

Ideological Infection*

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Abstract

Many policy problems are inherently dynamic, and outcomes worsen if policy is not adapted to changing circumstances. However, even if everyone agrees on how to address the problem, negotiations do not occur in a vacuum. Thus, ideological conflicts can infect even common-value issues, distorting negotiation dynamics and generating inefficiency. We develop a dynamic bargaining model to study when and how this *ideological infection* emerges. We find that dynamic policy problems are vulnerable to ideological infection precisely because the costs of inaction compound over time. Furthermore, inefficiency is inevitable when players anticipate that the conflict will rapidly intensify on the ideological dimension. Our findings thus offer a stark warning: even issues with clear common ground may be unable to escape political contagion.

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1 Introduction

Many policy problems are inherently dynamic. A crisis left unaddressed worsens over time; poorly maintained infrastructure deteriorates; and technological innovations continually reshape society, presenting governments with new and evolving challenges.

Given the growing price of inaction on these issues, one might expect little disagreement among policymakers. Yet, even on issues characterized by shared preferences, bargaining parties often fail to strike efficient agreements, instead adopting half measures that do not effectively adjust policy to a changing environment. Scholars of American politics, for example, lament Congress’ apparent inability to act efficiently on common-value issues in spite of underlying bipartisan agreement both in Congress and among voters ([Layman, Carsey and Horowitz, 2006](#)).¹ Outside the realm of domestic politics, cooperation between the United States and China on climate policy has fared no better, despite China’s Foreign Minister Wang Yi describing the issue as an “oasis,” due to the joint interests of the United States and China in addressing climate change during the Biden administration ([Stanway, 2021](#)). Why do bargaining parties so often fail to reach solutions to these shared problems?

In this paper, we show that, rather than foster agreement, the deteriorating nature of these issues is precisely what makes them susceptible to being engulfed in political conflict. The urgency of a shared problem can be used in bargaining as leverage to extract concessions on other issues that are divisive or ideological. In turn, this creates dynamic incentives to allow the problem to worsen in order to extract a better deal on ideological dimensions in the future.

We characterize two forms of distortion that can arise from these dynamic incentives. First, they can induce the players to develop divergent preferences over the common-value issue, generating what we call ideological infection. Second, they can lead the players to implement an inefficient policy. As we show below, distinguishing between these two forms of distortion is important: in our setting, ideological infection is sufficient but, surprisingly, not necessary for the equilibrium policy to be inefficient.

To see why worsening problems are uniquely vulnerable to these distortions, contrast them with a common-value issue where an inefficient policy is costly, but this cost does not increase over time. In this case, an agenda setter can still exploit the inefficiency on the common-value issue to secure a more favorable outcome on a divisive policy dimension in the future. However, any future gain the agenda setter might achieve by pursuing an inefficient policy today comes at an immediate cost to both bargaining parties. When the costs of

¹Even in an era of high polarization in the United States, there still exists bipartisan agreement on many issues ([Orth, Bialik and McKown-Dawson, 2022](#)).

inefficiency do not escalate over time, this immediate loss outweighs the potential future ideological gains. As a result, no incentives to distort policy emerge, ideological infection never arises, and the shared problem is addressed without delay.

Instead, suppose that the costs of inefficiency compound over time, as in the case of a worsening crisis. Due to this compounding, maintaining an inefficient policy today imposes a smaller immediate cost on both players while generating a larger future gain for the agenda setter. As a result, dynamic considerations may lead to infection of the players' preferences and prevent the bargaining parties from coming to an efficient agreement, leaving the shared problem unresolved.

We develop a formal theory to explore this intuition. Our model allows us to pinpoint the strategic mechanisms that prevent the bargaining players from efficiently addressing a shared problem and characterize the conditions under which this distortion materializes. In particular, our analysis shows that both infection and inefficiency become inescapable when compounding costs on the common-value dimension are paired with rapidly increasing polarization on the dimension of conflict, as this guarantees that future ideological concessions are sufficiently valuable. Strikingly, this implies that actors delay reaching efficient agreements today even as they anticipate their opponents becoming more ideologically rigid tomorrow — paradoxically narrowing the scope for compromise. Conversely, when polarization does not increase rapidly, infection can never emerge. Yet, surprisingly, compounding costs combined with dynamic bargaining constraints may still generate inefficient policy outcomes.

As noted at the start, many important policy areas exhibit compounding costs of inefficiency, where problems worsen over time if policy is not adapted to address them. Poorly maintained infrastructure deteriorates rapidly, becoming increasingly unreliable and costly to fix, a disease spreads faster and faster as more individuals become infected, and damages from climate change become harder to reverse. Moreover, issues are rarely negotiated in a vacuum. Bundling unrelated issues into a larger omnibus bill is a standard tool used in the U.S. Congress (Krutz, 2001; Clinton and Lapinski, 2006), and issue linkage is a common tactic in international negotiations (Tollison and Willett, 1979; Keohane, 1984). Consequently, the mechanism for inefficiency that we identify is inherent to the evolving nature of common-value policy problems and strategic bargaining. In contrast, existing explanations for why parties may fail to address common-value issues focus on external forces such as elections, exogenous constraints on the policy agenda, or issues which may become conflictual in the future (rather than imposing greater *common* costs). As such, our theory offers a novel explanation which implies distinct policy remedies, a point we return to in the conclusion.

The risk of conflict on ideological dimensions spilling over onto common-value issues is not merely theoretical. Although the Chinese Foreign Minister described climate policy as

an “oasis,” he went on to caution that: “surrounding the oasis is a desert, and the oasis could be desertified very soon. China-U.S. climate co-operation cannot be separated from the wider environment of China-U.S. relations.” (Stanway, 2021). Likewise, in the United States, rising ideological polarization is frequently blamed for legislative inaction even on issues with broad underlying agreement (Lee, 2009), leaving policy that “does not adjust to changing economic and demographic circumstances” (Barber et al., 2015, p. 41).

The remainder of the paper proceeds as follows. In Section 1.1, we offer an overview of the model and connect our contribution to the game-theoretic literature on bargaining. Next, in Section 1.2 we illustrate our theory with an application to legislative policymaking in the United States. We discuss how our results offer a novel explanation for why increasing polarization has coincided with Congress’ inability to legislate effectively even on issues which are not fundamentally ideological. We then present the full setup and analysis of our model in Sections 2 and 3, respectively. Section 4 describes two extensions to the baseline model. In the first, we analyze the effect of turnover in proposal power. In contrast to previous theories where an incumbent chooses an inefficient policy to insure against losing power, turnover in our model can mitigate the problem and help restore efficiency. In the second, we extend the analysis to an infinite-horizon model. We show that if the costs of inefficiency and polarization are both rapidly increasing, then players implement an inefficient policy in every period. Finally, we conclude with further discussion of the implications of our theory in Section 5.

1.1 Related Models of Bargaining

In our model, two players repeatedly bargain over both an ideological issue and a common-value one and, on each issue, the policy that is implemented today becomes the status quo tomorrow. As such, our research contributes to the extensive literature that explores bargaining with an endogenous status quo (see Eraslan, Evdokimov and Zápál (2022) for a review of this literature).²

Previous works in this literature have analyzed bargaining in one dimension, and found inefficiencies stemming from motives such as insurance against turnover (Buisseret and Bernhardt, 2017) or the possibility of developing future conflict on the issue (Riboni and Ruge-

²Previous papers in this literature that incorporate multiple policy dimensions have focused on issues of existence (Duggan and Kalandrakis, 2012) and indeterminacy (Anesi and Duggan, 2018) of equilibria. Closer to our work, Penn (2009) allows for multiple dimensions and characterizes how continuing policies can distort preferences. However, proposals are exogenous in her model. In contrast, the endogeneity of proposals is a crucial determinant of preferences in our model. Chen and Eraslan (2017) also analyzes dynamic bargaining with multiple policy dimensions, but assumes parties can only address one issue at a time.

