

# Policy Anchors, Voter Behavior, and the Dynamics of Party Platforms\*

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## Abstract

Voters are neither omniscient information processors nor perfectly rational decision makers. This does not mean they are irrational. Voting is a difficult problem, and even well-meaning voters must use shortcuts and heuristics to make decisions. In this paper we develop a model of voting with policy anchors. To evaluate a candidate, voters use both the current party platform and past positions. We show the use of policy anchors leads to two types of preference reversals in voting, within and across elections. We then incorporate policy anchors into a canonical model of electoral competition and show that anchors fundamentally change policy outcomes, with platforms evolving continuously where they otherwise would be largely stable. Moreover, this movement occurs in asymmetric cycles—either large jumps inward followed by incremental movement outward, or vice versa. We connect these results to broader questions about voter welfare and political representation, and observed patterns in platform dynamics.

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# 1 Introduction

A half century of research into political behavior has established all too clearly that voters fall short of any notion of a democratic ideal. Voters are neither omniscient information processors nor perfectly rational decision makers.

It does not follow from these shortcomings, however, that voters are irrational or, worse still, indifferent to political outcomes. The problem voters face is a difficult one, and given the cognitive limitations of the human mind, it is to be expected that even well-meaning decision makers must adopt a myriad of shortcuts and rules-of-thumb in the pursuit of effective decision making (Simon, 1955).

Voting is a particularly difficult problem because the policy space is *scale free*. What does it mean to an average voter, for example, if Republicans offer a tighter immigration policy? How much should the voter value this shift? In particular, how does she compare this policy that may lie to her right with the Democrats' position that is on her left? Relative to voting, choice problems in other domains are easy. Consumer goods, for instance, come with a price that allows direct comparison to other goods. In contrast, policy positions stand alone, with no natural metric for evaluation, making comparison difficult.

We propose that voters make sense of their decision by using past policies as guides. Specifically, that voters evaluate and respond to policy positions based, in part, upon policies that have been offered by parties in earlier elections. A moderate policy position from the Democrats will be more appealing to a centrist voter if the Democrats' previous position was more extreme, and vice versa. To provide a concrete example, the "compassionate conservatism" of George W. Bush was attractive to voters as it provided a positive contrast to the sharper-edged conservatism of an earlier era. The new policy offering can be seen by the voter as more moderate because relative to past policies it is. In this way, past policy positions of the parties serve as *anchors* in the voting decision.

We first formalize a theory of voting with policy anchors. We show that policy anchors lead to two distinct types of preference reversals relative to the standard model of spatial voting. First, *within* an election, a voter using a policy anchor may vote for the party that is more distant from her in the policy space, even though, all else equal, she prefers policies that are closer to her ideal point. Second, *across* elections, a voter may switch her preference over the same pair of policy positions from one election to the next. Although both preference reversals suggest that standard spatial voting is violated, we show that both types of behavior are natural consequences of spatial voting when voters evaluate positions with policy anchors.

We then explore how policy anchors shape politics more broadly. We ask two questions: How do parties respond strategically to voters using policy anchors? And what does this mean for policy outcomes? We build a model of electoral competition to answer these questions. The

model we build incorporates policy anchors into the classic model of two party competition with uncertainty of Wittman (1983) and Calvert (1985) while extending it dynamically to an arbitrary number of elections.

Anchoring by voters changes the incentives of the parties. Because a party will appeal more to voters if it moves toward the center from one election to the next, policy anchoring gives the parties an extra incentive to converge. The strength of this incentive depends on the location of the anchor itself, and we identify a non-monotonicity in how equilibrium platforms respond to the anchor.

A key insight of the model is that the nature of the policy response depends crucially on the shape of the voters' anchoring function. If voters evaluate a large change in position from an anchor as more significant than a series of small changes, the equilibrium platforms are single-peaked in the anchor. This means that centrist platforms emerge from centrist and extreme anchors, whereas moderate anchors lead to the most extreme platforms. This relationship flips, and equilibrium platforms are single-dipped in the anchor, when a series of small changes in position is relatively more important to voters than one large change.

We show that this asymmetry in response to anchors leads, over time, to cycles in policy platforms, and that these cycles are necessarily asymmetric. Over time, the parties either creep in incrementally toward the median voter before making a large jump outward, or the reverse, making a large jump inward followed by incremental deviations outward.

It is, in a sense, surprising that the parties would ever move *away* from the median voter when voters are anchoring. A shift away from the median voter makes a party *less* attractive to voters than in a standard environment without anchoring. The shift itself is not surprising as when the anchor is centrist, the policy benefit of shifting outward outweighs the anchor cost. The parties shift outward despite, not because of, voter anchoring.

The surprising part is that the anchor can be centrist in the first place given it must have been the policy platform in the previous election. How can it be worthwhile for the parties to shift so far to the center in one election if it is valuable only to shift back outward in the next election? The answer to this question is the reason why party platforms cycle in equilibrium. The parties have an incentive to converge given an initial extreme anchor, but once they do, their anchors reset at the new centrist locations and the benefit of converging disappears. From there, the parties have an incentive to shift back outward to improve the policy outcome. In a sense, the parties overshoot in their response to policy anchors, and they do so because voters evaluate small versus large movements relatively differently. Cycling does not depend on whether small or large changes are more important, only that they are evaluated differently. This differential treatment induces, in effect, a time-inconsistency in the incentives of the political parties and this drives cycles in their policy platforms.

Across time, these incentives lead to rich dynamics and a policy trajectory that is highly

path dependent. We show that anchoring creates much more policy change than otherwise would arise. Specifically, for an environment that without anchoring would exhibit large periods of stability and only periodic change, we show that with anchoring the party platforms are instead in a constant state of change, with policy flowing back and forth, mixing incremental shifts with larger jumps. This pattern of voter and party positioning offers a rationale for the finding that much party movement in the ideological space is detached from changes in voters’ policy opinions (Page and Shapiro, 1992; Budge et al., 2010; Druckman and Leeper, 2012).<sup>1</sup>

Underlying the effects of voter anchoring are deep questions about welfare as well as measurement. Do the movements in party platforms make voters better off if voters’ ideal policies are fixed and unchanging? Should we even measure voter welfare by these ideal points—what behavioral economists would refer to as voters’ *true* or *fundamental* preferences—or should welfare include the anchoring effect, the *decision utility* voters actually experience when casting their votes?

This question surfaces a fundamental ambiguity in the notion of political representation. Representative democracy requires that parties *represent* the preferences of voters. If we take voters’ true preferences as the standard, then the constant shifting of party platforms in our model implies a failure of political representation. But the party movements are in response to the voters’ anchoring—their decision utility from voting—and, in that sense, are a pure instantiation of political representation. Our paper offers a behavioral perspective on the role of representation and whether policymakers should act as delegates or trustees (see, e.g., Pitkin, 1967). Our theory emphasizes the importance of voters’ cognitive limitations to this question rather than their lack of expertise, which is the focus of the existing literature. We return to this and other thorny questions after presenting the formal analysis.

## Policy Anchors and Constructed Preference

A long-standing perspective in the psychology of choice is that preferences are constructed rather than inherent (Lichtenstein and Slovic, 2006). That is, a decision maker, in facing a choice set, does not consult a list of utility scores for each alternative and select the highest one. Rather, she constructs her preference from the choice set through comparison to other features of the environment. Such context-dependent choice can be influenced by the choice set itself, past choice sets, or other external features of the environment.

Voting is a natural domain for preference to be constructed rather than inherent. In

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<sup>1</sup>The extent of voter movement is an ongoing debate in the academic literature (e.g., Boxell et al., 2024), although a consensus does hold that voters have shifted ideological ground *less* than have elites. Importantly, our model highlights a distinction between the location of the voters’ ideal points and their preferences including anchoring. Here, our results resonate with observed patterns insofar as surveys of public opinion measure the former, which remain fixed over time in our model, and not the latter, which evolve endogenously. The next paragraph delves into the implications of this distinction for voter welfare.

addition to the scale-free structure of the policy space, the vote decision is extraordinarily broad, covering many distinct issue domains, with information that is dispersed and often hard to integrate. Politics is difficult to comprehend, even for experts. On top of this, voters are called upon to vote only at widely spaced intervals and receive minimal feedback on the quality of the decisions they make. It is natural, therefore, that voters lack well-tuned preferences and that they instead construct their preference from the environment as they go.

Our focus is on how past choice sets—the platforms parties offered in previous elections—influence the vote decision, what Tversky and Simonson (1993) refer to as the *background context*. The foundational premise of our theory is that a party’s platform will appeal more to a voter if it is closer to the voter than was the party’s platform in the last election. This premise underlies psychological phenomena in a variety of domains, from the Weber-Fechner Law of the 19<sup>th</sup> century (Thaler, 1980), to “range effects” (Parducci, 1965), and context-dependent models of choice (Tversky and Simonson, 1993).

In the context of politics, this premise is validated *within* a single election by Simonovits (2017) using a survey experiment based on US elections.<sup>2</sup> He demonstrates how the addition of an extreme candidate on one side of the spectrum makes the more moderate candidate from the same side more appealing.<sup>3</sup> In fact, when asked to place the moderate conservative on the policy space, subjects exposed to the extreme conservative positioned the moderate as more centrist. Similarly, exposure to an extreme liberal candidate induced respondents to place the moderate liberal candidate closer to the center of the spectrum.<sup>4</sup> In effect, the voters react to the presence of the extreme conservative by rescaling the right side of the policy space to fit him in, and to the extreme liberal by rescaling the left side. Notably, this rescaling is directional in that a moderate liberal candidate is immune from comparison to an extreme conservative on the other side of the political landscape, and vice versa. We theorize that this directional effect also applies *across* elections through comparison of parties’ past and present platforms.

In what follows, we develop a formal representation of the vote decision when voters use policy anchors to evaluate party platforms. It is important to note that the mathematical formulation is meant to *represent* voter preference rather than be a literal description of how a voter forms her preference. Following the method of mathematical psychology, it is not that the voter literally calculates direct and indirect evaluations of the party and then sums them together to arrive at her utility. Rather, it is that her behavior in practice can be represented,

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<sup>2</sup>See also Waismel-Manor and Simonovits (2017) and Wang and Chen (2019) for related evidence from Israel and Taiwan, respectively.

