > 0$ then for (22) to hold requires $\lambda_1 = \lambda_2 = 0$, but $\lambda_2 = 0$ contradicts that $\overline{x}(x,y) = \hat{x}_2^P$. Second, if $v'_2(\hat{y}_1) = 0$ then it must be that $\hat{y}_1 = \hat{y}_2$. Thus, the policy will be stuck at the first-period proposal x in the second period, which implies that $x = \hat{x}_2^P$. In this case, for condition (21a) to hold requires:

$$u'_{P1}(\hat{x}_2^P) + \lambda_1 \Big[u'_{V1}(\hat{x}_2^P) + u'_{V2}(\hat{x}_2^P)\Big] + \lambda_2 u'_{V2}(\hat{x}_2^P) = 0. \tag{24}$$

Because $\hat{x}_2^P \geq \hat{x}_1^P$ the LHS of (24) is strictly negative by assumption that P is always further to the right than V, which contradicts that (x,y) solves (3) and completes the argument. □

Lemma 2. *On the Y dimension $\hat{y}_d^P \leq \hat{y}_1 \leq \hat{y}_d^V$. On the X dimension $\hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{x}_1^P \leq \hat{x}_d^P$.*

Proof. First, we prove the result for the proposer's dynamic ideal point. If P chooses (x, y) such that $x_2^*(x, y) = \overline{x}(x, y)$ then the necessary condition $(\hat{x}_d^P, \hat{y}_d^P)$ solves is:

$$u'_{P1}(x) + \frac{\partial \overline{x}}{\partial x} u'_{P2}\big(\overline{x}(x, y)\big) = 0 \tag{25}$$

$$v'_1(y) + \frac{\partial \overline{x}}{\partial y} u'_{P2}\big(\overline{x}(x, y)\big) = 0. \tag{26}$$

Recall that $\overline{x}(x, y)$ solves (2), the implicit function theorem then yields:

$$\frac{\partial \overline{x}}{\partial x} = \frac{u'_{V2}(x)}{u'_{V2}\big(\overline{x}(x, y)\big)}, \text{ and } \frac{\partial \overline{x}}{\partial y} = \frac{v'_2(y)}{u'_{V2}\big(\overline{x}(x, y)\big)}.$$

As such, (25) can be written as: $u'_{P1}(x) + \frac{u'_{V2}(x)}{u'_{V2}(\overline{x})} u'_{P2}\big(\overline{x}(x, y)\big)$, note we will sometimes suppress dependence of $\overline{x}$ on the first-period policy (x, y). Furthermore, in equilibrium we must have $\hat{x}_2^V < x \leq \overline{x}(x, y) \leq \hat{x}_2^P$, which implies $\frac{u'_{V2}(x)}{u'_{V2}(\overline{x})} > 0$ and $u'_{P2}\big(\overline{x}(x, y)\big) \geq 0$. Therefore, $x > \hat{x}_1^P$ is necessary for (25) to hold. Next consider equation (26). After substituting for $\frac{\partial \overline{x}}{\partial y}$, (26) becomes:

$$v'_1(y) + \frac{u'_{P2}\big(\overline{x}(x, y)\big)}{u'_{V2}\big(\overline{x}(x, y)\big)} v'_2(y) = 0.$$

Because $\overline{x}(x, y) \in (\hat{x}_2^V, \hat{x}_2^P]$ we have $\frac{u'_{P2}(\overline{x})}{u'_{V2}(\overline{x})} \leq 0$. Thus, if $y \in (\hat{y}_1, \hat{y}_2]$ then (26) is strictly negative. Clearly $y > \hat{y}_2$ is never optimal. Hence, a necessary condition for (26) to hold is that $y \leq \hat{y}_1$.

Suppose instead that $(\hat{x}_d^P, \hat{y}_d^P)$ is such that $x_2^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$. In this case, if $x_2^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$ is optimal absent the veto player's first-period acceptance constraint, then clearly we must have $(\hat{x}_d^P, \hat{y}_d^P) = (\hat{x}_1^P, \hat{y}_1)$. Thus, it is always the case that $\hat{x}_d^P \geq \hat{x}_1^P$ and $\hat{y}_d^P \leq \hat{y}_1$.

Now consider the veto player. V's dynamic ideal point $(\hat{x}_d^V, \hat{y}_d^V)$ solves:

$$\max_{x,y} u_{V1}(x) + v_1(y) + u_{V2}\big(x_2^*(x, y)\big).$$

If $x_2^*(x, y) = \overline{x}(x, y)$ then $(\hat{x}_d^V, \hat{y}_d^V)$ must solve:

$$u'_{V1}(x) + u'_{V2}(x) = 0$$

$$v'_1(y) + v'_2(y) = 0.$$

Thus, $\hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{y}_d^V \geq \hat{y}_1$, as required. If instead it is optimal for V to choose $(\hat{x}_d^V, \hat{y}_d^V)$ such that $x_2^*(x, y) = \hat{x}_2^P$, then clearly it must be that $\hat{x}_d^V = \hat{x}_1^V$ and $\hat{y}_d^V = \hat{y}_1$, completing the argument. $\square$

Proposition 1. *If $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$, then there is no ideological infection, $\hat{y}_d^P = \hat{y}_d^V = \hat{y}_1$. Furthermore, the policy outcome is efficient, $y_1^* = \hat{y}_1$.*

Proof.

Part 1. To start, we prove part 1, that there is no ideological infection. For a contradiction, assume $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$, but $\hat{y}_d^i \neq \hat{y}_1$ for some $i \in \{V, P\}$.

First, consider player P and suppose $\hat{y}_d^P < \hat{y}_1$. Lemma A.1 implies that if $y_d^P \neq \hat{y}_1$ then $x^*(x,y) = \overline{x}(x,y)$. Thus, from our analysis in Lemma 2, it is necessary that P's dynamic ideal point (x_d^P, y_d^P) solves:

$$u_{P1}'(x) + \frac{u_{V2}'(x)}{u_{V2}'(\overline{x})} u_{P2}'(\overline{x}(x,y)) = 0 \tag{27}$$

$$v_1'(y) + \frac{v_2'(y)}{u_{V2}'(\overline{x})} u_{P2}'(\overline{x}(x,y)) = 0. \tag{28}$$

Using that $y \neq \hat{y}_1$, we can combine conditions (27) and (28) and rearrange to obtain that $(\hat{x}_d^P, \hat{y}_d^P)$ must satisfy:

$$\frac{v_2'(y)}{v_1'(y)} = \frac{u_{V2}'(x)}{u_{P1}'(x)}. \tag{29}$$

From the proof of Lemma 2, if (x,y) is such that $x_2^*(x,y) = \overline{x}(x,y)$ then $\hat{x}_d^P > \hat{x}_1^P$. Recall that $u_{P1}(x)$ is a translation of $u_{V1}(x)$ and u_{i1} is concave, thus, $u_{V1}'(x) < u_{P1}'(x) < 0$ for $x > \hat{x}_1^P$. Furthermore, by Assumption 2, $u_{V2}'(x) \leq u_{V1}'(x)$. Hence, $u_{V2}'(x) < u_{P1}'(x) < 0$. However, this implies $\frac{u_{V2}'(x)}{u_{P1}'(x)} > 1$ and by assumption $\frac{v_2'(y)}{v_1'(y)} \leq 1$. Therefore, $\frac{u_{V2}'(x)}{u_{P1}'(x)} > \frac{v_2'(y)}{v_1'(y)}$ and (29) cannot hold, contradicting that $\hat{y}_d^P \neq \hat{y}_1$.