Murcia, 2008; Zápál, 2011; Dziuda and Loeper, 2016; Austen-Smith et al., 2019).³ In our setting, instead, policymaking distortions arise due to the multidimensionality of the policy space combined with the evolution of preferences on each dimension. As such, distortions can emerge even when the proposer is guaranteed to remain in power indefinitely, and on issues where players anticipate becoming even *more* aligned over time due to the deteriorating situation.

The ability to bundle a common-value dimension with an ideological one relates our work to multidimensional bargaining where players can bundle policy with transfers (e.g., Austen-Smith and Banks, 1988; Diermeier and Merlo, 2000) or quality (e.g., Hirsch and Shotts, 2015; Hitt, Volden and Wiseman, 2017; Hirsch and Shotts, 2018). Similar to a transfer, our proposer can use the common-value dimension to obtain favorable policy on the partisan dimension. However, the common-value dimension in our model differs from a transfer because both players benefit from moving policy to the common-value ideal point, whereas a transfer benefits one player at a cost to the other. Furthermore, these papers focus on one-shot bargaining, whereas our key results emerge due to dynamic incentives. Consequently, the inefficiencies we find also do not arise in one-shot models of bundled bargaining, or if bargaining concludes once an agreement is reached (e.g., Fershtman, 1990; Jackson and Moselle, 2002; Chen and Eraslan, 2013; Salam, 2020).

Closer to our work, a small number of other papers have also analyzed how dynamic incentives can lead to inefficient agreements and delay when players bargain over multiple issues (Acharya and Ortner, 2013; Callander and Martin, 2017; Lee, 2020).⁴ Callander and Martin (2017) studies an endogenous status quo bargaining model where policies have an ideological and a quality component. In their model quality decays over time, but can be restored, while polarization is unchanging. As such, a key contribution of our paper is to highlight the role of intensifying ideological polarization in generating inefficiency on common-value issues.

Parties do delay coming to efficient agreements in Acharya and Ortner (2013) and Lee (2020). However, the features of the environment that drive delay in these papers differ from our setting, which yields different implications for policymaking and conditions for inefficiency. In particular, the focus on evolving policy problems is unique to our work, as neither of these previous papers considers issues where the costs of inefficiency compound

³This further differentiates our work from papers that find strategic polarization on a single dimension that can exhibit conflict (Dziuda and Loeper, 2018), or that study how the potential for future changes in polarization impact the selection of procedural rules (Diermeier, Prato and Vlaicu, 2020).

⁴Fox and Polborn (2024) analyzes how different veto institutions alter parties' incentives to inefficiently bundle common-value policies with divisive policies. However, in Fox and Polborn (2024) policies do not continue across periods. As such, the incentive to maintain leverage for tomorrow, which is crucial in our model, does not exist in their setting.

over time. Instead, the emergence of inefficient agreements in these models depends crucially on exogenous restrictions to the agenda, specifically the assumption that not all issues (or goods) are available for the players to bargain over in every period. Under this assumption, inefficient delay may occur because the veto player cannot find enough policy concessions to *statically* appease the proposer today under the limited menu of issues available.

Our characterization of the ideological policies that accompany inefficient choices on the common-value issue further highlights the distinct mechanism underlying our theory. Interestingly, we show that, under certain conditions, the first-period common-value policy falls short of the efficient outcome *and* the ideological policy is more moderate than the proposer’s first and second-period ideal points. Note, that this result contrasts with [Acharya and Ortner \(2013\)](#) and [Lee \(2020\)](#). The proposer in our model offers an inefficient policy even though the veto player is able to offer more ideological concessions today in exchange for more efficiency.⁵ Inefficiency emerges not because the proposer does not need more leverage, but precisely because the proposer lacks enough leverage to pull policy very far today. Thus, our theory can explain why we observe parties failing to address a worsening crisis, even when there still exist opportunities for parties to strike a bargain. Indeed, rarely, if ever, do we observe parties achieve all of their policy goals on divisive issues.

1.2 Congress, Conflict Expansion, and Polarization

Our model applies broadly to environments in which actors bargain over multiple issues as conditions evolve. To illustrate its implications, we focus on policymaking in the United States Congress.

American politics has undergone two key developments in recent decades. First, political parties have become increasingly polarized. Partisan polarization in Congress remained low until the mid-1970s but has risen sharply since then ([Barber et al., 2015](#)).⁶ Second, partisan conflict, while still rooted in traditional ideological cleavages, increasingly encompasses issues that we would consider common-value. [Lee \(2005\)](#) documents divisions over disaster relief and transportation—domains in which we might expect the preferences of “both parties and all voters [to be] located at a single point” ([Stokes, 1963](#), p. 372).⁷ As [Barber et al.](#)

⁵In the Appendix, we formalize this difference. We shut down compounding costs on the common-value issue but allow the players to weight the dimensions differently (as in [Acharya and Ortner \(2013\)](#) and [Lee \(2020\)](#)). We show that, while policies can be inefficient, the proposer only ever maintains leverage on the common-value dimension if it also pulls the ideological policy at least as far as its first-period ideal point.

⁶Although measures of polarization based on DW-NOMINATE do not distinguish underlying preferences from strategic behavior (see, e.g., [Clinton, Jackman and Rivers, 2004](#)), our theory requires only that the observed increase reflects *some* actual increase in ideological polarization on *some* dimension. The model then shows how this increase can make behavior appear polarized even on unrelated issues.

⁷The model’s insights would still apply if there were some disagreement over, for example, the optimal

(2015) observes, these areas have been “absorbed into” the broader partisan divide (p. 23). Indeed, Lee (2009) finds that one-third of the increase in congressional polarization between 1981 and 2004 is attributable to votes on issues with no clear ideological content. Parties thus appear polarized across virtually all policy dimensions, including those with little or no ideological content. In terms of our paper, there is ideological infection: partisan conflict on other dimensions has induced parties to adopt divergent stances over common-value issues.

Our theory suggests that the concurrence of these developments is no accident. A key implication is that inefficiency on a worsening common-value problem is inevitable when polarization on ideological issues rapidly intensifies, because growing polarization strengthens incentives to preserve inefficiency as a source of future leverage. The result is legislative dysfunction: Congress struggles to act efficiently even on issues over which there is little underlying disagreement (Layman, Carsey and Horowitz, 2006), and appears unable or unwilling to address even pressing national challenges, instead choosing to “kick the can down the road (...) and govern by (artificial) crises” (Barber et al., 2015, p. 41).

Previous Theories of Polarization and Congressional Dysfunction

Our theory is not the only one to draw a connection between these two developments in American politics. The combination of rising polarization and declining congressional productivity has attracted substantial attention, with two mechanisms standing out as potential explanations (for further discussion, see McCarty, 2016).

The first is the logic of the *gridlock interval* (e.g., Mayhew, 1991; Krehbiel, 1996). As parties’ ideal points move farther apart on the left–right dimension, the range of status quos over which they cannot agree on policy change expands, stalling legislation. This logic can explain paralysis on ideological issues where polarization has increased over time. It cannot, however, explain why parties appear polarized on ostensibly non-ideological issues where underlying disagreement remains limited—that is, where the status quo consistently lies outside the gridlock interval.

Theories of strategic disagreement offer a second explanation (Gilmour, 1995). Political actors may strategically support or oppose policies to damage the electoral reputations of their opponents, particularly the president. Electoral incentives can therefore generate gridlock and ineffective policymaking (Groseclose and McCarty, 2001; Lee, 2009). In a model of veto bargaining, Groseclose and McCarty (2001) shows how this “blame game” can induce Congress to propose extreme policies despite anticipating a presidential veto. Similarly, Lee (2009) argues that electoral posturing gives the opposition party incentives to block the president’s agenda, helping explain gridlock on non-ideological issues.

degree of disaster relief, provided that a worsening disaster shifted preferences in the same direction.