<sup>3</sup>Callander and Wilson (2006) provide observational evidence that the choice set of candidates within an election affects the turnout decision of voters.

<sup>4</sup>Rotter and Rotter (1966) provide similar experimental evidence, albeit with dated methods, on the 1964 U.S. presidential election examining the effect of Wallace on Goldwater’s vote total.

or described, by a particular utility function.

### **Other Related Ideas and Literature**

The idea that the external political environment plays an important role in shaping political preferences has a long history in political science (see Druckman and Lupia (2000) for a review). Numerous studies have documented effects on behavior that are difficult to square with the classic *homo economicus* decision making model. However, as emphasized by Druckman and Lupia (2000), this does not mean that voters do not hold preferences nor that those preferences have some form of invariant structure.

Our theory of policy anchors is not the only reason why voters might look to past policy platforms to evaluate a candidate. For example, a reputational theory of candidate positioning might suggest that a candidate’s true policy position is a weighted average of the positions he has held. Such a theory would lead to an opposite effect to what drives our theory, namely that a candidate’s platform today is less appealing if his platform yesterday was worse. Such a reputational approach represents a distinct mechanism and fundamentally different underlying psychological process that is best grounded in rational belief updating. The theories also differ in other ways. For instance, voter anchoring is not candidate specific as a reputational theory would be. Indeed, consistent with the evidence of Simonovits (2017) and the large psychological literature, the comparisons that drive anchoring do not need to come from the same person or entity, and need only be in the same direction as the platform under consideration.<sup>5</sup>

Our theory of voting is also conceptually distinct from theories of retrospective voting in which voters also look backward in making their vote choice. Underlying retrospective voting is a model of political accountability. As such retrospective voters look to past performance for guidance and not past policy platforms, as do voters in our model. Moreover, past performance is the guiding determinant of vote choice, whereas in our theory it conditions evaluation of the present platform, which is determinative of vote choice

Anchoring appears in many decision contexts and in a variety of forms. In political science, Arceneaux and Nicholson (2024) explore its implications for preferences over specific policy proposals across a variety of issue domains. As they argue, “To revisit tax policy, liberals may prefer higher rates than conservatives, but what counts as a high or low rate is largely arbitrary, set by an initial value (the status quo or the opening gambit of an agenda setter).” (page 754) Interestingly, and in contrast to evidence from psychology and economics, they find no evidence in politics that arbitrary or irrelevant anchors shape decision making. This accords with our theory that it is the past platform of a party that shapes evaluation of the current platforms and not extraneous political information.

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<sup>5</sup>To make the distinction even clearer, our theory can also apply to voting on ballot measures without an individual or entity attached.

Our paper fits within a broader but still relatively small literature that may be described as behavioral formal theory. Specifically, we fit within the niche of behavioral work that amends the utility function of voters to capture decision making more realistically and that then explore how this changes the incentives of party elites. Closest to our model is Callander and Wilson (2006) that shows how the entry and exit of parties within a single election affects evaluation of the remaining candidates. In contrast, we study two party elections and choice set effects across elections; specifically, how the background context of past elections affects evaluations in the current election.<sup>6</sup>

Our results on the dynamics of policy platforms connects with the important literature on path dependence (Pierson, 2000). Page (2006) points out that path dependence, as used in the literature, is not a single concept and can take several different forms. This is the case in our model as well. Following Page’s (2006) terminology, the cycles we identify in policy platforms are *path*-dependent but are not *outcome*-dependent. This means that the outcome in any election depends on the history—specifically, on the policy platforms offered by the parties in the previous election—but that the distribution of outcomes over the long-run is independent of the history.

## 2 Policy Anchors and Voter Behavior

In this section, we introduce policy anchors to the spatial theory of voting. We then explore the implications of anchors for voting behavior.

### 2.1 A Theory of Voting with Anchors

We consider a standard spatial voting environment. Throughout, we take the policy space to be single dimensional and given by  $[-1, 1]$ , with a generic policy platform at time  $t$  given by  $p_t$ .

Voters possess ideal points in  $[-1, 1]$  and evaluate policies spatially. For now, consider a single voter with an ideal policy of  $x \in [-1, 1]$  who at time  $t$  is forming an opinion of a policy platform to the right of her ideal point, which we denote  $r_t > x$ . Her evaluation depends on both the direct distance of the platform to her ideal policy *and* how that platform looks relative to the previous policy offering to the right of her ideal point,  $r_{t-1} > x$ . Likewise, when evaluating a policy to the left of her ideal point, denoted  $d_t < x$ , she compares it to the previous platform that was offered to her left,  $d_{t-1} < x$ . The policies  $r_{t-1}$  and  $d_{t-1}$  are the

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<sup>6</sup>See also Callander and Wilson (2008). The focus on choice set effects distinguishes our work from applications of prospect theory to politics that focus on loss aversion and the asymmetric treatment of gains and losses of policy outcomes; see Grillo (2016); Alesina and Passarelli (2019); Lockwood and Rockey (2020), and Grillo and Prato (2023).

*anchors* that the voter uses to help her evaluate platforms and construct her preferences over policies.

The relative distance of the policy platform  $p_t \in \{d_t, r_t\}$  is captured by the following:

$$\Delta p_t = |p_{t-1} - x| - |p_t - x|.$$

When the voter considers a platform and anchor to the right of her ideal policy, this relative distance is given by  $\Delta r_t = r_{t-1} - r_t$ , while if the platform is to the left of  $x$  it reduces to  $\Delta d_t = d_t - d_{t-1}$ . Thus, a positive value of  $\Delta p_t$  means the policy is relatively more appealing to the voter than it was previously, and a negative value that it is less appealing.<sup>7</sup>

The voter's preference combines this intertemporal evaluation with a direct evaluation of the party, and can be represented by the following utility function:

$$u_x(p_t|p_{t-1}) = \underbrace{-|p_t - x|}_{\text{direct effect}} + \underbrace{f(\Delta p_t)}_{\text{indirect effect}}, \quad (1)$$

The direct component of utility follows preference in the standard way, with platforms closer to the voter's ideal policy receiving higher utility than those further away. We assume this component is linear in policy to highlight clearly the interaction with the indirect component.

How the indirect evaluation impacts utility depends on the shape of the function,  $f$ . We assume  $f$  is twice-differentiable. Consistent with our theory, we take  $f$  to be strictly increasing in  $\Delta p_t$ ,  $f'(\Delta p_t) > 0$ , so that relative movements away from the voter are viewed unfavorably. We also assume that the effect of the anchor is neutral in the following sense:

$$f(0) = 0, \quad \text{and} \quad f(a) = -f(-a). \quad (2)$$

Thus, if the party platform does not shift, the indirect effect is zero, and the boost from shifting toward the voter is exactly matched by an equal shift away. Our results do not rely on this symmetry. We impose it for clarity and to highlight that the asymmetry in gains versus losses that drives prospect theory is not necessary for our results.

We will also focus our analysis on two particular classes of functions given by the following definition.

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<sup>7</sup>The dependence on  $\Delta p_t$  implies that the effect of a movement of certain size is independent of the distance the anchor is from the voter. This can be generalized to allow the absolute distance of the platforms from the voter to impact the evaluation through the use of *skew-symmetric* functions (Fishburn, 1982).



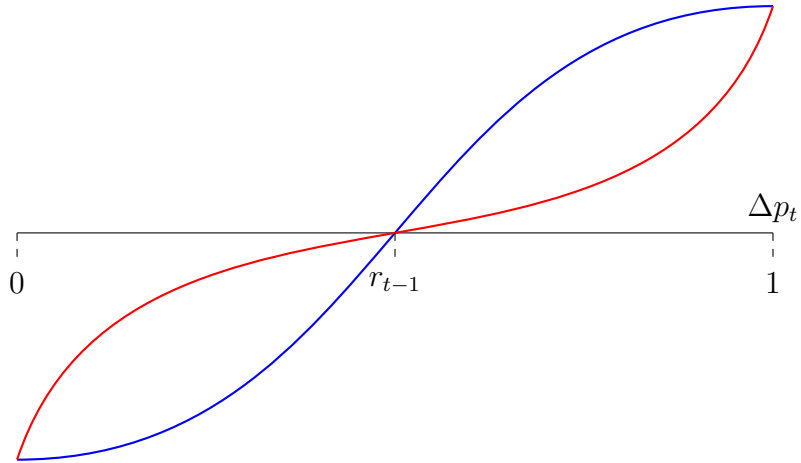


Figure 1: Anchor Functions: s-shaped (blue) and reflected s-shaped (red)

**Definition 1**

- $f$  is s-shaped if  $f''(\Delta p_t) > 0$  for  $\Delta p_t < 0$  and  $f''(\Delta p_t) < 0$  for  $\Delta p_t > 0$ .
- $f$  is reflected s-shaped if  $f''(\Delta p_t) < 0$  for  $\Delta p_t < 0$  and  $f''(\Delta p_t) > 0$  for  $\Delta p_t > 0$ .

The two forms of the  $f(\cdot)$  function differ in how voters evaluate small versus large changes. For the reflected s-shape, the voter views larger changes as relatively more significant than smaller changes. For example, compare two policy shifts toward the voter’s ideal point, if one shift is twice as large as the other, the voter views the larger shift more than twice as favorably as the smaller shift. For movements away from the voter, the same relative evaluation holds, although in terms of costs rather than benefits, with the larger jump more than twice as costly as the smaller step. These relative evaluations are reversed for an s-shape anchoring function. Figure 1 depicts examples of both shapes, with an s-shape anchoring function in blue and a reflected s-shape in red.

**Remark.** Formalizing the use of policy anchors requires us to make assumptions on how they are used. In our theory, voter anchoring is direction-specific. Voters compare today’s platform to her left to yesterday’s platform that was to her left, and not to a polity-wide variable such as the last election’s outcome, or an individual-specific variable such as a person’s current wealth level. As discussed in the introduction, this approach is consistent with the direction-specific contextual evaluations found by Simonovits (2017). For simplicity, anchoring is only to the platform from the previous election rather than the full policy history. Although one can imagine the effect of long-past platforms lingering in voters’ memories, recency bias points toward the most recent platform being the dominant anchor point. In Section 5 we discuss the implications of allowing the voters to have a longer memory.