Second, we show that L's preferences are also not infected. Suppose not, so $\hat{y}_d^V > \hat{y}_1$. A similar argument as for P in Lemma A.1 yields that if $\hat{y}_d^V \neq \hat{y}_1$ then $\overline{x}(\hat{x}_d^V, \hat{y}_d^V) \leq \hat{x}_2^P$. Thus, consider (x,y) such that $x_2^*(x,y) = \overline{x}(x,y)$. In this case, $(\hat{x}_d^V, \hat{y}_d^V)$ solves:

$$u_{V1}'(x) + u_{V2}'(x) = 0 \tag{30}$$

$$v_1'(y) + v_2'(y) = 0. \tag{31}$$

The assumption that $\frac{v_2'(y)}{v_1'(y)} \leq 1$ for all $y \leq \hat{y}_1$ implies that $\hat{y}_1 = \hat{y}_2$, otherwise the assumption would fail at $y = \hat{y}_1 < \hat{y}_2$. Therefore, if $y > \hat{y}_1 = \hat{y}_2$ then $v_1'(y) < 0$ and $v_2'(y) < 0$, which violates (31).

Part 2. Now we show that the policy outcome must also be efficient. To derive a contradiction, suppose that $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$ but $y_1^* \neq \hat{y}_1$. By Lemma A.1 if $y_1^* \neq \hat{y}_1$ then $\overline{x}(x_1^*, y_1^*) \leq \hat{x}_2^P$. Thus, the optimal policy proposal must solve system (21). To prove the result we now consider different cases depending on which constraints are binding.

Case 1: To start, assume $\lambda_2 > 0$, this implies that $\overline{x}(x, y) = \hat{x}_2^P$, which cannot be optimal by Lemma A.2.

Case 2: Second, assume $\lambda_1 = 0$ and $\lambda_2 = 0$. By $\lambda_1 = 0$ the proposer's unconstrained optimal policy is accepted by V. Thus, by Proposition 1 $y_1^* = \hat{y}_d^P = \hat{y}_1$, as required.

Case 3: Finally, consider the case where $\lambda_1 > 0$ and $\lambda_2 = 0$. Solving (21a) and (21b) for λ_1 implies that (x, y) must solve:

$$\frac{v_1'(y) + \frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})} v_2'(y)}{v_1'(y) + v_2'(y)} = \frac{u_{P1}'(x) + \frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})} u_{V2}'(x)}{u_{V1}'(x) + u_{V2}'(x)}. \tag{32}$$

Rearranging condition (32), we have that any optimal (x, y) must solve:

$$v_1'(y)u_{P1}'(x) - v_1'(y)u_{V1}'(x) + v_2'(y)u_{P1}'(x) - v_1'(y)u_{V2}'(x) \\ - \frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})}\Big(v_2'(y)u_{V1}'(x) - v_1'(y)u_{V2}'(x)\Big) = 0. \tag{33}$$

To obtain a contradiction, we show that if $y \neq \hat{y}_1$ then the LHS of (33) is strictly positive. Suppose $y < \hat{y}_1 = \hat{y}_2$, recalling that if $v_1'(y) \geq v_2'(y)$ for all $y \leq \hat{y}_1$ then $\hat{y}_1 = \hat{y}_2$ (an analogous argument proves the case $y > \hat{y}_1 = \hat{y}_2$). Thus, $v_1'(y) > 0$ and $v_2'(y) > 0$.

First, we show that the last term in (33) is always positive. To see this, note that $\frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})} < 0$, by $\overline{x}(x, y) \in (\hat{x}_2^V, \hat{x}_2^P]$. Thus, a sufficient condition for the last term on the LHS of (33) to be positive is that:

$$v_2'(y)u_{V1}'(x) - v_1'(y)u_{V2}'(x) > 0. \tag{34}$$

Clearly, in equilibrium, $x \geq \hat{x}_2^V$. Thus, if $x < \hat{x}_1^V$ then $v_2'(y)u_{V1}'(x) > 0$ and (34) holds. Instead, suppose that $x \geq \hat{x}_1^V$. By Assumption 2 $u_{V2}'(x) < u_{V1}'(x) < 0$. Additionally, by assumption, $v_1'(y) \geq v_2'(y) > 0$. As such, $v_2'(y)u_{V1}'(x) > v_1'(y)u_{V2}'(x)$, and (34) holds. Therefore, $-\frac{u_{P2}'(\overline{x})}{u_{V2}'(\overline{x})}\Big(v_2'(y)u_{V1}'(x) - v_1'(y)u_{V2}'(x)\Big) \geq 0$, as claimed.

Second, consider the term: $v_1'(y)u_{P1}'(x) - v_1'(y)u_{V1}'(x)$. By $y < \hat{y}_1$, this term is positive if and only if $u_{P1}'(x) \geq u_{V1}'(x)$, which holds by concavity of $u_1(x - \hat{x}^i)$ and $\hat{x}_1^V < \hat{x}_1^P$.

Finally, to complete the argument that the LHS of (33) is strictly positive, we show that

$v_2'(y)u_{P1}'(x) - v_1'(y)u_{V2}'(x) > 0$. This holds if and only if:

$$v_2'(y)u_{P1}'(x) > v_1'(y)u_{V2}'(x). \tag{35}$$

By our previous argument showing that (34) is positive, we have $v_2'(y)u_{V1}'(x) > v_1'(y)u_{V2}'(x)$. Thus, a sufficient condition for (35) to hold is that $v_2'(y)u_{P1}'(x) \geq v_2'(y)u_{V1}'(x)$, which again follows from concavity of u_1 and $\hat{x}_1^V < \hat{x}_1^P$. Therefore, the LHS of (33) is strictly positive, contradicting that $y < \hat{y}_1$ is optimal. □

Proposition 2.

1. *V's preferences are infected if and only if* $u_{V2}(\hat{x}_2^P) < \overline{u}_V$*; and*

2. *P's preferences are infected if and only if* $u_{V2}(\hat{x}_2^P) < \overline{u}_P$*.*

Furthermore, $\overline{u}_P < \overline{u}_V$.