Although electorally motivated strategic disagreement is likely an important source of polarization on common-value issues, we argue that it offers an incomplete explanation. Empirically, as discussed below, Congress often fails to legislate effectively on common-value issues even under unified government and in domains where empirical studies find little evidence of strategic disagreement. Theoretically, the opposition party has incentives to obstruct the president even when polarization is low. As Lee notes, “Reactions to presidential leadership are thus likely to provide a constant source of party conflict in Congress” (Lee, 2009, p. 96). Thus, although polarization may intensify these incentives (McCarty, 2016), theories of electorally motivated strategic disagreement may better explain the baseline level of conflict on non-ideological issues than the effects of increasing polarization.⁸

Our model complements these accounts by showing how and when the debilitating effects of rising polarization spill over onto dimensions not directly characterized by ideological conflict. It also identifies a distinct mechanism, rooted in the evolving nature of policy problems, that can generate congressional inaction even where electorally motivated strategic disagreement is unlikely to do so.

Example: Infrastructure Spending and Congressional Dysfunction

To illustrate these contributions, we now turn to the case of infrastructure spending, which we use as a running example throughout the paper. Naturally, it is difficult to find conclusive evidence from politicians that they deliberately enacted an inefficient policy. However, we show that this case aligns with the central features of our model, and that it is also difficult to reconcile with existing explanations.

Infrastructure is often cited as a rare area of bipartisan agreement. Politicians from both parties, along with the public at large, overwhelmingly support greater infrastructure funding (Orth and Bialik, 2024). In other words, increased spending in this domain falls outside the gridlock interval of the parties in Congress. In principle, then, congressional action to ensure efficient infrastructure investment should be a “no-brainer” (Newport, 2020). Yet the reality is quite different. Although, as predicted by our model, Congress periodically passes bills approving infrastructure expenditures, the U.S. continues to chronically underinvest in this area. Current investments amount to barely half of what is needed to maintain and improve critical systems (Walpole, 2021). Even after Congress agreed to a significant spending increase during the Biden administration, the most optimistic estimates still project a \$3.7 trillion shortfall over the next decade (Webber and Phillis, 2025).

Importantly, this inefficiency cannot be fully explained by the electoral incentives posited

⁸Moreover, the model in Groseclose and McCarty (2001) requires ideological conflict between Congress and the president, making its application to common-value issues less immediate.

by theories of strategic disagreement. As Lee (2009) notes, incentives to strategically disagree are stronger for issues on the president’s agenda, where party brands should be more directly tied to policy outcomes in that domain. To evaluate the effect of these incentives on voting behavior in the Senate, she compares levels of roll-call vote differences across parties—holding the issue area constant—between Congresses in which the issue was on the presidential agenda and those in which it was not.⁹ While her results highlight that strategic disagreement increases polarization in most areas, she finds no such effect for infrastructure-related issues. This suggests that strategic disagreement does not account for persistent inefficiency in this domain.

Moreover, if the failure to pass effective infrastructure bills were due to the majority party’s desire to avoid giving the president a political win—as theories of strategic disagreement suggest—we would expect to see increased infrastructure spending under unified government, when the ruling party could claim credit for a popular policy.¹⁰ However, infrastructure spending does not systematically increase during periods of unified government (Buchheim and Fretz, 2020).

Our model offers a potential explanation. As policymakers continue to underinvest in infrastructure, the costs of this underinvestment grow increasingly larger. Experts estimate that each dollar of deferred maintenance on roads and bridges leads to 4–5 dollars in future repair costs (Orr, 2006). As maintenance or repairs are deferred, the cost of addressing them later increases, and the infrastructure becomes increasingly unreliable.¹¹ In the language of our model, then, the costs of underspending on infrastructure compound over time. Our theory argues that this property is exactly what leads to inefficient bargains over infrastructure investments.

Specifically, as infrastructure continues to deteriorate, legislators anticipate that the growing urgency will compel a larger deal later. Thus, underinvestment today can provide greater leverage in future negotiations. Importantly, these incentives to underinvest persist in our model even in periods in which one party controls all branches of government, as they are rooted in a dynamic tradeoff born out of the anticipation of *future* bargaining.¹² This may help rationalize why infrastructure spending does not increase under unified government.

Furthermore, in line with the logic of our model, infrastructure spending is routinely bundled with unrelated, more divisive issues. Omnibus appropriation bills are a clear exam-

⁹An issue is coded as being on the president’s agenda if it was mentioned in the State of the Union address immediately preceding the vote.

¹⁰Note that budget reconciliation offers the majority party a tool to circumvent the Senate filibuster and increase infrastructure spending.

¹¹<https://www.doi.gov/deferred-maintenance-and-repair>

¹²In our model with turnover, underinvestment may be induced by either the proposer or the veto player, and can therefore be sustained regardless of which party holds power in a given period.

ple, with funding for infrastructure negotiated in conjunction with issues such as healthcare, workers’ protections, and foreign policy (Luhby and Lobosco, 2022). We also note that trends in infrastructure spending in the last decades are consistent with our theoretical expectations that increased partisan conflict on ideological dimensions exacerbates inefficiency on common-value issues. Infrastructure spending as a share of GDP has declined since the 1970s, a period which has also witnessed a rapid increase in ideological polarization (Kane, Tomer and Swedberg, 2025).

As noted above, compounding costs are a natural feature of many critical, evolving issues, which may help explain the pervasive inefficiency in Congress. Conversely, our framework also sheds light on the rare instances in which Congress has acted efficiently to address a shared problem. For example, the Americans with Disabilities Act (ADA) of 1990 saw strong bipartisan support. This bipartisan support persisted even after Supreme Court decisions, such as *Sutton v. United Airlines* (1999), and *Toyota Motor Manufacturing, Kentucky, Inc. v. Williams* (2002), narrowed the definition of “disability,” thus reducing the scope of the policy. Following these rulings, the Consortium for Citizens with Disabilities worked to build consensus within the disability community on an appropriate legislative fix (Feldblum, Barry and Benfer, 2007). This agreement was reached over the course of 2006,¹³ and subsequent deliberations with business associations led to a joint policy proposal in May 2008 (Benfer, 2009). Congress passed legislation closely reflecting this proposal with bipartisan support less than four months later.

Although delays in upholding these policies are costly, the costs do not compound over time in the same way they do for crumbling roads or bridges. For example, the number of individuals protected today does not depend on whether protections were in place in the past. Therefore, in this domain, delay does not create leverage for extracting larger ideological concessions, which makes efficient legislative action far more attainable.

2 The Model

Players and policies. There are two players, a proposer (P) and a veto player (V), who interact over two periods, $t \in \{1, 2\}$. The policy space is composed of an ideological dimension X and a common-value dimension Y . In every period t , the players bargain to determine a policy outcome $(x_t, y_t) \in X \times Y$.

¹³In 2006, the CCD circulated a report titled “Failing to Fulfill the ADA’s Promise and Intent: The Work of The Courts in Narrowing Protection Against Discrimination on the Basis of Disability,” which criticized the Supreme Court’s restrictive interpretations that limited protections for individuals with disabilities. The report is unpublished but was repeatedly cited in congressional hearings (<https://www.govinfo.gov/content/pkg/CHRG-110shrg39388/html/CHRG-110shrg39388.htm>).

Preferences. The stage utility to player $i \in \{P, V\}$ in period t from a policy outcome (x, y) is $u_{it}(x) + \alpha_t v(y)$, where we define $u_{it}(x) = -(x - \hat{x}_t^i)^2$ and $v(y) = -(y - \hat{y})^2$. Thus, player i 's statically optimal policy in period t is given by its ideal point $(\hat{x}_t^i, \hat{y})$.

The policy space on the conflict dimension is given by the real line, $X = \mathbb{R}$. In each period, player P 's preferred ideological policy is to the right of player V , $\hat{x}_t^V < \hat{x}_t^P$, and we allow conflict on the ideological dimension to (weakly) increase over time, $\hat{x}_2^V \leq \hat{x}_1^V < \hat{x}_1^P \leq \hat{x}_2^P$.

On the common-value dimension, the shared ideal point $\hat{y}$ remains fixed across periods. However, the weight the players place on the issue may change over time, i.e., we allow $\alpha_1 \neq \alpha_2$, which captures the evolution of the common-value issue. Additionally, we take the common-value policy dimension to be $Y = (-\infty, \hat{y}]$.¹⁴ To reduce notation, moving forward we normalize $\alpha_1 = 1$ and define $\alpha \equiv \alpha_2$.