## 2.2 Implications for Voting Behavior

Policy anchors do not upset the basic principle of spatial voting that, all else equal, a policy platform closer to the voter is more favorable than a platform further away. Furthermore, they do not overturn the standard single-crossing property, i.e., policy preferences are well-ordered across voters. Lemma 1 formalizes this property.

**Lemma 1**  $u_x(p_t|p_{t-1})$  is single-peaked in  $p_t$ , with unique maximizer at the voter’s ideal policy,  $x$ . Moreover, it satisfies the strict single-crossing property in  $(x, p_t)$ .

Policy anchors do, however, violate the standard implication of spatial voting that the voter votes for the policy platform that is closest to her ideal. To see this violation, consider the situation depicted in Figure 2 of a voter deciding between two parties,  $D$  and  $R$ . Party  $R$ ’s platform is closer to the voter’s ideal policy at  $x$  than is party  $D$ ’s. However, party  $R$  has not moved from its platform the previous election whereas party  $D$  has moved toward the voter.

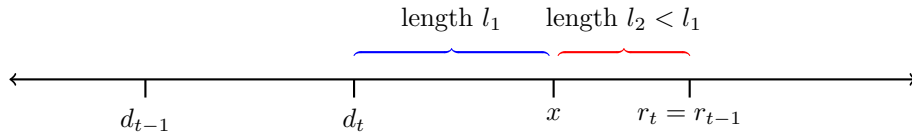


Figure 2: Within-Election Preference Reversal

Party  $D$ ’s shift has made it more appealing to the voter. If the boost from the shift is significant enough, it can induce the voter to support  $D$  over  $R$  despite the greater distance of  $D$ ’s platform in this election. Such a switch represents a preference reversal relative to the standard spatial voting theory.<sup>8</sup> We refer to it as the *within-election* preference reversal.

**Within-Election Preference Reversal** A voter may vote for a party other than the one with a platform closest to her ideal policy.

This same effect can lead to a preference reversal across elections. Even if the parties do not change their platforms from one election to the next, a voter may switch her vote from one party to the other. This possibility is depicted in Figure 3 that continues the example from Figure 2 to one more election. From election  $t - 1$  to election  $t$ ,  $D$ ’s initially extreme position gave it a boost in the eyes of the voter at election  $t$ . But after election  $t$ , the anchors reset, and the boost disappears, such that  $D$  loses this individual’s vote at  $t + 1$  because it

<sup>8</sup>A voter with asymmetric preferences around her ideal policy may also prefer  $d_t$  over  $r_t$ , although the spatial violation would go away with an appropriate rescaling of the policy space. Preference reversals due to the use of anchors are possible with any scaling of the policy space.

is further from the voter than party  $R$ . We refer to this as the *across-elections* preference reversal.

**Across-Elections Preference Reversal** For pairs of policy platforms that are unchanged from one election to the next,  $d_{t+1} = d_t$  and  $r_{t+1} = r_t$ , a voter may switch her vote from election  $t$  to election  $t + 1$ .

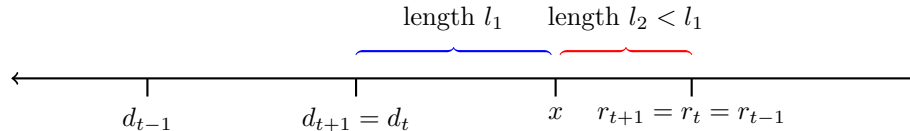


Figure 3: Across-Elections Preference Reversal

### 3 Electoral Competition with Anchors

We turn now to how voters’ use of anchors impacts politics more broadly. It is natural that those competing for voters’ affection adjust and adapt to the voters’ use of anchors. To study these follow-on effects, we develop a model of electoral competition and characterize how voters’ use of anchors affects the policy platforms offered by political parties.

The model builds on the classic Calvert-Wittman framework with policy-motivated parties, extending it dynamically. To highlight the role of anchors, we otherwise remain faithful to their framework.

In each period  $t = 1, 2, \dots$ , two parties  $D$  and  $R$  compete in an election by simultaneously offering platforms,  $d_t$  and  $r_t$ , respectively, in the policy space  $[-1, 1]$ . The anchor for a party is its previous policy offering,  $d_{t-1} \leq 0$  for party  $D$  and  $r_{t-1} \geq 0$  for party  $R$ . The anchors for the first election,  $d_0$  and  $r_0$ , are exogenous, whereas they are endogenous for all later elections. Voters observe the proposals, cast their votes in a majority rule election, and the winning party implements its platform.

By Lemma 1 voter utility functions are single-peaked over  $[-1, 1]$  and satisfy the single-crossing property in policies and ideal points. Therefore, the median voter is decisive over pairwise comparisons of policy platforms (Rothstein, 1991; Gans and Smart, 1996). As such, moving forward we focus on the vote of the median voter (or “the voter”), and we assume she has an ideal policy in the center of the distribution at 0.

In addition to policy, the voter cares about the valence of each party.<sup>9</sup> Normalizing the valence of the left party to 0,  $R$ ’s valence is given by  $\theta_t \sim U(-\psi, \psi)$ . The valence  $\theta_t$  is

<sup>9</sup>The valence shock is common and does not affect Lemma 1.

unknown to the parties until after they have chosen their policy platforms, but it is realized before voting occurs.

Thus, the voter's utility for electing  $D$  is

$$u_0(d_t|d_{t-1}) = -|d_t| + f(d_t - d_{t-1}), \quad (3)$$

while the voter's utility from electing  $R$  is

$$u_0(r_t|r_{t-1}) + \theta_t = -|r_t| + f(r_{t-1} - r_t) + \theta_t. \quad (4)$$

The parties are motivated purely by policy outcomes. The ideal policy of party  $D$  is  $-1$  and of party  $R$  is  $+1$ . Their utility functions from a policy  $p$  are

$$v_D(p) = -|p + 1| \quad (5)$$

$$v_R(p) = -|p - 1|. \quad (6)$$

**Intertemporal linkages.** The key linkage between elections is that the anchor points evolve endogenously: today's platform becomes tomorrow's anchor point. To focus on this, we shut down other linkages across time. Specifically, we assume that the parties (and voters) are myopic, so that they do not consider how today's platform choice will influence their utility in future periods. Similarly, the valence draw in each election is independent, such that there is no learning about voter preferences over time.

**The electoral environment.** We will compare the dynamics of party platforms in a political environment that is fixed versus one that changes periodically. We model changes in the political environment through the support of the valence shock. Specifically, when we reach that section we subscript  $\psi$  with  $t$ , and suppose that  $\psi_t \in \{\psi_l, \psi_h\}$  and evolves as a Markov chain with a transition probability  $\rho \in (0, 1)$ . This means that the state is either  $\psi_l$  or  $\psi_h$  and in each period it switches to the other state with probability  $\rho$ . With probability  $1 - \rho$  the state is unchanged. A small  $\rho$ , therefore, represents a very stable environment. We set  $\psi_0 = \psi_h$  to begin and impose  $1 + 2f(1) < \psi_l < \psi_h < 2$ , so that each realization of  $\psi_t$  satisfies Assumption 1 below. In Section 5 we discuss how our results extend if  $\psi_t$  is drawn from a distribution with continuous support.

**The anchoring function.** Finally, we impose the following assumptions on the anchoring function:

**Assumption 1** *For any  $\Delta p$  we assume:*

1.  $1 + 2f(1) < \psi$ .

2.  $\psi < 2$  and  $f(1) < 1/2$ .

3.  $\frac{f''(\Delta p)}{1+f'(\Delta p)} < 1$ .

Part 1 ensures that there is always an interior probability of winning the election. Part 2 implies that the standard Calvert-Wittman equilibrium is interior to the parties' ideal points, while still being consistent with part 1. Finally, part 3 guarantees existence and uniqueness of a symmetric equilibrium.

## 4 Results

We begin by describing the equilibrium party platforms *within* an election and then turn to platform dynamics *across* elections.

### 4.1 Party Platforms Within an Election

Consider the first election at  $t = 1$ . The voter votes for party  $R$  if  $u_0(r_1|r_0) + \theta_1 \geq u_0(d_1|d_0)$ , and for party  $D$  otherwise. Given her ideal point of 0, and that  $\theta_1$  is drawn uniformly over  $[-\psi, \psi]$ , the probability  $R$  wins the election given platforms  $(r_1, d_1)$  and anchors  $(r_0, d_0)$  is:

$$P(r_1, d_1|r_0, d_0) \equiv \frac{1}{2} - \frac{r_1 + d_1 + f(d_1 - d_0) - f(r_0 - r_1)}{2\psi}.$$

As the parties are policy motivated, they balance the probability of winning against the policy outcome. Party  $R$ 's maximization problem is given by:

$$\max_{r_1} P(r_1, d_1|\cdot)v_R(r_1) + (1 - P(r_1, d_1|\cdot))v_R(d_1), \quad (7)$$

Party  $D$ 's problem is analogous. As is standard, we focus on symmetric equilibria. In our setting, this requires the additional condition that the anchors are symmetric as well,  $r_0 = -d_0$ . Given this, a unique symmetric equilibrium exists and is given by the following.

**Proposition 1** *There exists a unique symmetric equilibrium  $r_1^* = -d_1^* \in (0, 1)$  where*

$$r_1^* = \frac{\psi}{2(1 + f'(r_0 - r_1^*))}. \quad (8)$$

The effect of the policy anchor is evident in the denominator of Equation (8). This result encompasses the canonical model in which the anchor term disappears (equivalent to the slope of  $f(\cdot)$  being zero everywhere). In that case, the equilibrium in Proposition 1 reduces to the classic result of Calvert and Wittman.

**Corollary 1** *Without voter anchoring, the equilibrium platforms are  $r_{cw}^* = -d_{cw}^* \in (0, 1)$  where*

$$r_{cw}^* = \frac{\psi}{2}. \quad (9)$$

The equilibrium without anchors is independent of the history. Thus, wherever the parties were previously located, they jump to the locations given by Equation (9).