Proof. To start, we prove part 1 of the proposition. Define $\overline{u}_V$ as:

$$\overline{u}_V \equiv u_{V1}(\hat{x}_\alpha^V) + v_1(\hat{y}_\alpha^V) + u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V),$$

where $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ solves:

$$\begin{aligned} u_{V1}'(x) + u_{V2}'(x) &= 0 \\ v_1'(y) + v_2'(y) &= 0. \end{aligned} \tag{36}$$

Part 1. We begin by showing that if $u_{V2}(\hat{x}_2^P) < \overline{u}_V$ then V's preferences are infected. In this case, $u_{V2}(\hat{x}_2^P) < \overline{u}_V < u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V)$ which implies $\overline{x}(\hat{x}_\alpha^V, \hat{y}_\alpha^V) < \hat{x}_2^P$. Therefore, V's dynamic payoff from $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ is $\overline{u}_V$. By construction $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ maximizes $u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$ and hence maximizes V's dynamic payoff among all policies (x, y) such $x_2^*(x, y) < \overline{x}(x, y)$. Finally, the best possible dynamic payoff to V from any policy (x, y) such that $x_2^*(x, y) = \hat{x}_2^P$ is $u_{V2}(\hat{x}_2^P)$ which is strictly less than the dynamic payoff from $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ by assumption that $u_{V2}(\hat{x}_2^P) < \overline{u}_V$. From inspection of (36), $\hat{y}_\alpha^V > \hat{y}_1$, and V's preferences are infected.

Next, we prove that if $u_{V2}(\hat{x}_2^P) \geq \overline{u}_V$ then V's preferences are not infected. Because $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ solves $\max_{x,y} u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$ we have that $\overline{u}_V \geq u_{V1}(\hat{x}_1^V) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1) = u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1)$. Therefore, the assumption $u_{V2}(\hat{x}_2^P) \geq \overline{u}_V$ also yields that $u_{V2}(\hat{x}_2^P) \geq u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1)$. Thus, the dynamic payoff to V from $(\hat{x}_1^V, \hat{y}_1)$ is $u_{V2}(\hat{x}_2^P)$, which is the greatest possible payoff from any first-period policy such that $x_2^*(x, y) = \hat{x}_2^P$.

Instead the best policy for V such that $\overline{x}(x,y) \leq \hat{x}_2^P$ solves:

$$\max_{x,y} u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$$
$$\text{s.t. } u_{V2}(x) + v_2(y) \geq u_{V2}(\hat{x}_2^P).$$

Recall that (x_α^V, y_α^V) solves this problem when the constraint does not bind, however, $u_{V2}(\hat{x}_2^P) \geq \overline{u}_V$ and hence the constraint must be binding at the solution. Therefore the best policy (x,y) for V such that $\overline{x}(x,y) \leq \hat{x}_2^P$ must set $\overline{x}(x,y) = \hat{x}_2^P$, and clearly $u_{V1}(x) + v_1(y) + u_{V2}(\hat{x}_2^P) < u_{V1}(\hat{x}_1^V) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_2^P)$. Consequently, $(\hat{x}_d^V, \hat{y}_d^V) = (\hat{x}_1^V, \hat{y}_1)$ and V's preferences are not infected.

Part 2. Now we prove part 2 of the proposition. Recall that $\overline{u}_P = u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$.

We first show that if $u_{V2}(\hat{x}_2^P) \geq \overline{u}_P$ then P's preferences are not infected. By definition of $\overline{u}_P$, if P chooses $(\hat{x}_1^P, \hat{y}_1)$ in the first period it can implement $(\hat{x}_2^P, \hat{y}_2)$ in the second. As this is the unique sequence of policies that yields P its first-best payoff, P's preferences are not infected.

Next, we show that if $u_{V2}(\hat{x}_2^P) < \overline{u}_P$ then P's preferences are infected. To show a contradiction, assume that $\hat{y}_d^P = \hat{y}_1$. We break the argument into two parts. First, let $\overline{x}(x, \hat{y}_d^P) = \hat{x}_2^P$, then from system (21) for (x,y) to satisfy the KKT conditions it must be that $x < \hat{x}_1^P$. However, this implies $u_{V2}(x) + v_2(\hat{y}_1) \leq u_{V2}(\hat{x}_2^P)$ and $u_{V2}(x) + v_2(\hat{y}_1) > u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1) = \overline{u}_P$, which contradicts that $u_{V2}(\hat{x}_2^P) < \overline{u}_P$.

Second, let $\overline{x}(\hat{x}_d^P, \hat{y}_d^P) < \hat{x}_2^P$. By System (21) $\hat{x}_d^P$ must solve:

$$\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} v'_2(\hat{y}_1) = 0. \tag{37}$$

However, $\overline{x}(x,\hat{y}_1) \in (\hat{x}_2^V, \hat{x}_2^P)$, thus $\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} < 0$. Additionally, by Assumption 1 $v'_2(\hat{y}_1) > 0$. Therefore, the LHS of (37) is strictly less than 0, which contradicts that $\hat{y}_2^P = \hat{y}_1$.

Part 3. To conclude the proof we now demonstrate that $\overline{u}_P < \overline{u}_V$. We again note that $(\hat{x}_\alpha^V, \hat{y}_\alpha^V)$ is the unique maximizer of $u_{V1}(x) + v_1(y) + u_{V2}(x) + v_2(y)$ and thus $\overline{u}_P = u_{V1}(\hat{x}_1) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1) < u_{V1}(\hat{x}_1) + v_1(\hat{y}_1) + u_{V2}(\hat{x}_1^V) + v_2(\hat{y}_1) < u_{V1}(\hat{x}_\alpha^V) + v_1(\hat{y}_\alpha^V) + u_{V2}(\hat{x}_\alpha^V) + v_2(\hat{y}_\alpha^V) = \overline{u}_V$. □

Proposition 3.

1. *Assume V's preferences are infected but P's preferences are not. If $U_q \leq u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ then the equilibrium policy is efficient. Otherwise, the equilibrium policy is*

inefficient for almost all $(U_q, \hat{x}_1^V)$.

2. *If both players' preferences are infected then the equilibrium policy is inefficient for almost all* $(U_q, \hat{x}_1^V)$.

Proof. We first prove the second part of the result. Assume that the preferences of both players are infected, thus, $\overline{u}_P > u_{V2}(\hat{x}_2^P)$ by Proposition 2.

By Lemmas A.1 and A.2 if the equilibrium policy is such that $x_2^*(x_1^*, y_1^*) = \hat{x}_2^P$ then $x_1^* < \hat{x}_1^P$ and $y_1^* = \hat{y}_1$. However, by $x_1^* < \hat{x}_1^P$ we have $u_{V2}(x_1^*)+v_2(\hat{y}_1) > u_{V2}(\hat{x}_1)+v_2(\hat{y}_1) = \overline{u}_P$. Thus, the assumption $\overline{u}_P > u_{V2}(\hat{x}_2^P)$ implies $u_{V2}(x_1^*) + v_2(\hat{y}_1) > u_{V2}(\hat{x}_2^P)$, which contradicts that $\overline{x}(x_1^*, \hat{y}_1) \geq \hat{x}_2^P$. Therefore, the equilibrium policy (x_1^*, y_1^*) must be such that $\overline{x}(x_1^*, y_1^*) < \hat{x}_2^P$ and thus solve system (21). From (21) if $y = \hat{y}_1$ then x must solve:

$$u_{V1}(x) + u_{V2}\big(\overline{x}(x, \hat{y}_1)\big) = U_q \tag{38}$$

The LHS of (38) is strictly decreasing in x for $x > \hat{x}_1^V$, thus, there must be a unique $x' > \hat{x}_1^V$ that solves this equality.