In our baseline model, the sequence of ideal policies on the conflict dimension, and the values of α_1 and α_2 , are common knowledge. Thus, the evolution of preferences is deterministic and the parties in our baseline model face no uncertainty.

Player i 's payoff in the dynamic game is given by:

$$\sum_{t \in \{1, 2\}} -(x_t - \hat{x}_t^i)^2 - \alpha_t (y_t - \hat{y})^2,$$

where for simplicity we assume no discounting.

Political environment. At the start of each period $t \in \{1, 2\}$ player P makes a proposal $(x_t, y_t) \in X \times Y$, which consists of a policy on the ideological issue, $x_t \in X$, and a policy on the common-value issue, $y_t \in Y$. Next, player V decides whether to accept or reject the proposal. If the proposal is accepted then the policy outcome in period t is (x_t, y_t) . If the proposal is rejected then the policy outcome in period t is (x_t^q, y_t^q) , where (x_t^q, y_t^q) is the status quo in period t . That is, proposals on the two dimensions are bundled together.

The policy outcome in the current period becomes the status quo in the subsequent period. Thus, if (x_1, y_1) is the policy outcome in period 1, then the status quo in period 2 is $(x_2^q, y_2^q) = (x_1, y_1)$. The status quo at the beginning of the game is exogenously set at (x_1^q, y_1^q) . We assume that policy on the common-value dimension is initially inefficient, $y_1^q < \hat{y}$, and that, absent bundling, the players cannot find a mutually beneficial agreement on the ideological dimension, which in our dynamic setting implies $x_1^q \in (\frac{\hat{x}_1^V + \hat{x}_2^V}{2}, \frac{\hat{x}_1^P + \hat{x}_2^P}{2})$.

¹⁴Imposing this upper bound avoids trivial multiplicities that would otherwise arise because, with symmetric utility, both players are indifferent (static and dynamically) between any two policies in Y that are equidistant from $\hat{y}$.

A definition of efficiency. In our analysis below, we will use the following terminology:

Definition 1. *A policy outcome (x_t, y_t) in period t is efficient if and only if it sets the common-value policy at $y_t = \hat{y}$.*

According to our definition, an inefficient outcome is always Pareto inefficient as well (both statically and dynamically). Specifically, static Pareto efficiency requires $(x_t, y_t) \in [\hat{x}_t^V, \hat{x}_t^P] \times \{\hat{y}\}$, i.e., efficiency on Y and policy in the current gridlock interval on X . Given our focus on understanding when players cannot agree on the common-value issue, our definition sidesteps the issue that the outcome can still be Pareto inefficient even if $y = \hat{y}$.

2.1 Discussion of the Model

Preference Evolution in a Changing World. The two key features of our model are the multidimensionality of the policy space and the possibility that these two dimensions may evolve over time. Here, we provide a few brief examples to clarify how to interpret our assumptions about the sources and nature of this evolution.

First, consider the evolution of the conflict on the X dimension. Suppose the bargaining players anticipate that their future preferences will change due to the evolution of a policy-relevant state of the world. Then player i 's preferences in period t are given by $-(x - \gamma_i \cdot \omega_t)^2$, where $\omega_t \geq 0$, $\gamma_P > 0$ and $\gamma_V < 0$. Here, the two players agree on the fundamental state of the world ω_t , but disagree on the implications for policymaking. Our assumption of weakly increasing conflict reflects a situation where players anticipate ω_t increasing over time, increasing the distance between their ideal points. For example, this can capture the observation that globalization has contributed to an increase in polarization, with an intensifying conflict between left- and right-wing parties on issues such as immigration or economic protectionism.¹⁵

Within the context of legislative bargaining, shifting ideal points can also model (in reduced form) situations in which parties expect increasing polarization among their respective constituents or members, continuing the trend of previous decades. In this view, the party leadership acts as a delegate of the members. As a result, if the party anticipates an evolution in the preferences of its base, it will act *as if* it expects its own preferences to change. As long as the party leadership aims at maximizing the long-run welfare of the party, it

¹⁵See Rodrik (2021) for a review of the literature on this topic, discussing how globalization and the resulting labor market shocks can increase polarization by driving a “greater wedge between winners and losers” of this process (p. 164).

will face the dynamic incentives we describe in the model (this is true even if the members themselves do not foresee the preference evolution).

Next, consider the evolution of the common-value dimension. In our model, changing circumstances affect how costly it is to implement an inefficient policy on the common-value issue in each of the periods. For instance, a gradually deteriorating infrastructure network or a deepening crisis becomes more and more costly over time if they remain unaddressed. By contrast, some problems may partially abate even without direct intervention, in which case the cost of inefficiency declines over time. Under quadratic-loss utility, the marginal cost of inefficiency is parametrized by the weight that players place on the common-value issue. We thus allow for both patterns, a worsening crisis or a stabilizing problem, by assuming this weight may differ across periods. When this weight rises, $\alpha_2 = \alpha > \alpha_1 = 1$, inefficiency becomes more costly in the second period, as in the case of a worsening crisis. When it weakly falls, $\alpha \leq 1$, inefficient policies are (weakly) less costly in the second period, corresponding to situations that gradually stabilize. It is useful to introduce the following definition:

Definition 2. *The costs of inefficiency are compounding over time if $\alpha > 1$.*

Throughout the analysis, we use the example of federal infrastructure spending in the United States Congress to offer a concrete interpretation of the model. In this case, P and V can be thought of, respectively, as the majority party and the minority party, with the minority having veto power when it controls the presidency or enough seats to effectively filibuster.

Under this interpretation, the common-value policy y captures the level of investment in infrastructure maintenance and upgrades, while dimension X is a more divisive issue — such as taxes, healthcare or regulatory policies. A policy (x, y) is then a level of infrastructure spending and a proposal on a more conflictual policy issue, bundled together in an omnibus bill. Crucially for our theory, poorly maintained infrastructures rapidly decay, becoming increasingly unreliable. In the context of our model, then, fixing deteriorated infrastructure grows more salient over time: $-\alpha(y - \hat{y})^2 < -(y - \hat{y})^2$, and the costs of inefficiency compound across periods.¹⁶

In turn, growing polarization on X reflects the trend of increasing in partisan conflict over the last decades.

¹⁶In our model, even if the first-period policy is efficient the players must pass policy again tomorrow to address the issue. We could instead assume that if $y_1 = \hat{y}$ then the problem is solved, e.g., efficient spending today also prevents infrastructure from deteriorating tomorrow. This would not impact our results because, in equilibrium, if the efficient policy is passed in the first period, $y_1 = \hat{y}$, it remains in place in the second.

Model Assumptions. In the baseline model, we abstract from proposer turnover. This assumption allows us to isolate the mechanism driving our results, while shutting down a feature that has been shown to generate inefficiency in prior work. We also adopt a two-period framework to obtain sharper characterizations. Neither assumption, however, is essential to the model’s core mechanism. In the last section of the paper, we consider extensions relaxing each of these assumptions.

In order to more clearly illustrate our results, we assume the players share exactly the same ideal policy on the Y dimension. However, our intuitions apply broadly to cases in which players face some disagreement on this dimension, but the status quo is outside of the gridlock interval. In other words, the crucial feature of the Y dimension is that players always agree on the optimal *direction* of policy change. For example, Democrats and Republicans may disagree on exactly *how much* infrastructure spending is optimal, but they agree that increased investments are necessary.

In contrast, the assumption that $\hat{x}_2^V \leq \hat{x}_1^V < \hat{x}_1^P \leq \hat{x}_2^P$ ensures that the X dimension *always* features conflict and thus distinguishes it from a common-value dimension. Absent this assumption, a policy that is in the gridlock interval on X in the first period can become unstuck, even without the Y dimension, if the ideal points of both players move closer together or shift in the same direction in the second period.

3 Analysis

Moving to the analysis, our solution concept is subgame perfect equilibrium and we proceed by backward induction.