When voters use policy anchors, equilibrium locations depend on the history. A first observation is that anchoring provides a convergent force on party locations. To see this, recall the classic incentives of policy motivated parties: By moderating toward the median voter, a party increases its probability of winning but wins with a less attractive policy. Without policy anchoring, these forces are in perfect balance for party  $R$  at  $r_{cw}^*$ . With policy anchoring, and regardless of where the anchor is, moderating towards the median gives  $R$  a slightly larger boost in its probability of winning as the new platform will look more attractive (or less unattractive) relative to the anchor. This is evident formally in Equation (8) as the slope of  $f(\cdot)$  is everywhere strictly positive. It follows that the equilibrium with anchoring is, all else equal, always more convergent than it is without anchoring.

**Corollary 2** *The equilibrium platforms with voter anchoring satisfy  $r_1^* \in (0, r_{cw}^*)$ .*

Despite this result, it does not follow that the use of policy anchors induces parties to always moderate in order to look better than they did previously. For anchors that are very moderate, the parties will in fact adopt platforms that are more extreme than their anchors, and they do this despite knowing that the comparison will be viewed poorly by voters.

To see why, return again to the classic trade-off parties face, this time turning it around: by choosing a more extreme platform, a party lowers its probability of winning but wins with a more appealing policy. This incentive is strong when starting from a very moderate position, so even though a more extreme platform comes with the additional cost of a poor comparison to the policy anchor, it is still worth it.

A key threshold in the logic of policy positioning with anchors is the policy that is the fixed point of Equation (8), which we denote by  $\bar{r}$ . When the policy anchor for party  $R$  is at  $\bar{r}$ , its optimal choice is  $\bar{r}$  itself. Formally,  $\bar{r}$  satisfies the following:

$$\bar{r} = \frac{\psi}{2(1 + f'(0))}. \quad (10)$$

When the anchoring function is reflected s-shape,  $\bar{r}$  is an upper bound on the the equilibrium platforms of the right-wing party. Proposition 2(a) describes how the equilibrium varies in the location of the anchor around  $\bar{r}$ .

**Proposition 2 (a)** *Suppose the anchoring function is reflected s-shape. If:*

1.  $r_0 < \bar{r}$  then  $r_1^* \in (r_0, \bar{r})$ , and is increasing in  $r_0$  at rate less than one.
2.  $r_0 > \bar{r}$  then  $r_1^* < \bar{r}$ , and decreasing in  $r_0$ .
3.  $r_0 = \bar{r}$  then  $r_1^* = \bar{r}$ .

Figure 4(a) depicts the key equilibrium properties. For an anchor more moderate than  $\bar{r}$  the equilibrium platforms are more extreme than the anchor, whereas for anchors more extreme than  $\bar{r}$  the platforms are more moderate. The most extreme policy platforms arise when the anchor is  $\bar{r}$  and Party  $R$  locates exactly there.

The equilibrium platforms relative to the anchor are not symmetric around  $\bar{r}$ . For moderate anchors, the platforms are incrementally more extreme, with the gap decreasing the closer the anchor is to  $\bar{r}$ . For more extreme anchors, the platforms are substantially more moderate than the anchor, with the gap increasing the more extreme is the anchor. As such, the equilibrium policy platforms are non-monotonic in the anchor, with the most moderate policy positions coming from moderate and extreme anchors.

This pattern is the consequence of a reflected s-shape anchoring function. Small differences with the anchor lead to relatively small benefits and costs whereas larger differences lead to relatively large benefits and costs. Thus, when the anchor is moderate and the parties wish to offer more extreme policies, their incentives are tempered by a marginal anchoring cost that is increasing in the relative movement. Conversely, when the anchor is extreme and the parties wish to offer more moderate policies, the benefit of doing so is increasing in the shift, and so the parties overshoot, offering even more moderate policies than they otherwise would. The asymmetric reaction of platforms to the anchor reflects this logic.

This logic persists but is flipped when the anchoring function is s-shaped. This leads to very different equilibrium policy platforms that are more extreme than the threshold  $\bar{r}$ . Moreover, the platforms are again non-monotonic in the location of the anchor, but they now form a single-dipped function such that centrist and extreme anchors give rise to local maxima in the equilibrium platforms. Proposition 2(b) describes the equilibrium properties.

**Proposition 2 (b)** *Suppose the anchoring function is s-shape. If:*

1.  $r_0 < \bar{r}$  then  $r_1^* > \bar{r}$ , and decreasing in  $r_0$ .
2.  $r_0 > \bar{r}$  then  $r_1^* \in (\bar{r}, r_0)$ , and is increasing in  $r_0$  at rate less than one.
3.  $r_0 = \bar{r}$  then  $r_1^* = \bar{r}$ .

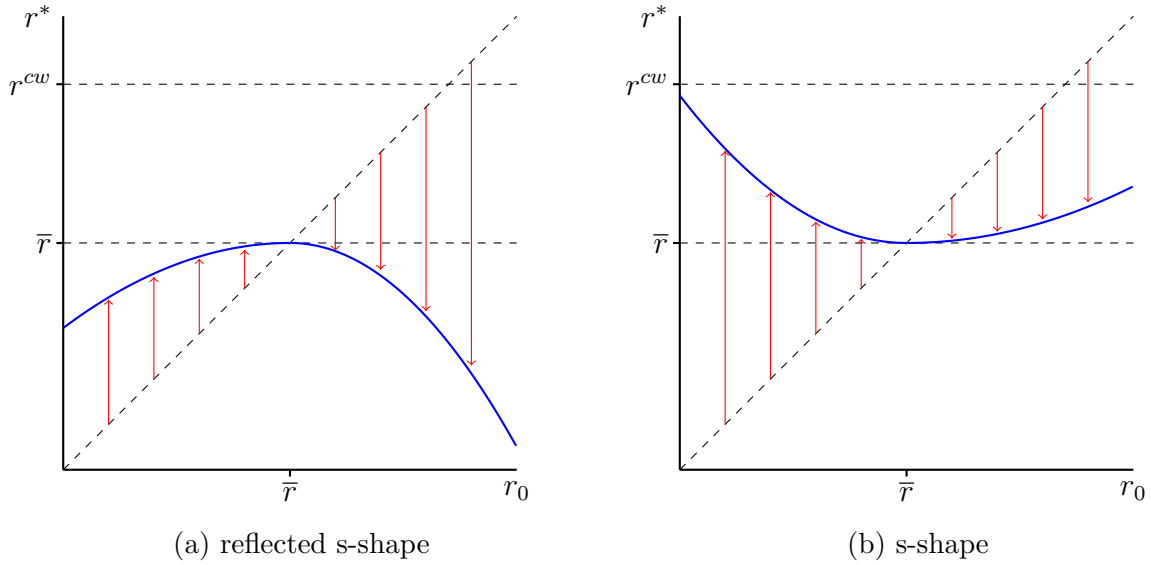


Figure 4: Equilibrium Platforms

The s-shaped anchoring function reverses the parties' incentives. In this case, the marginal cost of shifting away from the median voter is decreasing in the size of the shift, and so starting from centrist anchors, the parties shift outward more than they otherwise would, overshooting the threshold  $\bar{r}$ . Starting from an extreme anchor, the incentive to shift inward is similarly reflected. The marginal benefit is decreasing in the size of the shift, so platforms do not shift all the way to  $\bar{r}$ . The equilibrium platforms are depicted in Figure 4(b).

## 4.2 The Dynamics of Party Platforms

Over time, the policy anchors evolve with political competition. Voters use the previous positions of the parties to evaluate their current platforms, and the current platforms then become the anchors for the following election. In this section we study the dynamics in party platforms that this recursive process generates. We begin with a political environment that is unchanging and then turn to the more realistic setting in which the environment itself changes over time.

### 4.2.1 Party Platforms in an *Unchanging* World

In an unchanging environment, the competitive incentives of the parties recur from one election to the next. Without policy anchors, equilibrium behavior is straightforward: In the first election the parties locate at  $d_{cw}$  and  $r_{cw}$  and remain there for each election thereafter.

With policy anchors, equilibrium behavior is very different. The policy platforms trace a rich dynamic of positions, changing from one election to the next, possibly reversing direction,



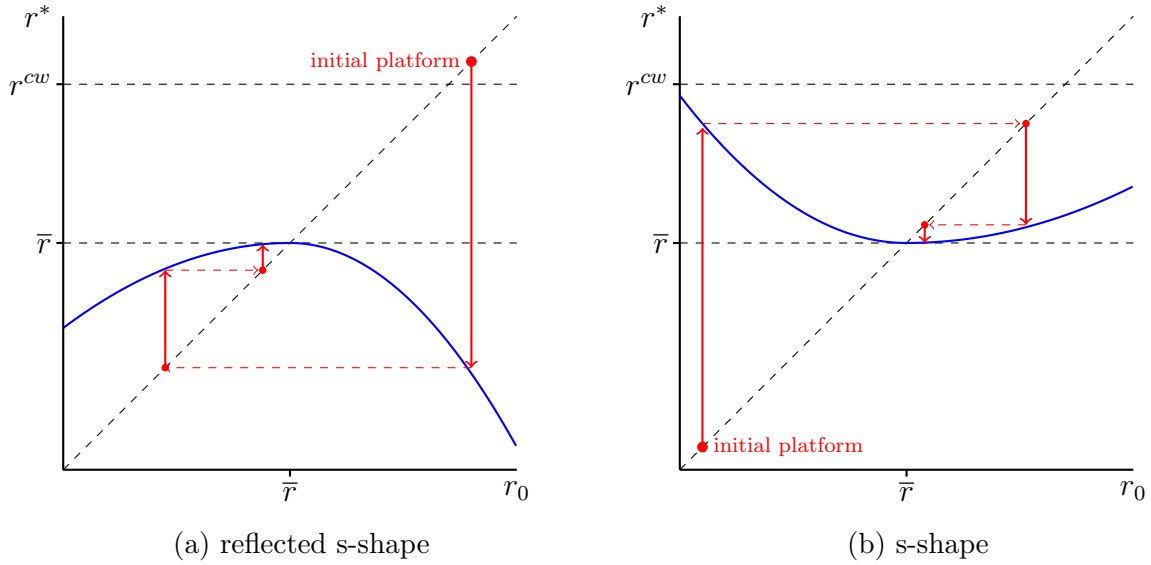


Figure 5: Equilibrium Platform Dynamics

and converging toward but never settling down to stable locations. It is only if the initial anchor is precisely at the threshold  $\bar{r}$  that equilibrium platforms are ever stable.