Solving for λ_1 from (21a) and (21b) and rearranging we have that the equilibrium x must also solve:

$$\frac{u'_{P2}(\overline{x}(x, \hat{y}_1))}{u'_{V2}(\overline{x}(x, \hat{y}_1))} - \frac{u'_{P1}(x)}{u'_{V1}(x)} = 0. \tag{39}$$

Using this condition, define $f : \mathbb{R}^2 \to \mathbb{R}$ as $f(x, \hat{x}_1^V) = \frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} - \frac{u'_{P1}(x)}{u'_{V1}(x)}$. Then

$$Df(x, \hat{x}_1^V) = \left(\frac{\partial \overline{x}}{\partial x} \frac{u''_{P2}(\overline{x})u'_{V2}(\overline{x}) - u'_{P2}(\overline{x})u''_{V2}(\overline{x})}{u'_{V2}(\overline{x})^2} - \frac{u'_{V1}(x)u''_{P1}(x) - u'_{P1}(x)u''_{V1}(x)}{u'_{V1}(x)^2}, -\frac{u'_{P1}(x)u''_{V1}(x)}{[u'_{V1}(x)]^2} \right),$$

Notice that if $x = \hat{x}_1^P$ then (39) cannot hold. Moreover, $-\frac{u'_{P1}(x)u''_{V1}(x)}{[u'_{V1}(x)]^2} = 0$ if and only if $x = \hat{x}_1^P$. Therefore, if $(x, \hat{x}_1^V)$ is such that $f(x, \hat{x}_1^V) = 0$ then $Df(x, \hat{x}_1^V)$ has rank $1 = \min\{1, 2\}$. Thus, 0 is a regular value of f and by the Transversality Theorem (De la Fuente, 2000, Theorem 2.5) the set of x that solve (39) is measure 0 for almost all $\hat{x}_{V1}$. Fix such a $\hat{x}_{V1}$. The unique solution to (38) is strictly increasing in U_q, while solutions to (39) do not change in U_q.[21] Thus for almost all $(U_q, \hat{x}_{V1})$ the solution to (38) does not coincide with any solutions to (39).

Next, we prove the first part of the proposition. Suppose only V's preferences are infected,

[21]Note we can change U_q without changin anything in (39) by changing the initial status quo policy.

thus, $\overline{u}_V > u_{V2}(\hat{x}_2^P) > \overline{u}_P$ by Proposition 2. For $U_q \leq u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ clearly $(x, y) = (\hat{x}_1^P, \hat{y}_1)$ is optimal, as P passes its ideal point in each period. Next, assume $U_q > u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$. Thus, V rejects $(\hat{x}_1^P, \hat{y}_1)$, which implies P chooses (x, y) such that $x_2^*(x, y) = \overline{x}(x, y)$ and $\lambda_1 > 0$. For a contradiction suppose that $y_1 = \hat{y}_1$. By $U_q > u_{V1}(\hat{x}_1^P) + u_{V2}(\hat{x}_2^P)$ the veto player must also reject any policy $(x_1, \hat{y}_1)$ where $x_1 > \hat{x}_1^P$. Thus, $x_1 < \hat{x}_1^P$. Solving for λ_1 and rearranging x must also solve

$$\frac{u'_{P2}(\overline{x}(x, \hat{y}_1))}{u'_{V2}(\overline{x}(x, \hat{y}_1))} - \frac{u'_{P1}(x)}{u'_{V1}(x)} = 0, \tag{40}$$

and the same argument as above yields that the equilibrium policy can be efficient for only a measure zero set of parameters $(U_q, \hat{x}_1^V)$. □

Proposition 4. *Assume $u_{V2}(\hat{x}_2^P) < \overline{u}_P$. There exists an open interval $(\underline{U}_q, \overline{U}_q)$, such that, if $U_q \in (\underline{U}_q, \overline{U}_q)$ then $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$.*

Proof. Since $u_{V2}(\hat{x}_2^P) < \overline{u}_P$ the optimal proposal must solve system (21), and the implicit function theorem delivers that we can view solutions (x_1^*, y_1^*) as continuous functions of U_q. For U_q sufficiently large we have $x_1^*(U_q) < \hat{x}_1^P$ and $y_1^*(U_q) > \hat{y}_1$, and for U_q sufficiently small we have $x_1^*(U_q) > \hat{x}_1^P$ and $y_1^*(U_q) < \hat{y}_1$, because the equilibrium policy must be close to V's and P's dynamic ideal points, respectively. Thus, there must exist some U'_q such that $y = \hat{y}_1$. Specifically, let U'_q be the first such value of U_q where $y_1^* = \hat{y}_1$.

First, we show that $U'_q > u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$. Suppose not, so that $U'_q \leq u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$. The veto constraint must bind, otherwise P could choose its unconstrained optimal which sets $y < \hat{y}_1$. Thus, if $y = \hat{y}_1$ then, from our earlier analysis, the equilibrium proposal must satisfy:

$$\frac{u'_{P2}(\overline{x}(x, \hat{y}_1))}{u'_{V2}(\overline{x}(x, \hat{y}_1))} = \frac{u'_{P1}(x)}{u'_{V1}(x)}. \tag{41}$$

Furthermore, because $U'_q \leq u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$ we must have $x > \hat{x}_1^P$, otherwise if $x < \hat{x}_1^P$ then P could profitably deviate to $x = \hat{x}_1^P$, which the veto player would accept. Thus, if $y = \hat{y}_1$ then $x > \hat{x}_1^P$. If $x > \hat{x}_1^P$ then $\frac{u'_{P1}(x)}{u'_{V1}(x)} > 0$. However, $\frac{u'_{P2}(\overline{x}(x,\hat{y}_1))}{u'_{V2}(\overline{x}(x,\hat{y}_1))} < 0$, contradicting that (41) holds.

Therefore, there is some $U'_q > u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}(x_2^*(\hat{x}_1^P, \hat{y}_1))$ such that $y < \hat{y}_1$ for $U_q < U'_q$. Furthermore, for any $U_q \in \Big(u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big), U'_q\Big)$ if $y < \hat{y}_1$ and $x > \hat{x}_1^P$ then it is profitable for V to reject because $U'_q > u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big)$. Thus, $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}_1$ for $\Big(u_{V1}(\hat{x}_1^P) + v(\hat{y}_1) + u_{V2}\big(x_2^*(\hat{x}_1^P, \hat{y}_1)\big), U'_q\Big)$, as claimed. □

B Proofs for Turnover Extension

Proposition 6. *If $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$, then both players' preferences are infected for almost all values of $\rho \in (0,1)$.*

Proof. We consider the case of player P. A similar argument extends the result to player V. The argument proceeds as follows. First, we show that if $(\hat{x}_d^P, \hat{y}_d^P)$ is such that $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \overline{x}_V(\hat{x}_d^P, \hat{y}_d^P)$ or $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \underline{x}(\hat{x}_d^P, \hat{y}_d^P)$ then it must be that $\hat{y}_d^P \neq \hat{y}_1$ for almost all values of ρ. Second, we show that if $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$ and $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^V$ then the only possible solution for player P's preferences to not be infected requires $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = (\hat{x}_1^P, \hat{y}_1)$, however, this solution is not feasible by $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$.