In the second period, players only consider their static payoffs. Thus, V accepts any policy (x, y) such that:

$$-(x - \hat{x}_2^V)^2 - \alpha(y - \hat{y})^2 \geq -(x_2^q - \hat{x}_2^V)^2 - \alpha(y_2^q - \hat{y})^2. \quad (1)$$

V is only willing to accept an ideological policy that moves farther away from its ideal point on X if the proposal improves on the common-value status quo. Consequently, if the inherited status quo is inefficient, $y_2^q \neq \hat{y}$, then P can extract concessions on the conflict dimension by proposing a bundle that moves the common-value policy closer to $\hat{y}$.

The acceptance constraint in (1) indicates that, in our setting, ideological riders can pass only by riding on top of sufficiently attractive improvements on a shared policy problem. In the infrastructure application, this condition corresponds to the minority party accepting a package that shifts tax, healthcare, or regulatory policy away from its preferred position only

if the bill also delivers enough additional infrastructure spending relative to the inherited baseline.

Anticipating this constraint, in equilibrium P chooses its proposal to maximize $u_{P2}(x) + v_2(y)$ subject to (1). Let $\bar{x}(x_2^q, y_2^q)$ be the upper solution to:

$$-(x - \hat{x}_2^V)^2 - \alpha(y - \hat{y})^2 = -(x_2^q - \hat{x}_2^V)^2 - \alpha(y_2^q - \hat{y})^2. \quad (2)$$

Lemma 1 characterizes P 's optimal second-period proposal.

Lemma 1. *In the second period P proposes $y_2^* = \hat{y}$ and $x_2^*(x_2^q, y_2^q) = \min \left\{ \hat{x}_2^P, \bar{x}(x_2^q, y_2^q) \right\}$.*

Proposing $y = \hat{y}$ maximizes V 's utility from the offer on the common-value dimension, and thus maximizes V 's willingness to accept a worse payoff on the ideological dimension. As such, the efficient policy $y = \hat{y}$ both maximizes P 's payoff on the common-value dimension *and* the extent to which P can move the outcome towards its ideal policy $\hat{x}_2^P$. Therefore, the equilibrium policy outcome is always efficient in the last period, emphasizing that the ability to bundle dimensions does not lead to inefficiency absent dynamic motives.

Lemma 1 highlights that the second-period equilibrium outcome depends on the policy implemented in the previous period. Turning to the first period, then, the players must balance their static preferences against their dynamic incentives. The players' dynamically optimal policies induced by these strategic considerations are central to our concept of ideological infection. It is therefore useful to introduce the following definition of a player's dynamic ideal point.

Definition 3. *Player i 's dynamic ideal point $(\hat{x}_d^i, \hat{y}_d^i)$ solves*

$$\max_{x,y} -(x - \hat{x}_1^i)^2 - (y - \hat{y})^2 - (x_2^*(x, y) - \hat{x}_2^i)^2.$$

Thus, player i 's dynamic ideal point is the policy that i would choose to implement today, anticipating bargaining tomorrow.

Lemma 2 provides an initial characterization of these dynamic ideal points.

Lemma 2. *On the Y dimension $\hat{y}_d^P \leq \hat{y} = \hat{y}_d^V$. On the X dimension $\hat{x}_d^V \in [\hat{x}_2^V, \hat{x}_1^V]$ and $\hat{x}_d^P \in [\hat{x}_1^P, \hat{x}_2^P]$.*

Although both players prefer the efficient common-value policy today, their incentives to influence future policy outcomes can lead to divergent dynamic preferences. From equation (1), we see that moving y_2^q further from $\hat{y}$ increases $\bar{x}(x_2^q, y_2^q)$, i.e., it increases the proposer's second-period leverage. As such, the proposer's second-period equilibrium payoff increases

when the inherited common-value policy is more inefficient, i.e., further from $\hat{y}$, as shown in Figure 1. Consequently, P may prefer an inefficient policy that preserves its bargaining power for the second period.

Returning to our example, P 's dynamic ideal point reflects not only the immediate desire to repair roads and bridges, but also the value of keeping some unmet need on the table so that future infrastructure packages can be used to extract ideological concessions from the opposing party.

Conversely, the veto player's second-period payoff increases when y_2^q moves closer to $\hat{y}$, and is maximized exactly at $y_2^q = \hat{y}$, as this erases the proposer's leverage in the second period. The veto player prefers the efficient $\hat{y}$ both *statically* and *dynamically*.

Notice that the players' dynamic preferences on the conflict dimension can also be distorted from their static ideal policy. For player i a policy closer to $\hat{x}_2^i$ improves its equilibrium policy payoff tomorrow and worsens the other player's payoff. As such, each player's dynamic ideal point lies between its first and second-period optimum.

It is important to note that this distortion of preferences on X would emerge even in a model without the Y dimension. Absent the Y dimension, policy is always stuck (in the gridlock interval) and P 's dynamic ideal point is again in $(\hat{x}_1^P, \hat{x}_2^P)$, as this balances its payoff from today versus tomorrow. In contrast, in a model with only the Y dimension there is no difficulty in agreeing to the efficient policy today and tomorrow. Consequently, any preference divergence on the common-value dimension is due solely to the existence of multiple dimensions.

We now formally introduce the concept of ideological infection.

Definition 4. *There is ideological infection if and only if $\hat{y}_d^P < \hat{y}$.*

If the proposer's incentives to extract future ideological concessions are strong enough, then it prefers an inefficient common-value policy in the first period. We refer to this distortion of the proposer's preferences as *ideological infection*. Notice that we explicitly distinguish between ideological infection and inefficiency (Definition 1). Inefficiency concerns the policies implemented in equilibrium, whereas infection refers to an *induced* conflict between the players' preferences on the common-value dimension, and thus would manifest observationally as disagreement in their *stated* preferences. This distinction is theoretically important because, as we show below, one can arise without the other. While infection always leads to inefficiency, inefficiency may arise even in the absence of ideological infection.

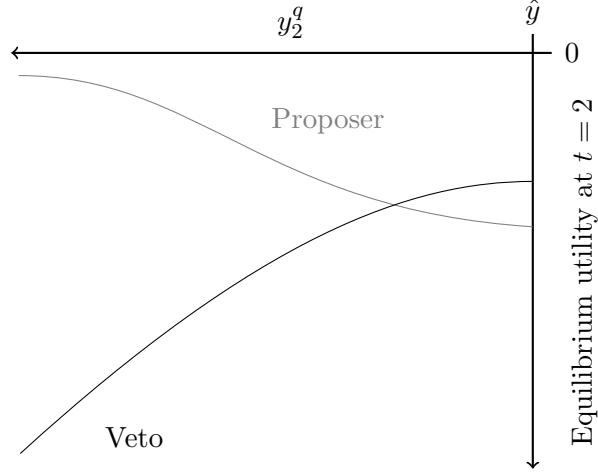


Figure 1: Second-period equilibrium payoffs as a function of the second-period status quo on the common-value dimension.

3.1 The Role of Compounding Costs of Inefficiency

In our analysis, we are interested in unpacking the conditions under which (a) ideological infection emerges and (b) the equilibrium policy outcome is inefficient. Our first result shows that compounding costs of inefficiency are *necessary* for both infection and inefficiency.

Proposition 1. *If the costs of inefficiency do not compound over time, $\alpha \leq 1$, then there is no ideological infection, $\hat{y}_d^P = \hat{y} = \hat{y}_d^V$. Furthermore, the equilibrium policy outcome is efficient, $y_1^* = \hat{y}$.*

To see the importance of compounding, recall that an inefficient policy which undershoots $\hat{y}$ imposes costs on both parties in the first period *and* increases the cost to the veto player for maintaining the status quo in the second period. In turn, this allows P to pull x_2 closer to $\hat{x}_2^P$. Indeed, in equilibrium the veto player is made indifferent between keeping the status quo and accepting the proposer's offer in the second period; therefore, the amount of concessions the veto player will have to grant on the conflict dimension is exactly $\alpha(y_2^q - \hat{y})^2$.¹⁷

Suppose the costs of inefficiency do not compound over time, $\alpha \leq 1$. This implies that $\alpha(y_1 - \hat{y})^2 \leq (y_1 - \hat{y})^2$ for any $y_1 < \hat{y}$. The cost that the proposer can impose on the veto player in the second period is *lower* than the loss from an inefficient policy that *both* players bear in the first period. As a result, the ideological concessions the proposer would extract in the second period are smaller than the immediate cost of the inefficiency. The

¹⁷This is always true in equilibrium if the first-period policy is inefficient. If the policy is efficient, then the veto constraint may not bind in the second period.