We again begin by describing equilibrium behavior for reflected s-shape anchor functions.

**Proposition 3 (a)** *Suppose the anchoring function is reflected s-shape. In the election at  $t = 1, 2, \dots$ , if:*

1.  $r_0 < \bar{r}$  then  $r_t^* \in (r_{t-1}, \bar{r})$  for all  $t$ , and  $r_t^* \rightarrow \bar{r}$  as  $t \rightarrow \infty$ .
2.  $r_0 > \bar{r}$  then  $r_1^* < \bar{r}$ , and dynamics then follow case (1).
3.  $r_0 = \bar{r}$  then  $r_t^* = \bar{r}$  for all  $t$ .

Figure 5(a) traces out a sample path of the platform dynamics generated by the behavior in Proposition 3(a). Starting from an extreme anchor, the red arrows and dashed lines trace through the equilibrium path of  $R$ 's policy platform. Initially,  $R$ 's platform makes a large jump inward as the shape of the anchoring function increases the reward the larger the movement relative to the anchor. From there,  $R$ 's platform creeps outward, with  $R$  suffering a cost each election from a platform that is unappealing relative to the anchor.  $R$ 's platform converges toward the threshold  $\bar{r}$  but its steps get smaller at a faster rate and its platform never quite reaches  $\bar{r}$ .

Policy dynamics for a s-shaped anchoring function follow a similar albeit reflected logic.

**Proposition 3 (b)** *Suppose the anchoring function is s-shape. In the election at  $t = 1, 2, \dots$ , if:*

1.  $r_0 > \bar{r}$  then  $r_t^* \in (\bar{r}, r_{t-1})$  for all  $t$ , and  $r_t^* \rightarrow \bar{r}$  as  $t \rightarrow \infty$ .
2.  $r_0 < \bar{r}$  then  $r_1^* > \bar{r}$ , and dynamics then follow case (1).
3.  $r_0 = \bar{r}$  then  $r_t^* = \bar{r}$  for all  $t$ .

For an s-shape anchoring function and a centrist policy anchor, equilibrium platforms initially jump outward, across the  $\bar{r}$  threshold, before reversing direction and iterating toward  $\bar{r}$  but never reaching it. This is depicted in Figure 5(b).

Reversals in direction of party platforms represent a time-inconsistency in party incentives that is due to voter anchoring. The parties have an incentive to converge starting from an extreme position as doing so improves the policy outcome *and* makes them look more attractive voters relative to their previous positions. When the anchoring function is reflected s-shape, the marginal benefit to party  $R$  of creeping inward is increasing, and thus it overshoots  $\bar{r}$ . Once at the new location, however, the anchor resets and the high marginal benefit of convergence disappears. Instead there is a low marginal anchoring cost for small steps outward. Consequently, the party shifts outward to a better policy, but does so in small steps to avoid the higher marginal cost of a large jump. The reverse pattern holds for an s-shape anchoring function as then the parties make large jumps outward, overshooting  $\bar{r}$  on the high side, and creeping back toward it. It is exactly at  $\bar{r}$  that party  $R$ 's policy incentive to diverge to a better policy is exactly in balance with the anchoring benefit of converging.

Before moving on, it is worth emphasizing that the behavior in Propositions 3(a) and (b) is not because the parties are forward-looking. When a party moves away from the median voter, it is not that it is suffering a cost today to benefit from a more attractive anchor in the future. Rather, in each election the parties myopically optimize, choosing the platform that is best for them that period. The asymmetry in movement, whether large jumps inward and incremental movement outward or vice versa, derives purely from voter anchoring and the shape of the anchoring function.

#### 4.2.2 Party Platforms in a *Changing* World

In this section we allow for the environment itself to change over time, where the state of the world is the support of the valence shock, either  $\psi_l$  or  $\psi_h$ . The size of  $\psi$  affects the incentives of parties to converge or diverge. For the smaller value,  $\psi_l$ , an incremental movement toward the center increases the probability of winning by more than for the high value,  $\psi_h$ . This incentive, in turn, leads to a lower value of the threshold  $\bar{r}$ , and we denote the level associated with each value of  $\psi$  by  $\bar{r}_l$  and  $\bar{r}_h$  where  $\bar{r}_l < \bar{r}_h$ .

Without policy anchors, equilibrium behavior is once again straightforward. For each state, the equilibrium platforms are constant, and when the state changes, the equilibrium shifts instantly and completely from one value to the other. When the state switches from  $\psi_l$  to  $\psi_h$  the platforms shift from moderate to extreme, and vice versa when the state switches back. The platforms shift, therefore, only when the state changes and by a fixed and constant amount each time.

With policy anchors, the platform dynamics are much richer. The platforms never stabilize. They endlessly evolve with incremental shifts and moderate jumps in one direction, and larger occasional jumps in the other directions. This pattern holds even when the state changes infrequently ( $\rho$  is small). In an environment that is mostly stable, policy changes in a constant flow, cycling between moderate and extreme positions.

To describe equilibrium behavior, we begin once again with the reflected s-shape anchoring function. Recall that the environment is initially in state  $\psi_h$ .

**Proposition 4 (a)** *Suppose the anchoring function is reflected s-shape and  $r_0 < \bar{r}_h$ . In the election at  $t = 1, 2, \dots$ , the following holds:*

**Unchanged state:**  $\psi_t = \psi_{t-1}$ .

- In state  $\psi_h$ ,  $r_t^* \in (r_{t-1}, \bar{r}_h)$  and  $r_t^* \rightarrow \bar{r}_h$  from below as  $t \rightarrow \infty$ .
- In state  $\psi_l$ ,  $r_t^* \in (r_{t-1}, \bar{r}_l)$  and  $r_t^* \rightarrow \bar{r}_l$  from below as  $t \rightarrow \infty$ .

**Changed state:**  $\psi_t \neq \psi_{t-1}$ .

- $\psi_t = \psi_h$ :  $r_t^* \in (r_{t-1}^*, \bar{r}_h)$ , where  $(r_t^* - r_{t-1}^*)$  is decreasing in  $r_{t-1}^*$ .
- $\psi_t = \psi_l$ :  $r_t^* < \bar{r}_l$ , where  $(r_{t-1}^* - r_t^*)$  is increasing in  $r_{t-1}^*$ .

The path of policy over time is depicted in Figure 6(a). If a state persists from one election to the next, the party platforms creep outward toward the threshold  $\bar{r}_l$  or  $\bar{r}_h$ . When the state shifts from  $\psi_l$  to  $\psi_h$ , policy continues in the same direction although it makes a larger jump outward. Balancing this outward movement is the response when the state shifts from  $\psi_h$  to  $\psi_l$ . In that case, policy reverses course and makes a substantial jump toward the center. This jump renews the process outward once more.

The size of the platform jumps and the amplitude of the dynamic path is not a constant. Instead, they vary in the number of elections between a change in state. The more time spent in state  $\psi_l$ , the closer the platform is to  $\bar{r}_l$  and, therefore, the closer it also is to  $\bar{r}_h$ . Consequently, the smaller is the jump when the state switches to  $\psi_h$ . In contrast, the more time the political environment has spent in state  $\psi_h$ , the further is  $R$ 's platform from  $\bar{r}_l$ , and the larger the jump inward when the state switches to  $\psi_l$ . In this way, the asymmetric changes

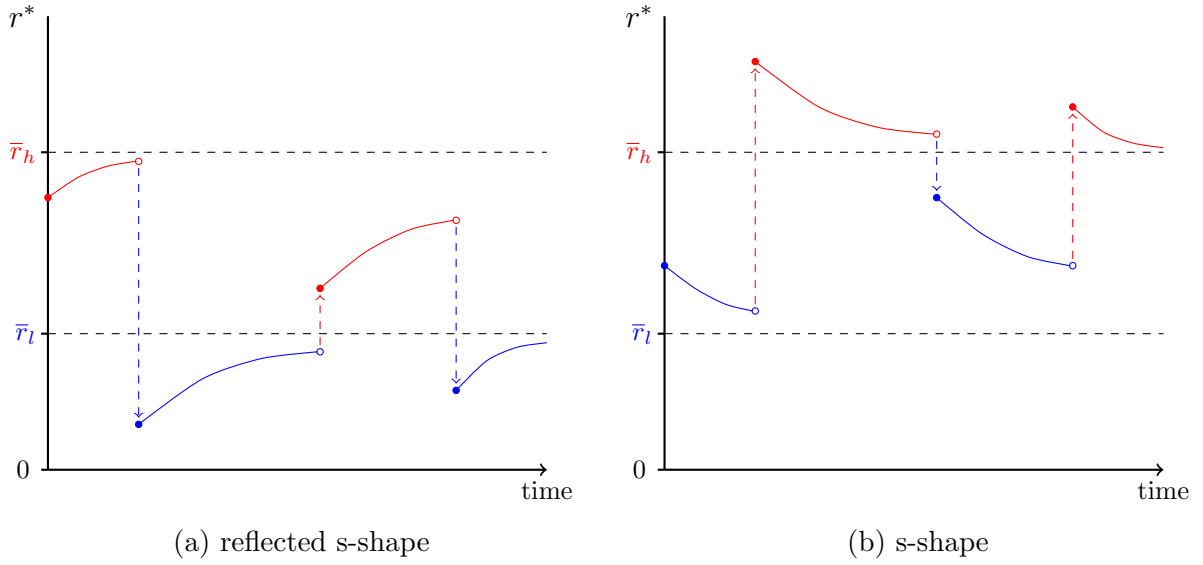


Figure 6: Platform Dynamics in a Changing World

around the  $\bar{r}$  threshold in an unchanging world manifest in a changing world as asymmetric sized jumps when the environment switches from one state to the other.

Proposition 4(b) describes the inverted equilibrium behavior that emerges for the s-shape anchoring function.

**Proposition 4 (b)** *Suppose the anchoring function is s-shape and  $r_0 > \bar{r}_h$ . In the election at  $t = 1, 2, \dots$ , the following holds:*

**Unchanged state:**  $\psi_t = \psi_{t-1}$ .

- In state  $\psi_h$ ,  $r_t^* \in (\bar{r}_h, r_{t-1}^*)$  and  $r_t^* \rightarrow \bar{r}_h$  from above as  $t \rightarrow \infty$ .
- In state  $\psi_l$ ,  $r_t^* \in (\bar{r}_l, r_{t-1}^*)$  and  $r_t^* \rightarrow \bar{r}_l$  from above as  $t \rightarrow \infty$ .