Step 1. We argue that if $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \overline{x}_V(\hat{x}_d^P, \hat{y}_d^P)$ or $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \underline{x}(\hat{x}_d^P, \hat{y}_d^P)$ then $\hat{y}_d^P \neq \hat{y}_1$ for almost all values of ρ. We break the argument into three parts, depending on if (x,y) is such that one of the players can pass $\hat{x}_2^i$ in the second period.

Part 1. Consider (x,y) such that $u_{V2}(\hat{x}_2^P) \leq u_{V2}(x) + v_2(y)$ and $u_{P2}(\hat{x}_2^V) \leq u_{P2}(x) + v_2(y)$. Then if the proposer's dynamic ideal point satisfies these inequalities it must solve:

$$\max_{(x,y)} u_{P1}(x) + v_1(y) + \rho u_{P2}\big(\overline{x}_V(x,y)\big) + (1-\rho)\big(u_{P2}(x) + v_2(y)\big).$$

Letting $a = -x$ and $b = -y$ we can rewrite P's problem as:

$$\max_{(a,b)} u_{P1}(-a) + v_1(-b) + \rho u_{P2}\big(\overline{x}_{V2}(a,b)\big) + (1-\rho)\big(u_{P2}(-a) + v_2(-b)\big).$$

Taking cross-partials of the objective function yields:

$$\frac{\partial^2}{\partial a \partial b} = \rho\Big(\frac{\partial^2 \overline{x}_V}{\partial a \partial b} u'_{P2}\big(\overline{x}_{V2}(-a,-b)\big) + \frac{\partial \overline{x}_V}{\partial a} \cdot \frac{\partial \overline{x}_V}{\partial b} u''_{P2}\big(\overline{x}_V(-a,-b)\big)\Big)$$
$$\frac{\partial^2}{\partial a \partial \rho} = \frac{\partial \overline{x}_V}{\partial a} u'_{P2}\big(\overline{x}_P(-a,-b)\big) + u'_{P2}(-a)$$
$$\frac{\partial^2}{\partial b \partial \rho} = \frac{\partial \overline{x}_V}{\partial b} u'_{P2}\big(\overline{x}_V(-a,-b)\big) + v'_2(-b)$$

We have $\frac{\partial^2 \overline{x}_{V2}}{\partial a \partial b} > 0$, $\frac{\partial \overline{x}_{V2}}{\partial a} < 0$, and $\frac{\partial \overline{x}_{V2}}{\partial b} > 0$, which yields, $\frac{\partial^2}{\partial a \partial b} > 0$ and $\frac{\partial^2}{\partial b \partial \rho} > 0$. Finally, $u'_{P2}(\overline{x}) < u'_{P2}(x) = u'_{P2}(-a)$ and $\frac{\partial \overline{x}_{V2}}{\partial a} \in (-1,0)$, thus $\frac{\partial^2}{\partial a \partial \rho} > 0$. Then the usual results on monotone comparative statics (Milgrom and Shannon, 1994) deliver that y_d^* is monotonic in ρ, and thus $y_d^* = \hat{y}_1$ for at most one value of ρ.

Part 2. Consider (x, y) such that $u_{V2}(\hat{x}_2^P) \geq u_{V2}(x) + v_2(y)$ and $u_{P2}(\hat{x}_2^V) \leq u_{P2}(x) + v_2(y)$. Then if the proposer's dynamic ideal point satisfies these inequalities it must solve:

$$\max_{(x,y)} u_{P1}(x) + v_1(y) + \rho u_{P2}\big(\hat{x}_2^P\big) + (1-\rho)\big(u_{P2}(x) + v_2(y)\big).$$

Thus, such a $(\hat{x}_d^P, \hat{y}_d^P)$ must solve:

$$\begin{aligned} u'_{P1}(x) + (1-\rho)u'_{P2}(x) &= 0 \\ v'_1(y) + (1-\rho)v'_2(y) &= 0, \end{aligned}$$

which clearly cannot be satisfied if $y = \hat{y}_1$ when $v'_2(\hat{y}_1) > v'_1(\hat{y}_1) = 0$ and $\rho \in (0, 1)$.

Part 3. Consider (x, y) such that $u_{V2}(\hat{x}_2^P) \leq u_{V2}(x) + v_2(y)$ and $u_{P2}(\hat{x}_2^V) \geq u_{P2}(x) + v_2(y)$. Then if the proposer's dynamic ideal point satisfies these inequalities it must solve:

$$\max_{(x,y)} u_{P1}(x) + v_1(y) + \rho u_{P2}\big(\overline{x}_V(x, y)\big) + (1-\rho)u_{P2}(\hat{x}_2^V).$$

Thus, such a $(\hat{x}_d^P, \hat{y}_d^P)$ needs to solve:

$$\begin{aligned} u'_{P1}(x) + \rho \frac{u'_{V2}(x)}{u'_{V2}\big(\overline{x}_V(x,y)\big)} u'_{P2}\big(\overline{x}_V(x,y)\big) &= 0 \\ v'_1(y) + \rho \frac{v'_2(y)}{u'_{V2}\big(\overline{x}_V(x,y)\big)} u'_{P2}\big(\overline{x}_V(x,y)\big) &= 0. \end{aligned}$$

Because $\frac{u'_{P2}\left(\overline{x}_V(x,y)\right)}{u'_{V2}\left(\overline{x}_V(x,y)\right)} < 0$ and $\rho \neq 0$, again it is clear that the second equality cannot be satisfied when $y = \hat{y}_1$ if $v'_2(\hat{y}_1) > v'_1(\hat{y}_1) = 0$.

Step 2. By the proof of step 1, if P's preferences are not infected then $(\hat{x}_d^P, \hat{y}_d^P)$ must be such that $x_P^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^P$ and $x_V^*(\hat{x}_d^P, \hat{y}_d^P) = \hat{x}_2^V$. Thus, if P is not infected then $(\hat{x}_d^P, \hat{y}_d^P)$ solves:

$$\max_{x,y} \; u_{P1}(x) + v_1(y) + \rho u_{P2}(\hat{x}_2^P) + (1-\rho)u_{P2}(\hat{x}_2^V) \tag{42}$$

$$\text{s.t.} \; u_{V2}(\hat{x}_2^P) \geq u_{V2}(x) + v_2(y) \tag{43}$$

$$u_{P2}(\hat{x}_2^V) \geq u_{P2}(x) + v_2(y) \tag{44}$$

P's dynamic ideal point $(\hat{x}_d^P, \hat{x}_2^V)$ needs to solve the KKT conditions of this problem, which are given by:

$$u'_{P1}(x) - \lambda_1 u'_{V2}(x) - \lambda_2 u'_{P2}(x) = 0 \tag{45}$$

$$v'_1(y) - \lambda_1 v'_2(y) - \lambda_2 v'_2(y) = 0 \tag{46}$$

$$\lambda_1 \big[u_{V2}(\hat{x}_2^P) - u_{V2}(x) - v_2(y)\big] = 0 \tag{47}$$

$$\lambda_2 \big[u_{P2}(\hat{x}_2^V) - u_{P2}(x) - v_2(y)\big] = 0. \tag{48}$$

First, suppose λ_1 or $\lambda_2 \neq 0$ and $y = \hat{y}_1$. Then equation (46) reduces to $-(\lambda_1 + \lambda_2)v'_2(\hat{y}_1) < 0$. Thus, if P's preference are not infected it must be that neither constraint is binding.