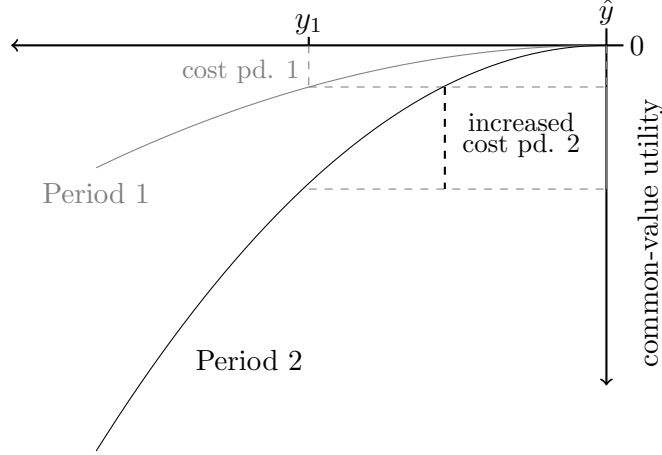


Figure 2: common-value utility in first and second period under compounding costs, $\alpha > 1$, as a function of y_1 .

proposer has no incentive to pursue an inefficient policy, and no ideological infection emerges. Furthermore, the equilibrium outcome is always efficient.

In contrast, when $\alpha > 1$, the cost of residual inefficiency increases across periods, creating the *wedge* illustrated in Figure 2, and potentially allowing the proposer to profit from choosing an inefficient policy in the first period.

In the case of infrastructure spending, this result clarifies that our explanation for chronic underinvestment hinges on the compounding nature of deferred maintenance. If failing to repair roads and bridges today simply shifted costs across years without making future deterioration more severe, and hence more costly, then even highly polarized parties would still agree to implement the efficient level of investment in the first period. With decaying infrastructure, by contrast, starting from an inefficiently low level of maintenance or investment means that the marginal benefit of an extra dollar of spending is higher tomorrow than today, which is captured in the model by $\alpha > 1$, so there is a greater cost to deviating from the efficient level $\hat{y}$. This increase in the marginal benefit of spending or, equivalently, in the marginal cost of underinvestment, is precisely what opens the door to inefficient agreements.

3.2 The Role of Increasing Extremism and Polarization

The results of the previous section show that compounding costs on the common-value dimension are necessary for ideological infection and inefficient policymaking to arise in equilibrium. We now complete the analysis by showing that whether infection and inefficiency actually occur also depends on how preferences evolve along the conflict dimension.

Throughout this section, we maintain the assumption that the costs of inefficiency are

compounding over time.

Assumption 1. *The costs of inefficiency are compounding over time: $\alpha > 1$.*

We begin by characterizing the conditions for ideological infection.

Lemma 3. *There exists a unique $\xi_{infect} > \hat{x}_1^P$ such that ideological infection emerges if and only if $\hat{x}_2^P > \xi_{infect}$. Furthermore, ξ_{infect} is decreasing in α .*

Recall that the proposer’s dynamic ideal point is the policy P would implement absent the veto constraint in the first period, and we say there is ideological infection whenever this dynamic ideal point is such that $y < \hat{y}$. If the proposer implements the efficient common-value policy today, then the conflict dimension policy will be gridlocked in the second period and whatever policy the proposer implements today will remain in place tomorrow. Therefore, absent the veto’s first-period constraint, if the proposer were to choose an efficient common-value policy today, it would pair it with an ideological policy $x_1 = \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$, which balances the first and second-period costs. Alternatively, the proposer could pursue an inefficient policy on the common-value dimension to maintain leverage for the future and pull the second-period ideological policy further to the right.

Increasing $\hat{x}_2^P$ increases the return to the proposer for moving the second-period outcome further to the right, due to concave utility. Infection therefore occurs if and only if $\hat{x}_2^P$ is sufficiently extreme. If P is relatively moderate then the gains from preserving leverage do not outweigh the costs of inefficiency. Importantly, the cutoff ξ_{infect} lies strictly above $\hat{x}_1^P$. If P ’s ideological preferences are stable over time, then clearly the policy $x = \hat{x}_1^P = \hat{x}_2^P$ and $y = \hat{y}$ maximizes the proposer’s utility today *and* tomorrow, as its dynamic and static preferences coincide. Thus, $\hat{x}_2^P > \hat{x}_1^P$ is necessary for the proposer’s preferences over Y to be infected: when the proposer anticipates needing more leverage in the future to adapt policy to changing ideological preferences, then it may favor an inefficient policy today that “fabricates” a future crisis.

Importantly, how much more extreme the proposer must become for its preferences to be infected depends on α . The parameter α measures how quickly the costs of inefficiency compound. If α is large, then an inefficient policy today generates a large amount of leverage for the proposer tomorrow. Consequently, greater values of α incentivize the proposer to prefer the inefficient policy.

As noted in the introduction, the result that the proposer’s preferences are sometimes infected distinguishes our theory from existing explanations for congressional inaction on common-value problems, which do not predict inefficiency when the majority party is unconstrained in policymaking (see, e.g., [Lee, 2009](#)). Our findings thus may help explain why

the majority may deliberately underinvest in infrastructure even under unified government. Doing so allows the party to later bundle a larger investment package with policies that better align with increasingly extreme ideological goals, for instance because of a polarizing electoral base or shifting circumstances.

Next, we move from preferences to outcomes. Proposition 2 characterizes the conditions under which the equilibrium policy is inefficient.

Proposition 2. *There exists a unique $\bar{\xi}$ such that the equilibrium policy is inefficient if and only if $\hat{x}_2^P > \bar{\xi}$. Furthermore, there exist unique $1 < \underline{\alpha} < \bar{\alpha}$ such that:*

- *If $\alpha > \bar{\alpha}$, then $\bar{\xi} = \xi_{infect}$;*
- *If $\alpha \in (\underline{\alpha}, \bar{\alpha})$, then $\bar{\xi} \in (\hat{x}_1^P, \xi_{infect})$;*
- *If $\alpha < \underline{\alpha}$, then there exist conditions under which $\bar{\xi} < \hat{x}_1^P$.*

Proposition 2 establishes that increasing the proposer's second-period extremism also leads to inefficient policy outcomes. In particular, whenever P 's second-period preferences are sufficiently extreme, inefficiency is inevitable in the first period. However, once again, the amount of extremism required is sensitive to α . When α is sufficiently high, infection of the proposer's preferences becomes the key driver of inefficiency, so inefficiency requires that P become sufficiently *more extreme over time*. By contrast, when α is low, policy can be inefficient even if the proposer's preferences remain stable, $\hat{x}_2^P = \hat{x}_1^P$.

This result is due to the dual role that α plays in shaping the proposer's incentives when we take into account the constraint from the veto player in the first period. On one hand, as previously discussed, a higher value of α makes preserving leverage more valuable. This is the *leverage effect*. On the other hand, a greater α also makes keeping the status quo more costly to the veto player in the first period, since V anticipates being made indifferent in period 2. This *constraint effect* weakens P 's incentive to propose an inefficient policy because V is already willing to make significant ideological concessions.

When α is close to 1, the veto player is very demanding in the first period and the constraint effect dominates. As such, even if P 's preferences are not infected, the costs of inefficiency are small enough that V will not accept P 's dynamic ideal point because it would pull policy too far right, specifically to $x = \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$. The proposer then faces two options. The first is to propose $y = \hat{y}$, which is efficient on the Y dimension, and lets P pull the ideological policy as far right as possible today, to some $x_0 < \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$, at which point policy is stuck in the second period. Alternatively, P can propose $y < \hat{y}$ to preserve leverage for the second period. Buying this leverage requires proposing a more moderate first-period position $x < x_0$ to compensate the veto player for today's common-value loss and tomorrow's



Figure 3: The cutoff $\bar{\xi}$ as a function of α . The equilibrium policy is inefficient whenever P 's second-period ideal point on the conflict dimension is above the curve.

concessions relative to the efficient policy. However, with compounding costs of inefficiency, this bundle grants the proposer enough additional leverage in the second period to secure a more extreme outcome $\bar{x}(x, y) > x_0$. The proposer thus faces a tradeoff between gaining on the common-value *and* ideological dimensions today against obtaining a better ideological payoff tomorrow. Under some conditions, P pursues an inefficient policy to compensate for having limited leverage today.