**Changed state:**  $\psi_t \neq \psi_{t-1}$ .

- $\psi_t = \psi_h$ :  $r_t^* > \bar{r}_h$ , where  $(r_t^* - r_{t-1}^*)$  is decreasing in  $r_{t-1}^*$ .
- $\psi_t = \psi_l$ :  $r_t^* \in (\bar{r}_l, r_{t-1}^*)$ , where  $(r_{t-1}^* - r_t^*)$  is increasing in  $r_{t-1}^*$ .

For an s-shape anchoring function, when the state is unchanged  $R$ 's platform converges toward  $\bar{r}_h$  in state  $h$  and toward  $\bar{r}_l$  in state  $l$  as elections pass. Policy jumps outward when the state shifts from  $\psi_l$  to  $\psi_h$  and by a larger amount than it jumps inward following a change in state from  $\psi_h$  to  $\psi_l$ . The change from  $\psi_l$  to  $\psi_h$  is the only time policy reverses direction, making a large jump outward that renews the process inward once more.

## 5 Discussion

In this section, we discuss the implications of our model and questions that it gives rise to. We also address informally several variations and extensions of the model.

### Political Representation

In our model, voters' ideal policies—their fundamental preferences—are unchanging, yet the policy platforms offered by the parties do change. Moreover, they change in response to how voters evaluate policies—their constructed preferences. If political representation requires that parties represent the preferences of voters, is this or is this not representation?

The scope of this question extends well beyond our model. Indeed, it lies at the heart of the classic debate over whether true representation requires elected officials to act as delegates or as trustees (Pitkin, 1967). In the usual framing of the problem, it is differences in expertise between voters and politicians that create a tension between whether representatives should act as delegates or trustees. Instead, in our model it is cognitive limitations on the part of voters that create a wedge between voters' behavior and their long-term interests. As such, it remains an open question whether policymakers are representing voter preferences even though the parties commit to platforms and have no private information.

The wedge in voters' interests turns on the difference between voters constructed preference—the decision utility they experience when voting—and voters' true or fundamental preferences that comes from their ideal point. Which quantity should we employ to evaluate the extent of successful representation?

One argument is that the fundamental preference should be used as it more accurately measures the real and persistent experience that comes from policies in action, whereas decision utility is ephemeral and exists only in the electoral moment. The counter argument is that decision utility is more real to voters and measures the choice they make rather than something that happens to them indirectly. Moreover, voters' experience in the electoral moment determines whether in their minds the electoral outcome is correct and, thus, is the foundation for democratic legitimacy

In our model, the parties are self-interested and respond to voters' decision utility. As we saw, this leads to continuous cycles in party platforms where the platforms would otherwise be largely stable. The fortuitous irony, however, is that appealing to voters' constructed preferences induces parties to converge closer to the ideal point of the median voter—recall Corollary 2—and this increases voter welfare as measured by their fundamental preferences.

### The Dynamics of Party Platforms

The movements of political parties have long fascinated political scientists. In contrast with implications from classic models of electoral competition, empirical scholars often point to

rich dynamics in the trajectory of electoral platforms, with parties moving in a seemingly chaotic way shifting back and forth between more moderate and more extreme positions (see, e.g., Budge et al., 2010). Understanding these dynamics — why and when parties make bold jumps versus incremental change, why they sometimes reverse course and other times continue moving in the same direction — is important to understanding the nature of political competition and policy outcomes.

Our model shows that a relatively small departure from classic models of spatial competition, that accounts for voters’ difficulties in evaluating platforms, can help rationalize these patterns. Platforms’ frequent movement, the whipsaw patterns that it features, and the asymmetric nature of these patterns, can all be accounted for by voters’ use of policy anchors.

Furthermore, our theory can help us understand the apparent mismatch between the volatility of party platforms and the relative stability of voters’ policy opinions (Page and Shapiro, 1992; Druckman and Leeper, 2012). In our model, while voters’ constructed preferences over the policy space evolve endogenously, the ideological location of their ideal points remains fixed. Our results thus resonate with observed patterns insofar as surveys of public opinion measure the latter, and not the former. For instance, survey questions that ask respondents to place their political views on a left-right scale or assess whether a given level of spending or government intervention is excessive or insufficient aim to capture the location of their ideal points. Prompts asking voters to express the extent to which they approve or disapprove of different parties, instead, capture their constructed preferences.

By making this distinction explicit, our theory underscores the importance of considering how public opinion is measured when assessing both the extent and the consequences of voter preference change. Ignoring this distinction risks conflating stable ideological bliss points with evolving, context-dependent evaluations—leading to misguided interpretations of electoral dynamics.

### **Variations and Extensions**

*More policy anchors.* In the model, we assume that voters look only to the most recent platform of each party to evaluate the current proposals. It is reasonable to think instead that a longer history of the platforms influences the voters’ evaluation. In the context of our model, this can be captured, for example, by assuming that voters anchor their evaluation of today’s proposals to a weighted average of past platforms over a finite history, with the weights potentially decreasing as older platforms fade in influence. In this case, voters’ longer memory creates a lag in how the anchoring function responds to the platforms proposed along the path. This lag slows down the platform dynamics emerging in our baseline model, however, it does need to alter their qualitative features. In particular, the same dynamics emerge in the long run, but they may be preceded by an initial period of seemingly more

erratic movement, during which platforms shift back and forth between more extreme and more moderate positions before settling into the patterns we characterized above.

Another possibility is that voters only remember the most extreme (or most moderate) platforms proposed by the parties over some finite past number of periods. As in the baseline, the parties' platforms evolve with long periods of small movements in one direction punctuated by larger jumps in the opposite direction. In contrast to the baseline model, however, temporary periods of stability emerge in this version of the game. This stability occurs because the same past policy may remain the most extreme platform for several periods in a row, until voters eventually use the more recent platform as an anchor. Interestingly, stability has an asymmetric feature, as only moves in one direction are followed by a finite number of periods of no change, after which the platforms resume their path as in the baseline model. Thus, similar to the previous case, voters forming anchors based on a longer history of platforms can slow the dynamics we characterized earlier, but does not overturn the long-run evolution of policies.

*Asymmetric equilibria.* For tractability, our analysis fixed the initial policy anchors for both parties symmetrically around the median voter's bliss point, allowing us to focus on symmetric equilibria. A natural question is how platform dynamics would change if parties started from asymmetric anchors. Here, parties face similar incentives as in the baseline model: to moderate when their anchor is extreme, and move more extreme when their anchor is moderate. As such, platform dynamics in an asymmetric equilibrium should have many of the same features we discuss above. Platforms continuously evolve, and parties make smaller moves in one direction followed by larger jumps in the other, depending on the shape of the anchoring function.

However, two crucial differences can emerge. First, in the symmetric equilibrium, it is always the case that *both* parties move more moderate or *both* parties move more extreme. In contrast, in an asymmetric equilibrium, there can be periods in which one party 'chases' the other. A party starting from an extreme anchor moves toward the median voter, while its opponent, with a very moderate anchor, shifts away. Importantly, this dynamic can lead to a within-election preference reversal for the median voter — an outcome that cannot arise when parties are always positioned symmetrically. Second, even when the parties both become more extreme or both become more moderate, the extent of this movement can differ across parties. Thus, for example, we can observe both parties becoming more extreme, but with one party shifting significantly more than the other.

*More than two states.* To clearly illustrate our results, our model with a changing world only considers a binary state. If instead  $\psi_t$  is drawn from a continuous support in periods where the state changes, then in almost all states there is some probability of observing platform

movement reverse direction, but the underlying logic and results remains unchanged. That is, for a given state  $\psi$  there is a cut-off  $\bar{r}_\psi$  that determines whether party platforms moderate or polarize. For example, consider the case where  $f$  is reflected s-shaped. Absent a state change, when the starting policy anchor is above  $\bar{r}_\psi$  the party platforms jump inward and then begin to slowly creep outward toward  $\bar{r}_\psi$ , as in the baseline model. As such, when platforms initially begin moving more extreme, the anchors are very moderate. Thus, if there is a state change,  $\psi_t \neq \psi_{t-1}$ , it is very likely that the new  $\bar{r}_{\psi_t}$  is still more extreme than the anchor, and platforms continue to creep outward. However, if platforms have repeatedly moved outward over a long period of time, then it becomes increasingly likely that a new draw of  $\psi_t$  yields a  $\bar{r}_{\psi_t}$  that is below the policy anchor. In turn, parties adopt platforms more moderate than  $\bar{r}_{\psi_t}$ , halting the outward movement, triggering a large inward shift toward the voter, and restarting the cycle.

Thus, just as in our two-state model, small shocks to the environment will drive platforms to cycle indefinitely in an asymmetric fashion. Moreover, the longer the state remains stable, the more dramatic the platform reversals will be in equilibrium.

## 6 Conclusion

Voters face a difficult problem when evaluating competing policy platforms. The policy space lacks a natural metric for evaluating the distance between policies, policies fall to both the left and right of voters' ideal points, and voting takes places at infrequent intervals. We argue that these complications lead voters to use previously seen policies as *anchors* to construct their preferences over the platforms that parties offer today.

In contrast to standard spatial models of elections, we show that the use of anchors generates rich dynamics in the movement of party platforms over time. Platforms cycle endlessly, with periods of gradually increasing extremism punctuated by dramatic movements to the center, or vice versa. This movement occurs despite stability in voters' underlying ideal points. Our results raise fundamental questions about the nature of political representation in a world where voters face cognitive limitations and rely on shortcuts to construct their preferences, and help reconcile theoretical predictions with observed patterns in platform dynamics.



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## A Proofs

**Proof of Lemma 1.** To start, we show that  $u_x(p_t|p_{t-1})$  is single-peaked at  $x$ . Consider any  $r_t, r'_t$  with  $x < r_t < r'_t$ . We show  $u_x(r_t|r_{t-1}) > u_x(r'_t|r_{t-1})$ . To see this, note that: (1)  $-|x - r_t| > -|x - r'_t|$ ; and (2)  $f(r_{t-1} - r_t) > f(r_{t-1} - r'_t)$ , since  $f$  is strictly increasing in  $\Delta r_t$ . The analogous argument yields that  $u_x(d_t|d_{t-1}) > u_x(d'_t|d_{t-1})$  for  $d'_t < d_t < x$ .