Second, let $\lambda_1 = \lambda_2 = 0$. Then, $(\hat{x}_d^P, \hat{x}_2^V)$ must solve $u'_{P1}(x) = 0$ and $v'_1(y) = 0$ and thus $(\hat{x}_d^P, \hat{x}_2^V) = (\hat{x}_1^P, \hat{y}_1)$. However, by assumption, either: (i) $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$; or (ii) $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$. If $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ holds this immediately yields that $(\hat{x}_1^P, \hat{y}_1)$ is not a feasible solution, specifically, if violates (43). If instead $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$ holds then note that $u_{P2}(\hat{x}_1^V) < u_{P2}(\hat{x}_1^P)$, which implies $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1) < u_{P2}(\hat{x}_1^P) + v_2(\hat{y}_1)$. Hence, $(\hat{x}_1, \hat{y}_1)$ is not feasible as it violates constraint (44). Analogous arguments yield that if $u_{V2}(\hat{x}_2^P) < u_{V2}(\hat{x}_1^P) + v_2(\hat{y}_1)$ or $u_{P2}(\hat{x}_2^V) < u_{P2}(\hat{x}_1^V) + v_2(\hat{y}_1)$ then V's preferences must be infected as well. □

Proposition 7. *Suppose each player i's dynamic ideal point is such that $\overline{x}_V(\hat{x}_d^i, \hat{y}_d^i) < \hat{x}_2^P$ and $\underline{x}_P(\hat{x}_d^i, \hat{y}_d^i) > \hat{x}_2^V$. Then, $\hat{y}_d^P$ is decreasing in ρ and $\hat{y}_d^V$ is increasing in ρ.*

Proof. The result follows from the proof of Step 1 Part 1 of Proposition 6. □

C Proofs for Long-run Outcomes Extension

Proposition 8. *If $\eta < \frac{1}{2}$ then there exists $\hat{t} < T$ such that the equilibrium policy outcome is $x_t^* = \hat{x}_t^P$ and $y_t^* = \hat{y}_t$ in every period $t \geq \hat{t}$. Furthermore, for γ sufficiently large $\hat{t} = 1$.*

Proof. First, we show that if t is sufficiently large then in equilibrium R proposes $x_t^* = \hat{x}_t^P$ and $y_t^* = \hat{y}_t$ whenever the status quo is such that $u_{V_t}(\hat{x}_t^P) + v_t(\hat{y}_t) \geq u_{V_t}(x_t^q) + v_t(y_t^q)$ and V accepts the proposal.

To start, we establish that V's static payoff in period t from getting P's ideal point $(\hat{x}_t^P, \hat{y}_t)$ is greater than its payoff from getting P's ideal point from the previous period $t-1$ $(\hat{x}_{t-1}^P, \hat{y}_{t-1})$ whenever t is sufficiently large. More precisely, we claim that for t sufficiently large:

$$-\big(t^\eta + t^\eta\big)^2 > -\big((t-1)^\eta + t^\eta\big)^2 - \big(\gamma(t-1) - \gamma t\big)^2. \tag{49}$$

To see that (49) holds for t sufficiently large, note that when $\eta < \frac{1}{2}$:

$$\lim_{t\to\infty} -\big(t^\eta + t^\eta\big)^2 + \big((t-1)^\eta + t^\eta\big)^2 = 0,$$

whereas, $-\big(\gamma(t-1) - \gamma t\big)^2 = -\gamma^2 < 0$ for all t. Thus, under the proposed strategies, for t sufficiently large if V accepts P's static ideal point in period t then it is willing to accept P's static ideal point in period $t+1$.

Clearly, P has no incentive to deviate given the proposed strategies. Next, consider V. V is willing to accept the proposal if rejecting today leads to P making the same form of proposal tomorrow, since given the strategies V accepts which thus yields the same dynamic utility to V and by construction it is statically optimal to accept. Thus, we need to show that if t is sufficiently large and $u_{V_t}(\hat{x}_t^P) + v_t(\hat{y}_t) \geq u_{V_t}(x_t^q) + v_t(y_t^q)$ then $u_{V_{t+1}}(\hat{x}_{t+1}^P) + v_{t+1}(\hat{y}_{t+1}) \geq u_{V_{t+1}}(x_t^q) + v_{t+1}(y_t^q)$.

Writing the condition as $u_{V_t}(\hat{x}_t^P) + v_t(\hat{y}_t) - u_{V_t}(x_t^q) - v_t(y_t^q) \geq 0$ we have that the same condition will hold in the next period if the LHS of the condition is increasing in t. Differentiating yields

$$2\eta t^{n-1}(x^q + t^n) - 2\gamma(y^q - \gamma t) - 8\eta t^{2n-1}.$$

Rearranging this is positive if and only if

$$y^q < \frac{1}{\gamma}\eta t^{\eta-1}\Big[x^q - 3t^\eta\Big] + \gamma t \tag{50}$$

As $t \to \infty$ the RHS of (50) goes to infinity when $\eta < \frac{1}{2}$ and $\gamma > 0$. Thus, as $y^q \leq \gamma T$ for all t there exists a t' such that for all $t \geq t'$ the statement holds.

This implies that P's unconstrained optimal policy in any period $t \geq t'$ is $(\hat{x}_t^P, \hat{y}^t)$. Thus, if P does not get its unconstrained optimal in a period then P must be choosing (x, y) to make V indifferent between accepting and rejecting. Therefore, if there is an equilibrium such that for all t party P does not propose $(\hat{x}_t^P, \hat{y}_t)$ then V's dynamic payoff is given by:

$$\sum_{t=0}^{\infty} u_{V_t}(x_1^q) + v_t(y_1^q).$$

However, by $\eta < 1/2$ there is some period t'' such that starting in t'' the dynamic payoff to V from getting $(\hat{x}_t^P, \hat{y}_t)$ every period is greater than this dynamic payoff from the initial status quo, which contradicts that this is an equilibrium.

Finally, note that for γ sufficiently large (50) holds for $t = 1$. □

Proposition 9. *Assume $\eta > 1$. There exists $\tilde{t} < T$ such that $y_t^* \neq \hat{y}_t$ in every period $t \in \{\tilde{t}, ..., T-1\}$. If γ is sufficiently small then $\tilde{t} = 1$.*

Proof. We break the argument into several steps.