In particular, similar to the logic of infection, the tradeoff breaks in favor of inefficiency if and only if $\hat{x}_2^P$ is sufficiently large. Furthermore, because all feasible policies are to the left of $\frac{\hat{x}_1^P + \hat{x}_2^P}{2}$, concavity implies that this cutoff is less demanding than the condition for infection, $\bar{\xi} < \xi_{infect}$. Indeed, when α is just above 1 there exist parameter values such that equilibrium policy is inefficient even if the proposer does not become more extreme in period 2, $\bar{\xi} = \hat{x}_1^P$. As α increases away from 1, however, V becomes willing to accept a more extreme x today, and so the constraint effect weakens P 's incentives to propose the inefficient policy. Consequently, when α is sufficiently high, $\alpha > \underline{\alpha}$, the threshold $\bar{\xi}$ always rises above $\hat{x}_1^P$.

Once the costs of compounding are very high, $\alpha > \bar{\alpha}$, the veto player's payoff from keeping the status quo is sufficiently bad that the proposer is unconstrained when choosing policy. Thus, if policy is inefficient it must be because the proposer's preferences are infected. Notice that this implies that the constraint effect of α is no longer relevant, only the leverage effect due to the compounding costs of inefficiency remains at play. In this region, then, $\bar{\xi}$ coincides with ξ_{infect} and is decreasing in α .

Overall, increasing α makes P less inclined to propose the inefficient policy via the constraint effect, but more inclined through the leverage effect. These contrasting effects imply that the conditions for inefficiency are non-monotonic in α , and they are most likely to hold

when α is either very large or very close to 1. Figure 3 depicts the equilibrium characterization of inefficiency.

Corollary 1. *For $\hat{x}_1^P$ sufficiently large, $\bar{\xi}$ is maximized at an intermediate value of α . Moreover, $\lim_{\alpha \rightarrow 1} \bar{\xi} < \hat{x}_1^P$ and $\lim_{\alpha \rightarrow \infty} \bar{\xi} = \hat{x}_1^P$.*

This result and the preceding discussion, when contrasted with Proposition 1, highlight that α affects incentives for inefficiency on both the extensive and intensive margins. On the extensive margin, $\alpha > 1$ is necessary for inefficiency because compounding costs generate incentives to distort policy. On the intensive margin, however, the strength of these incentives also depends on the degree of compounding. As a consequence, as Corollary 1 shows, lowering α close to 1 can actually exacerbate the problem.

Next, Corollary 2 provides a characterization of x_1^* that further illuminates the mechanism underlying inefficiency in our setting.

Corollary 2. *There exist parameter values such that, in equilibrium, $y_1^* < \hat{y}$ and $x_1^* < \hat{x}_1^P$.*

Corollary 2 shows that it is possible for the first-period equilibrium policy outcome to be inefficient on the common-value dimension *and* more moderate than the proposer's first-period ideal on the conflict dimension. This result is surprising. Recall that the veto player must always be compensated for the static and dynamic costs of inefficiency. Therefore, a more efficient y_1 would allow the proposer to pull the ideological policy closer to both its first- and second-period ideal points while also avoiding the cost on the common-value issue.

Why, then, does the proposer not choose a more efficient bundle? The reason is that a more efficient common-value policy does not take advantage of the compounding effect of α . It thus leads to a much more moderate ideological policy in the second period, despite being paired with a more extreme ideological policy in the first. Therefore, under some conditions, the proposer prefers to incur costs today on both dimensions in order to extract larger concessions tomorrow. In terms of policymaking in Congress, chronic inefficiencies do not arise because the majority party has already secured all of its desired concessions on divisive issues, but precisely because it is unable to do so.

In our analysis above, we characterized equilibrium outcomes by focusing on the extremism of the proposer's second-period preferences. Our final results complete the picture by examining the effects of the veto player's anticipated extremism as well. In Lemma 4, we restrict attention to the case $\alpha > \underline{\alpha}$, which is the region where the cutoff $\bar{\xi}$ is a function of $\hat{x}_2^V$ for all parameter values.

Lemma 4.

- If $\alpha \in (\underline{\alpha}, \bar{\alpha})$, then decreasing $\hat{x}_2^V$ decreases $\bar{\xi}$;
- If $\alpha > \bar{\alpha}$, then decreasing $\hat{x}_2^V$ increases $\bar{\xi}$;
- Furthermore, decreasing $\hat{x}_2^V$ increases both $\bar{\alpha}$ and $\underline{\alpha}$.

Decreasing $\hat{x}_2^V$ moves the veto player further away from the proposer, and thus makes V more resistant to changes on the ideological dimension in both periods. This has two contrasting effects on the emergence of inefficiency. First, tightening the first-period constraint strengthens P 's incentives to exploit the compounding effect of α and build more leverage for tomorrow, since V is very unwilling to accept changes on the X dimension today. At the same time, however, making V more stubborn in the second period reduces the benefit from creating leverage, lowering P 's willingness to incur the costs of an inefficient policy on Y . Thus, the net effect of a decrease in $\hat{x}_2^V$ can go in either direction and depends on α .

As discussed above, if α is very high then the proposer is unconstrained in the first period. In this case, $\hat{x}_2^V$ matters only through the second-period constraint, and a more extreme veto makes inefficiency harder to sustain. In contrast, when $\alpha < \bar{\alpha}$ the effect goes in the other direction, because changes in $\hat{x}_2^V$ primarily operate through the first-period constraint. In sum, for $\alpha > \bar{\alpha}$, a more moderate veto player unambiguously makes inefficiency more likely. For $\alpha \in (\underline{\alpha}, \bar{\alpha})$ the net effect is ambiguous. If the veto player becomes more moderate then $\bar{\xi}$ increases, making inefficiency harder to sustain, but $\bar{\alpha}$ also decreases, which may push the environment into the region where inefficiency is becoming easier to sustain.

Thus, while increasing polarization driven by the proposer's preferences always makes inefficiency more likely to emerge, polarization driven by the veto player may have the opposite effect. However, increasing polarization in the real world likely reflects *both* parties becoming more extreme. The overall effect of increasing polarization on inefficiency then depends on the relative impact of the proposer's and veto player's preferences. In our final result of this section, we set $\hat{x}_t^V = -\hat{x}_t^P \equiv \hat{x}_t$, and study the effect of a symmetric increase in polarization.

Proposition 3. *Let $\hat{x}_t^P = -\hat{x}_t^V \equiv \hat{x}_t$. The equilibrium policy is inefficient only if $\hat{x}_2$ is sufficiently large. Furthermore, for α sufficiently large, there exists $\xi^{pol} \in (\hat{x}_1, \infty)$ such that the equilibrium policy is inefficient if and only if $\hat{x}_2 > \xi^{pol}$.*

When both players become symmetrically more extreme, the effect through the proposer's preferences always dominates. Thus, increasing the amount of polarization the players anticipate in the second period always pushes towards inefficiency. Note that the same result

holds even if the veto player moves relatively more than the proposer, as long as the difference is not too large. Furthermore, analogous to Proposition 2, for α sufficiently large the relevant cutoff $\bar{\xi}^{pol}$ lies strictly above $\hat{x}_1$. That is, rising ideological polarization over time is both necessary and sufficient to generate inefficiency when the costs of inefficiency are rapidly compounding. Proposition 3 also suggests that the anticipation of depolarization can help restore efficiency today. If α is large, then any future depolarization will remove the distortion. When α is small, inefficiency may persist when players expect the amount of conflict to decrease, similar to the case where $\bar{\xi} = \hat{x}_1^P$ in Proposition 2. However, anticipating sufficient depolarization will still always correct for inefficiency in the first period.