Next, we prove that  $u_x(p_t|p_{t-1})$  satisfies the single-crossing property. Specifically, we show that for all  $x < x'$  and all  $p_t < p'_t$  if  $u_x(p'_t|p_{t-1}) > u_x(p_t|p_{t-1})$  then  $u_{x'}(p'_t|p_{t-1}) > u_{x'}(p_t|p_{t-1})$ .

First, take  $d_t < d'_t < x'$ . By single-peakedness of  $u_{x'}$  it follows that  $u_{x'}(d'_t|d_{t-1}) > u_{x'}(d_t|d_{t-1})$ . Second, by single-peakedness we cannot have  $x < r_t < r'_t$  and  $u_x(r'_t|r_{t-1}) > u_x(r_t|r_{t-1})$ .

Finally, take  $d_t < x < x' < r_t$ . Then  $x < x' < r_t$  yields:

$$\begin{aligned} u_{x'}(r_t|r_{t-1}) &= -|r_t - x'| + f(r_{t-1} - r_t) \\ &> -|r_t - x| + f(r_{t-1} - r_t) = u_x(r_t|r_{t-1}). \end{aligned}$$

Additionally,  $d_t < x < x'$  implies:

$$\begin{aligned} u_x(d_t|d_{t-1}) &= -|d_t - x| + f(d_t - d_{t-1}) \\ &> -|d_t - x'| + f(d_t - d_{t-1}) = u_{x'}(d_t|d_{t-1}). \end{aligned}$$

Thus, if  $u_x(r_t|r_{t-1}) > u_x(d_t|d_{t-1})$  then  $u_{x'}(r_t|r_{t-1}) > u_{x'}(d_t|d_{t-1})$ , as required. ■

**Lemma A1** *R's probability of winning in each election is interior,  $P(r_t, d_t|\cdot) \in (0, 1)$ .*

**Proof.** First, we show  $P(r_t, d_t|\cdot) < 1$ . This requires

$$\begin{aligned} \frac{1}{2} - \frac{r_t + d_t + f(\Delta d_t) - f(\Delta r_t)}{2\psi} &< 1 \\ \Leftrightarrow 0 &< r + d + f(\Delta d_t) - f(\Delta r_t) + \psi \end{aligned}$$

Note that  $r_t \geq 0$  and  $d_t \geq -1$ . Therefore, a sufficient condition for the above to hold is that:

$$\begin{aligned} 0 &< 0 - 1 + f(\Delta d_t) - f(\Delta r_t) + \psi \\ \Leftrightarrow 1 &< f(\Delta d_t) - f(\Delta r_t) + \psi. \end{aligned} \tag{11}$$

Finally, notice that  $f(\Delta d_t) \geq f(-1 - 0)$  and  $f(\Delta r_t) \leq f(1 - 0)$ . Thus, inequality (11) holds

if:

$$\begin{aligned} 1 &< f(-1 - 0) - f(1 - 0) + \psi \\ &\Leftrightarrow 1 + 2f(1) < \psi, \end{aligned}$$

where the second line follows from the assumption that  $f(a) = -f(-a)$  and the last inequality holds by Assumption 1.

Second, we show that  $P(r_t, d_t|\cdot) > 0$ . This requires

$$\begin{aligned} \frac{1}{2} - \frac{r_t + d_t + f(\Delta d_t) - f(\Delta r_t)}{2\psi} &> 0 \\ \Leftrightarrow \psi &> r_t + d_t + f(\Delta d_t) - f(\Delta r_t), \end{aligned}$$

which again holds by assumption that  $1 + 2f(1) < \psi$ . ■

**Lemma A2** *For any anchors  $(r_{t-1}, d_{t-1})$ , if  $(r_t^*, d_t^*)$  is an interior equilibrium then it solves:*

$$\left(\frac{-1 - f'(r_{t-1} - r_t)}{2\psi}\right)(r_t - d_t) + P(r_t, d_t|r_{t-1}, d_{t-1}) = 0 \quad (12)$$

$$\left(\frac{1 + f'(d_t - d_{t-1})}{2\psi}\right)(r_t - d_t) - 1 + P(r_t, d_t|r_{t-1}, d_{t-1}) = 0 \quad (13)$$

**Proof.** Follows by differentiating  $R$ 's expected utility in (7) with respect to  $r_t$ , and likewise for the analogous problem for  $D$ . ■

**Lemma A3** *Party  $i$ 's expected utility is strictly concave in its own policy choice.*

**Proof.** We prove the result for party  $R$ , the case of party  $D$  follows symmetrically. Fixing party  $D$ 's policy at  $d_t$  and differentiating yields:

$$\left(\frac{-1 - f'(r_{t-1} - r_t)}{2\psi}\right)(r_t - d_t) + 1 - \frac{r_t + d_t + f(d_t - d_{t-1}) - f(r_{t-1} - r_t) + \psi}{2\psi}. \quad (14)$$

Differentiating again and rearranging, the second derivative of  $R$ 's expected utility is negative if:

$$f''(\Delta r_t)(r_t - d_t) < 2(1 + f'(\Delta r_t)). \quad (15)$$

In each period,  $r_t, d_t \in [-1, 1]$ , therefore,  $r_t - d_t \leq 2$ . Thus, a sufficient condition for inequality (15) to hold is that  $2f''(\Delta r_t) < 2(1 + f'(\Delta r_t))$ , which holds by Assumption 1. ■

**Proof of Proposition 1.** We first show that any symmetric equilibrium must be interior,  $r_t^* \in (0, 1)$ . For the standard argument we cannot have an equilibrium with  $r_t^* = 0$ . Next, suppose that  $r_t^* = 1$  is an equilibrium. Fix  $D$ 's policy at  $d_t = -1$  and  $d_{t-1} = -r_{t-1}$ , and consider  $R$ 's best response. A necessary condition for  $r_t = 1$  to be a best response is that  $R$ 's utility is increasing as  $r_t \rightarrow 1$ . That is,

$$\lim_{r_t \rightarrow 1} \left( \frac{-1 - f'(\Delta r_t)}{2\psi} \right) (r_t + 1) + 1 - \frac{r_t - 1 + f(-1 + r_{t-1}) - f(r_{t-1} - r_t) + \psi}{2\psi} > 0 \quad (16)$$

$$\Leftrightarrow \left( \frac{-1 - f'(r_{t-1} - 1)}{2\psi} \right) 2 + 1 - \frac{\psi}{2\psi} > 0 \quad (17)$$

$$\Leftrightarrow \frac{-1 - f'(r_{t-1} - 1)}{\psi} + \frac{1}{2} > 0 \quad (18)$$

$$\Leftrightarrow \frac{\psi}{2(1 + f'(r_{t-1} - 1))} > 1. \quad (19)$$

However,  $\frac{\psi}{2(1+f'(r_{t-1}-1))} < \frac{\psi}{2}$  and, by assumption,  $\frac{\psi}{2} < 1$ , a contradiction. Thus, there cannot be an equilibrium  $r^* = 1$ .

Now consider an interior equilibrium,  $r_t^* \in (0, 1)$ . Substituting  $r_t = -d_t$  into  $R$ 's first-order condition (12), we obtain the following expression:

$$2r_t \left( -1 - f'(r_{t-1} - r_t) \right) + \psi = 0. \quad (20)$$

Thus, if  $r_t^*$  is a symmetric equilibrium, then it must solve

$$r_t = \frac{\psi}{2(1 + f'(\Delta r_t))}. \quad (21)$$

Finally, we prove that symmetric equilibrium exists and is unique. Define the function  $h : [0, 1] \rightarrow [-1, 1]$  as

$$h(r) = \frac{\psi}{2(1 + f'(r_{t-1} - r))} - r.$$

By equation (21), if  $r_t^*$  is a symmetric equilibrium then  $h(r_t^*) = 0$ . We first show that there exists a solution to the equation  $h(r) = 0$ .

At  $r = 0$  we have:

$$h(0) = \frac{\psi}{2(1 + f'(r_{t-1}))} > 0.$$

Instead, at  $r = 1$ :

$$h(1) = \frac{\psi}{2(1 + f'(r_{t-1} - 1))} - 1 < \frac{\psi}{2} - 1 < 0.$$

Furthermore, continuity of  $f'(\Delta r_t)$  implies continuity of  $h$ . Thus, by the intermediate value theorem, there exists an  $r_t^*$  such that  $h(r_t^*) = 0$ .

Next, we demonstrate that this solution is unique. Differentiating yields

$$h'(r) = \frac{\psi}{2} \frac{f''(r_{t-1} - r)}{1 + f'(r_{t-1} - r)} - 1.$$

By Assumption 1,  $\frac{\psi}{2} < 1$  and  $\frac{f''(r_{t-1} - r)}{1 + f'(r_{t-1} - r)} < 1$ . Hence,  $h'(r) < 0$  and there is only one  $r_t^*$  such that  $h(r_t^*) = 0$ .

Finally, we show that  $r_t^*$  is indeed  $R$ 's best response to  $D$  choosing  $-r_t^*$ . Consider  $R$ 's first-order condition with  $D$ 's policy fixed at  $-r_t^*$ :

$$\left( \frac{-1 + f'(r_{t-1} - r_t)}{2\psi} \right) (r_t + r_t^*) + 1 - \frac{r_t - r_t^* + f(-r_t^* - d_{t-1}) - f(r_{t-1} - r_t) + \psi}{2\psi} = 0. \quad (22)$$

By construction  $r_t^*$  solves  $R$ 's first-order condition. To conclude the argument note that, by Lemma A3,  $R$ 's expected utility is concave in  $r_t$ , thus,  $r_t^*$  must be the unique solution to  $R$ 's first-order condition. The symmetric argument proves that  $d_t^* = -r_t^*$  is  $D$ 's best response to  $R$  choosing  $r_t^*$ , as required. ■

**Proof of Corollary 2.** The result follows from Proposition 1, Corollary 1, and the observation that  $f'(\cdot) > 0$  everywhere. ■

#### Lemma A4

1. Assume  $f$  is reflected s-shaped. If  $r_0 < \bar{r}$  then  $r_1^* \in (r_0, \bar{r})$ . Otherwise, if  $r_0 > \bar{r}$  then  $r_1^* < \bar{r}$ .
2. Assume  $f$  is s-shaped. If  $r_0 < \bar{r}$  then  $r_1^* > \bar{r}$ . Otherwise, if  $r_0 > \bar{r}$  then  $r_1^* \in (\bar{r}, r_0)$ .
3. If  $r_0 = \bar{r}$  then  $r_1^* = \bar{r}$ .