Step 1. In Step 1 we argue that if T is sufficiently large then at $T-1$ the policy outcome must be inefficient. If $\eta > \frac{1}{2}$ then $\lim_{T\to\infty} -(T^\eta + T^\eta)^2 + \big((T-1)^\eta + T^\eta\big)^2 = -\infty$, while $-\big(\gamma(T-1) - \gamma T\big)^2 = -\gamma^2$ for all T. Therefore, $u_{VT}(\hat{x}_T^P) < u_{VT}(\hat{x}_{T-1}^P) + v_T(\hat{y}_{T-1})$ and $y_T^* \neq \hat{y}_T$ by Propositions 2 and 3.

Step 2. Now, we characterize the continuation payoffs beginning in period $T-1$. Note that P's optimal proposal will be constrained in period T. Thus, if P is constrained in $T-1$, $u_{VT-1}(\hat{x}_d^P) + v_{T-1}(\hat{y}_d^P) + u_{VT}(\hat{x}_d^P) + v_T(\hat{y}_d^P) \leq u_{VT-1}(\hat{x}^q) + v_{T-1}(\hat{y}^q) + u_{VT}(\hat{x}^q) + v_T(\hat{y}^q)$ then $\frac{\partial w_{T-1}^V}{\partial y^q} = v'_{T-1}(y^q) + v'_T(y^q)$ and $\frac{\partial w_{T-1}^V}{\partial x^q} = u'_{VT-1}(x^q) + u'_{VT}(x^q)$. On the other hand, if P is unconstrained then both of these derivatives are 0. Furthermore, the envelope theorem delivers that $w_{T-1}^P(x^q, y^q)$ is differentiable almost everywhere in x^q and y^q, with

$$\frac{\partial w_{T-1}^P}{\partial y^q} = \begin{cases} -\lambda_{T-1}^* \frac{\partial w_{T-1}^V}{\partial y^q} & \text{if P is constrained,} \\ 0 & \text{otherwise.} \end{cases}$$

$$\frac{\partial w_{T-1}^P}{\partial x^q} = \begin{cases} -\lambda_{T-1}^* \frac{\partial w_{T-1}^V}{\partial x^q} & \text{if P is constrained,} \\ 0 & \text{otherwise.} \end{cases}$$

Step 3. We now show that if the period t equilibrium policy is inefficient and continuation payoffs have the analogous properties to those characterized for the $T-1$ case in Step 2, then the equilibrium policy in period $t-1$ is inefficient and continuation payoffs have the same form as in $T-1$. The induction argument together with Steps 1 and 2 as the base case then delivers the proposition.

Suppose $y_t^* \neq \hat{y}_t$ and the derivatives for each player's continuation payoffs $w_t^V(x_t^q, y_t^q)$ and

$w_t^P(x_t^q, y_t^q)$ have the same form as at $T-1$. In period $t-1$ P's optimal proposal solves:

$$\max_{(x,y)} \; u_{Pt-1}(x) + v_{t-1}(y) + w_t^P(x,y)$$
$$\text{s.t. } u_{Vt-1}(x) + v_{t-1}(y) + w_t^V(x,y) \geq u_{Vt-1}(x_{t-1}^q) + v_{t-1}(y_{t-1}^q) + w_t^V(x_{t-1}^q, y_{t-1}^q).$$

We break the argument into two parts, depending on whether the veto player's constraint is binding at time $t-1$.

If P chooses a policy where the constraint does not bind then (x_{t-1}^*, y_{t-1}^*) solves:

$$u'_{Pt-1}(x) + \frac{\partial w_t^P}{\partial x} = 0 \tag{51}$$
$$v'_{t-1}(y) + \frac{\partial w_t^P}{\partial y} = 0. \tag{52}$$

Thus, for $\hat{y}_{t-1}$ to be optimal requires that $\frac{\partial w_t^P}{\partial y}|_{y=\hat{y}_{t-1}} = 0$ which implies that P is also unconstrained at time t. If P is unconstrained at time t then $\frac{\partial w_t^P}{\partial x} = 0$, and for (51) to also hold requires $x_{t-1}^* = \hat{x}_{t-1}^P$. However, this contradicts that P is unconstrained at period t, as P's unconstrained optimal policy at time t sets $x_t^* > \hat{x}_t^P > \hat{x}^P$ and $y_t^* > \hat{y}_t > \hat{y}_{t-1}$. Thus, at time t accepting such a (x_t^*, y_t^*) is strictly worse for V than rejecting to keep $(\hat{x}_t^P, \hat{y}_{t-1})$.

Next, suppose that P chooses a policy when the veto constraint is binding. Then $(x_{t-1}^*, y_{t-1}^*, \lambda_{t-1}^*)$ solves:

$$u'_{Pt-1}(x) + \frac{\partial w_t^P}{\partial x} + \lambda\Big[u'_{Vt-1}(x) + \frac{\partial w_t^V}{\partial x}\Big] = 0 \tag{53}$$
$$v'_{t-1}(y) + \frac{\partial w_t^P}{\partial y} + \lambda\Big[v'_{t-1}(y) + \frac{\partial w_t^V}{\partial y}\Big] = 0 \tag{54}$$
$$\lambda\Big[u_{Vt-1}(x) + v_{t-1}(y) + w_t^V(x,y) - u_{Vt-1}(x_{t-1}^q) - v_{t-1}(y_{t-1}^q) - w_t^V(x_{t-1}^q, y_{t-1}^q)\Big] = 0 \tag{55}$$

Since the constraint is binding we have $\lambda > 0$. Solving for λ and rearranging, for (53)-(55) to hold at $y = \hat{y}_{t-1}$ requires that x_{t-1}^* satisfies:

$$\frac{\partial w_t^V}{\partial y}\Big|_{y=\hat{y}_{t-1}}\Big(u'_{Pt-1}(x) + \frac{\partial w_t^P}{\partial x}\Big) - \frac{\partial w_t^P}{\partial y}\Big|_{y=\hat{y}_{t-1}}\Big(u'_{Vt-1}(x) + \frac{\partial w_t^V}{\partial x}\Big) = 0 \tag{56}$$
$$u_{Vt-1}(x) + w_t^V(x, \hat{y}_{t-1}) = u_{Vt-1}(x_{t-1}^q) + v_{t-1}(y_{t-1}^q) + w_t^V(x_{t-1}^q, y_{t-1}^q). \tag{57}$$

The LHS of (57) is decreasing in x, thus there is at most one x which solves it. Additionally, because u is quadratic, the LHS of (56) is linear in x. However, the first equality does not depend on the status quo, and thus any perturbation of the initial status quo only changes the solution to (57) and there cannot be an x^*_{t-1} that satisfies (56) and (57) for almost all (x^q_1, y^q_1).