**Proof.** We break the argument into two cases, depending on the shape of  $f$ . Recall that  $r_1^*$  must solve  $h(r) = 0$ , with  $h$  defined as in the proof of Proposition 1.

1. Assume  $f$  is reflected s-shaped.

First, suppose  $r_0 < \bar{r}$ , we show this implies that  $r_1^* \in (r_0, \bar{r})$ . If  $r = r_0$  then

$$h(r_0) = \frac{\psi}{2(1 + f'(0))} - r_0 = \bar{r} - r_0 > 0.$$

Instead, if  $r = \bar{r}$  then  $h(\bar{r}) = \frac{\psi}{2(1+f'(r_0-\bar{r}))} - \bar{r}$ . Because  $f$  is s-shaped,  $\frac{\psi}{2(1+f'(r_0-r))}$  is maximized at  $r = r_0$ . Therefore:

$$h(\bar{r}) = \frac{\psi}{2(1+f'(r_0-\bar{r}))} - \bar{r} = \frac{\psi}{2(1+f'(r_0-\bar{r}))} - \frac{\psi}{2(1+f'(0))} < 0.$$

Thus,  $h(r_0) > 0 > h(\bar{r})$ , which implies that the unique solution to  $h(r) = 0$  is given by  $r_1^* \in (r_0, \bar{r})$ .

Second, let  $r_0 > \bar{r}$ , we show that  $r_1^* < \bar{r}$ . At  $r = 0$  we have  $h(0) = \frac{\psi}{2(1+f'(r_0))} > 0$ . Instead, as in the previous case,  $h(\bar{r}) < 0$ . Thus,  $0 < r_1^* < \bar{r} < r_0$ , completing the argument.

2. Assume  $f$  is s-shaped.

First, let  $r_0 < \bar{r}$ . We show that  $h(r) > 0$  for all  $r < \bar{r}$  and, hence, it must be that  $r_1^* \geq \bar{r}$ . Notice that  $\frac{\psi}{2(1+f'(r_0-r))}$  is minimized at  $r = r_0$  because  $f$  is s-shaped. Therefore, for  $r < \bar{r}$ ,  $h(r) > \frac{\psi}{2(1+f'(0))} - r = \bar{r} - r \geq 0$ , as required.

Finally, suppose  $r_0 > \bar{r}$ . We demonstrate that if  $r_1^*$  solves  $h(r) = 0$  then  $r_1^* \in (\bar{r}, r_0)$ . Evaluating  $h$  at  $r = r_0$  yields  $h(r_0) = \frac{\psi}{2(1+f'(0))} - r_0 = \bar{r} - r_0 < 0$ . Instead, at  $r = \bar{r}$  we have  $h(\bar{r}) = \frac{\psi}{2(1+f'(r_0-\bar{r}))} - \bar{r} > 0$ , where the final inequality follows because  $\bar{r} = \frac{\psi}{2(1+f'(0))}$  and  $\frac{\psi}{2(1+f'(r_0-r))}$  is minimized at  $r = r_0$  when  $f$  is s-shaped. Thus,  $r_1^* \in (r_0, \bar{r})$ .

To conclude, note that  $r_1^*$  is continuous in  $r_0$  and, thus, if  $r_0 = \bar{r}$  then  $r_1^* = \bar{r}$ . ■

### Lemma A5

1. Assume  $f$  is reflected s-shaped. If  $r_0 < \bar{r}$  then  $r_1^*$  is increasing in  $r_0$ . Otherwise, if  $r_0 > \bar{r}$  then  $r_1^*$  is decreasing in  $r_0$ .
2. Assume  $f$  is s-shaped. If  $r_0 < \bar{r}$  then  $r_1^*$  is decreasing in  $r_0$ . Otherwise, if  $r_0 > \bar{r}$  then  $r_1^*$  is increasing in  $r_0$ .
3.  $\left| \frac{\partial r_1^*}{\partial r_0} \right| < 1$ .

**Proof.** Applying the implicit function theorem to (8):

$$\frac{\partial r_1^*}{\partial r_0} = \frac{\frac{-\psi f''(r_0-r_1^*)}{2(1+f'(r_0-r_1^*))^2}}{1 - \frac{\psi f''(r_0-r_1^*)}{2(1+f'(r_0-r_1^*))^2}} \quad (23)$$

By Assumption 1,  $\frac{\psi}{2} < 1$  and  $\frac{f''(r_0-r_1^*)}{1+f'(r_0-r_1^*)} < 1$ . Thus, the denominator of (23) is positive. Therefore,  $\frac{\partial r_1^*}{\partial r_0} > 0$  if and only if  $f''(r_0-r_1^*) < 0$ . We break the argument into two cases, depending on the shape of  $f$ .

1. Assume that  $f$  is reflected s-shaped. By Lemma A4,  $r_1^* > r_0$  if and only if  $r_0 < \bar{r}$ . Since  $f$  is reflected s-shaped, this implies  $f''(r_0 - r_1^*) < 0$  for  $r_0 < \bar{r}$  and  $f''(r_0 - r_1^*) > 0$  for  $r_0 > \bar{r}$ . Thus,  $\frac{\partial r_1^*}{\partial r_0} > 0$  if  $r_0 < \bar{r}$  and  $\frac{\partial r_1^*}{\partial r_0} < 0$  if  $r_0 > \bar{r}$ , as claimed.
2. Assume that  $f$  is s-shaped. By Lemma A4,  $r_1^* > r_0$  if and only if  $r_0 < \bar{r}$ . Since  $f$  is s-shaped, this implies  $f''(r_0 - r_1^*) > 0$  for  $r_0 < \bar{r}$  and  $f''(r_0 - r_1^*) < 0$  for  $r_0 > \bar{r}$ . Thus,  $\frac{\partial r_1^*}{\partial r_0} < 0$  if  $r_0 < \bar{r}$  and  $\frac{\partial r_1^*}{\partial r_0} > 0$  if  $r_0 > \bar{r}$ , as claimed.

To conclude, we show that  $|\frac{\partial r_1^*}{\partial r_0}| < 1$ . Clearly  $\frac{\partial r_1^*}{\partial r_0} < 1$ . Finally, by Assumption 1,  $\frac{\psi}{2} < 1 < 1 + f'(r_0 - r_1^*)$  and  $f''(r_0 - r_1^*) < 1 + f'(r_0 - r_1^*)$ , which implies  $\frac{\partial r_1^*}{\partial r_0} > -1$ . ■

**Proof of Propositions 2 (a) and (b).** Follow immediately from Lemmas A4 and A5. ■

**Proof of Propositions 3 (a) and (b).** We prove the result for Proposition 3 (a). The proof of Proposition 3 (b) follows an analogous argument.

To show part 1, first note that, by Proposition 2 (a), if at period  $\bar{t}$   $r_{\bar{t}-1}^* < \bar{r}$  then  $r_{\bar{t} \in (r_{\bar{t}-1}^*, \bar{r})}^*$ . Thus, the sequence of platforms  $(r_{\bar{t}-1}^*, r_{\bar{t}}^*, r_{\bar{t}+1}^*, \dots)$  is strictly increasing and bounded above by  $\bar{r}$ .

We now show that  $\bar{r}$  is the supremum of  $(r_{\bar{t}-1}^*, r_{\bar{t}}^*, r_{\bar{t}+1}^*, \dots)$ . Suppose instead that the supremum is given by some  $r' < \bar{r}$ . Then the monotone convergence theorem implies that  $\lim_{t \rightarrow \infty} r_t^* = r'$ . However, by Proposition 2 (a) if  $r_t^* = r' < \bar{r}$  then  $r_{t+1}^* > r'$ . Therefore, for all  $\epsilon > 0$  sufficiently small if  $r_t^* = r' - \epsilon$  then  $r_{t+1}^* > r'$ , contradicting that  $r' < \bar{r}$  is the supremum of the sequence of platforms.

Thus,  $\bar{r}$  is the supremum of  $(r_{\bar{t}-1}^*, r_{\bar{t}}^*, r_{\bar{t}+1}^*, \dots)$  and the monotone convergence theorem yields  $\lim_{t \rightarrow \infty} r_t^* = \bar{r}$ .

Parts 2 and 3 then follow immediately from Proposition 2 (a). ■

**Proof of Propositions 4 (a) and (b).** We prove the result for Proposition 4 (a). The proof of Proposition 4 (b) follows an analogous argument.

The proof for when the state does not change,  $\psi_t = \psi_{t-1}$ , follows immediately by Proposition 3 (a).

Next, suppose that  $\psi_{t-1} = \psi_l$  and  $\psi_t = \psi_h$ . First, by Proposition 3 (a)  $r_t^* \in (r_{t-1}^* - 1, \bar{r}_h)$  for any  $r_{t-1}^*$ , as claimed. Second, note that if  $\psi_{t-1} = \psi_l$  then  $r_{t-1}^* < \bar{r}_l < \bar{r}_h$  by Proposition 2 (a). Thus, by Proposition 2 (a) also yields  $\frac{\partial r_t^*}{\partial r_{t-1}^*} < 1$ , which implies that  $r_t^* - r_{t-1}^*$  is decreasing in  $r_{t-1}^*$ .

Finally, assume that  $\psi_{t-1} = \psi_h$  and  $\psi_t = \psi_l$ . Proposition 3 (a) implies that  $r_t^* < \bar{r}_l$  for any  $r_{t-1}^*$ . To conclude, note that  $|\frac{\partial r_t^*}{\partial r_{t-1}^*}| < 1$  by Lemma A5. Thus,  $r_{t-1}^* - r_t^*$  is increasing in  $r_{t-1}^*$ . ■