Finally, note that if the constraint is not binding, $u_{Vt-1}(x^*_{t-1})+v_{t-1}(y^*_{t-1})+w^V_t(x^*_{t-1}, y^*_{t-1}) > u_{Vt-1}(x^q_{t-1}) + v_{t-1}(y^q_{t-1}) + w^V_t(x^q_{t-1}, y^q_{t-1})$, then $\frac{\partial w^V_{t-1}}{\partial y^q_{t-1}} = \frac{\partial w^V_{t-1}}{\partial x^q_{t-1}} = 0$ and the envelope theorem delivers that also $\frac{\partial w^P_{t-1}}{\partial y^q_{t-1}} = \frac{\partial w^P_{t-1}}{\partial x^q_{t-1}} = 0$. Instead, if the constraint is binding then $w_{t-1}(x^q_{t-1}, y^q_{t-1}) = u_{Vt-1}(x^q_{t-1}) + v_{t-1}(y^q_{t-1}) + w^V_t(x^q_{t-1}, y^q_{t-1})$. Therefore, $\frac{\partial w^V_{t-1}}{\partial y^q_{t-1}} = v'_{t-1}(y^q_{t-1}) + \frac{\partial w^V_t}{\partial y^q_{t-1}}$ and $\frac{\partial w^V_{t-1}}{\partial x^q_{t-1}} = u'_{Vt-1}(y^q_{t-1}) + \frac{\partial w^V_t}{\partial x^q_{t-1}}$. Again the envelope theorem yields $\frac{\partial w^P_{t-1}}{\partial x^y_{t-1}} = -\lambda^* \frac{\partial w^V_{t-1}}{\partial x^q_{t-1}}$ and $\frac{\partial w^P_{t-1}}{\partial x^q_{t-1}} = -\lambda^* \frac{\partial w^V_{t-1}}{\partial x^q_{t-1}}$. Thus, the derivatives of the continuation payoffs at $t-1$ have the desired form as well.

□

D Different Weights on Dimensions

Here we consider the baseline model where $v(y) \equiv v_1(y) = v_2(y)$, so there is no change in preferences on the Y dimension, but we assume that the two players put different weights on the two dimensions. Specifically, assume that the proposer's stage utility is

$$u_{Pt}(x) + \theta v(y),$$

with $\theta > 0$. As in the baseline, the veto player's stage utility is instead

$$u_{Vt}(x) + v(y).$$

Further, assume that $\hat{y}_1 = \hat{y}_2 = \hat{y}$, so that the optimal policy on the Y dimension remains constant across periods.

Here, we show that a policy pair such that $x^*_1 < \hat{x}^P_1$ and $y^*_1 < \hat{y}$ can never be sustained in equilibrium. Clearly, if U_q is such that the proposer is unconstrained in the first period, the equilibrium policy must satisfy $x^*_1 > \hat{x}^P_1$.

Suppose instead that U_q is sufficiently high that the proposer is constrained in the first period. To establish a contradiction, suppose that $x^*_1 < \hat{x}^P_1$, $y^*_1 < \hat{y}$ is an equilibrium. Recall that $\bar{x}(x^*_1, y^*_1)$ solves $u_{V2}(x) = u_{V2}(x^*_1)+v_2(y^*_1)$. Further, (x^*_1, y^*) must solve $u_{V1}(x^*_1)+v(y^*_1)+u_{V2}(x^*_1)+v(y^*_1) = U_q$. Let $\tilde{x}$ denote the policy that solves $u_{V1}(x)+u_{V2}(x) = U_q$. Suppose that $u_{V2}(\tilde{x}) \le u_{V2}(x^*_1) + v(y^*_1)$. Then, it must be the case that a deviation to $(\tilde{x}, \hat{y})$ is profitable for the proposer, as this bundle is passable by definition and improves the proposer's payoff if this inequality is satisfied. Suppose instead $u_{V2}(\tilde{x}) > u_{V2}(x^*_1) + v(y^*_1)$. Then, the above equations imply that $u_{V1}(\tilde{x}) < u_{V1}(x^*_1) + v(y^*_1)$, otherwise $u_{V2}(\tilde{x}) > u_{V2}(x^*_1) + v(y^*_1)$ and

$u_{V1}(\tilde{x}) + u_{V2}(\tilde{x}) = U_q$ would imply $u_{V1}(x_1^*) + v(y_1^*) + u_{V2}(x_1^*) + v(y_1^*) > U_q$. Therefore, given concavity and the assumption that $\hat{x}_{V2} \leq \hat{x}_{V1}$, the following holds: $-v(y_1^*) < u_{V1}(x_1^*) - u_{V1}(\tilde{x}) < u_{V2}(x_1^*) - u_{V2}(\tilde{x})$. Thus, $u_{V2}(\tilde{x}) < u_{V2}(x_1^*) + v(y_1^*)$, a contradiction.

Finally, note that, as in Acharya and Ortner (2013) and Lee (2020), the proposer may still want to implement an inefficient policy on the common-values dimension in this setting, $\hat{y}_d^P < \hat{y}_1$. To see this, consider the first-order conditions that $(\hat{x}_d^P, \hat{y}_d^P)$ must solve, assuming $\overline{x}(\hat{x}_1^P, \hat{y}_1) < \hat{x}_2^P$ so that P cannot just achieve its ideal point both periods:

$$u'_{P1}(x) + \frac{u'_{P2}\big(\overline{x}(x,y)\big)}{u'_{V2}\big(\overline{x}(x,y)\big)} u'_{V2}(x) = 0 \tag{58}$$

$$\theta v'(y) + \frac{u'_{P2}\big(\overline{x}(x,y)\big)}{u'_{V2}\big(\overline{x}(x,y)\big)} v'(y) = 0 \tag{59}$$

There is always a solution to these necessary first-order conditions where $y = \hat{y}$ and x solves $u'_{P1}(x) + u'_{P2}(x)$. Indeed, if θ is sufficiently large this solution does maximize P's payoff, as maintaining inefficiency for tomorrow is costly. However, when θ is sufficiently small P weights the costs of inefficiency less than V and $y < \hat{y}$ can instead be better. In this case, from condition (59) we require that $\theta = -\frac{u'_{P2}\left(\overline{x}(x,y)\right)}{u'_{V2}\left(\overline{x}(x,y)\right)}$. When θ is small this implies that x and y are such that $\overline{x}(x,y)$ is close to $\hat{x}_2^P$. Additionally, this condition together with (58) implies that the policy on the X dimension solves $u'_{P1}(x) - \theta u'_{v2}(x) = 0$, and hence x is close to $\hat{x}_1^P$ for θ small. Thus, for θ sufficiently low the inefficient solution yields a better first-period policy on X, a better second-period policy on X, and the cost from the inefficiency is relatively negligible.

For example, suppose $u_{it}(x) = -(x - \hat{x})^2$, with $\hat{x}_1^P = -\hat{x}_1^V = 1$ and $\hat{x}_2^P = -\hat{x}_2^v = 2$. Additionally, let $v(y) = -(y-1)^2$ and $\theta = \frac{1}{8}$. Then $(\hat{x}_d^P, \hat{y}_d^P) \approx (1.43, .058)$ which gives a dynamic payoff of $\approx -.49$, versus the best efficient policy $(x, y) = (1.5, 1)$ which yields $-.5$.

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