Implications for Empirical Research. In concluding this section, we note that our model yields empirical predictions for when distortions should be most prevalent. Corollary 1 implies that, as long as the initial level of polarization is not too low, inefficiency is more likely to be observed on issues where the underlying problem deteriorates either very rapidly or very slowly when left unaddressed. Empirically, the rate of compounding costs could be captured, for instance, by estimating the additional repair costs from delaying bridge maintenance by one year, or the escalation of damages from postponing climate mitigation. Using such measures as proxies for the degree of compounding, the model predicts that the likelihood a crisis remains unresolved is greatest when the crisis worsens either very rapidly or very slowly. Furthermore, Proposition 3 shows that a sufficiently rapid (symmetric) increase in ideological polarization is sufficient for inefficiency, and necessary when compounding is rapid. Thus, underinvestment on worsening common-value problems should correlate with a trend of increasing polarization on ideological issues, but the relation is mediated by the degree of compounding.

In the United States, both “slow-burning” crises — such as climate change or the solvency of Social Security — and rapidly deteriorating ones — such as decaying infrastructure or recurring debt-ceiling standoffs and government shutdowns — are frequently met with delayed and incomplete responses. Additionally, as discussed in the introduction, worsening gridlock on common-value issues has gone hand in hand with rising ideological polarization on traditionally conflictual dimensions. To the best of our knowledge, however, the joint relationship between crisis severity, polarization, and the probability of timely resolution has not been systematically analyzed. Our framework thus offers a concrete set of implications for understanding trends in polarization and policymaking that can be used to form hypotheses in future empirical work.

4 Extensions

4.1 Turnover

Up to this point, we have assumed that player P remains the proposer in both periods, emphasizing that inefficiency and ideological infection in our setting do not stem from fear of the other player taking power. We now turn our attention to the implications of turnover for our mechanism. Specifically, we assume P is the proposer in period 1 and remains so with probability $\rho \in (0, 1)$ in period 2, while V becomes the proposer with probability $1 - \rho$. Thus, ρ characterizes the degree of *persistence* in political power.

For simplicity, we focus on analyzing when the proposer's preferences are infected. Note that infection is necessary and sufficient for the policy outcome to be inefficient whenever the first-period constraint from the veto player is not binding. This will be the case if the initial status quo y_1^q is sufficiently inefficient, e.g., a severe crisis has hit and so the prevailing policy is not well-suited to the circumstances. Alternatively, a party will not be constrained if it has a sufficiently large majority in the first period that it can pass policy without support from the minority party.

Our first result establishes that the qualitative conclusions from our baseline model remain robust in this richer setting. If there are compounding costs then inefficiency arises in equilibrium only if the proposer's second-period ideal point is sufficiently extreme. However, with turnover this is no longer sufficient. There must also be enough persistence in political power.¹⁸

Proposition 4. *There exists a unique $\hat{\rho} \in (0, 1]$ such that the proposer's preferences are infected if and only if $\rho > \hat{\rho}$. Moreover, $\hat{\rho} < 1$ if and only if $\hat{x}_2^P > \xi_{infect}$.*

If V is certain to be in power tomorrow ($\rho = 0$), then the proposer has no reason to choose an inefficient policy today. The leverage generated tomorrow from an inefficient status quo will just be used by V to pull policy further left on the ideological dimension. More generally, whenever the probability of being replaced is strictly positive, the proposer faces a tradeoff. On one hand, distorting today's policy preserves leverage in case P maintains power in the second period. On the other hand, the efficient policy acts as a form of insurance against

¹⁸The case of inefficiency when the proposer is constrained is more nuanced, as both the proposer's and veto player's preferences may become infected and drive inefficiency in equilibrium. In the Appendix, we show several numerical examples that suggest that the qualitative conclusions of Proposition 4 continue to hold. Even when the proposer is constrained, sufficiently high ρ is necessary for inefficiency to emerge in equilibrium. Although the veto player may prefer an inefficient policy when ρ is very low, the proposer uses its first-mover advantage to avoid an inefficiency that is costly today and likely to be used against it tomorrow.

turnover by limiting V 's future bargaining power should the proposer be ousted. As the risk of being ousted rises, the insurance motive strengthens. For a sufficiently low ρ , P always prefers to implement an efficient policy, and its preferences are not infected.

In contrast, if ρ is high, then P can prefer an inefficient policy. Similar to the baseline model, this occurs when $\hat{x}_2^P$ is sufficiently extreme, because this makes maintaining additional leverage more valuable to P tomorrow. However, the result with turnover is more subtle because this leverage is only valuable to P *conditional on remaining in power*. If instead V gains proposal power tomorrow, then this leverage will be used to pull policy further left, and hence increasing $\hat{x}_2^P$ makes an inefficient policy less attractive to P because $\hat{x}_2^P$ will be even further from the second-period policy. When ρ is high the proposer is likely to retain power. Consequently, if $\hat{x}_2^P$ increases, then P 's dynamic considerations are dominated by the incentive to preserve leverage, leading to infection and inefficiency.

We note that the cutoff $\hat{\rho}$ can be very close to $\frac{1}{2}$.¹⁹ Despite being risk-averse, the proposer may still choose to create inefficiency today, even when the resulting leverage is almost as likely to instead be used by V tomorrow.

In terms of infrastructure negotiations, Proposition 4 implies that, all else equal, the majority party is more likely to pursue an inefficiently low level of infrastructure spending when it has a strong electoral position. Electoral security pushes the party to dynamically prefer an inefficient policy. This finding complements our results from the previous section, and generates an additional empirical implication of our theory that future research may bring to the data.

Proposition 4 has important implications for understanding the relationship between political power and inefficient policymaking on common-value issues. Specifically, the findings of Proposition 4 run counter the classic logic of the stationary bandit theory (Olson, 1993). In Olson (1993), a ruler who expects to stay in power behaves like a “stationary bandit”, with incentives to invest in public goods to guarantee that future productivity remains high and he can continue extracting rents. Instead, it is the expectation of losing power that leads an autocrat to behave as a “roving bandit”: underinvesting, depleting public goods, and over-extracting today, since he does not internalize the future costs of these behaviors.

This result emerges because Olson considers a ruler with absolute power. In our model, instead, a constrained proposer has to “share” power with the veto player, and this reverses the logic of the stationary bandit. The cost of an inefficient status quo inherited from the first period is borne entirely by the second-period veto player and used as leverage on the ideological dimension. As a result, the prospect of losing power disciplines the first-period proposer because efficiency becomes a method to insure against the possibility that someone

¹⁹See the numerical simulations in Appendix A.2.

else will inherit and exploit the leverage created by today’s distortion.

This result also distinguishes our theory from existing work in political economy that finds the prospect of turnover may generate inefficient distortions because a statically suboptimal policy ties the government’s hands in the future. For example, a conservative incumbent may choose a higher-than-efficient level of government borrowing if it risks losing the next election, because debt limits the scope of future government spending if a more expansionary party gains power tomorrow (see Persson and Svensson, 1989).²⁰ In contrast, our analysis shows that when policy is multidimensional and inefficiency on one issue can be used as leverage on others, turnover has a positive effect on equilibrium outcomes.

4.2 Long-run Outcomes

In the model analyzed so far, the players interact over two periods, and due to end-period effects the efficient policy is always implemented in equilibrium in the second. In this section, we extend the analysis to an infinite time horizon and show that, under some conditions, the equilibrium outcome is inefficient in *every* period.

For simplicity, we assume that the players’ preferences on the conflict dimension evolve according to $\hat{x}_t^P = -\hat{x}_t^V = \beta^t$. The parameter β thus captures the speed at which ideological polarization increases over time. On the common-value issue, α_t is given by α^t , with $\alpha > 1$ to capture compounding costs. Future payoffs are discounted at a common rate $\delta \in (0, 1)$, and we assume that $\delta\beta^2 < 1$ and $\delta\alpha < 1$ to ensure that discounted sums converge. Our solution concept is stationary Markov perfect equilibrium (henceforth “equilibrium”).²¹

Proposition 5. *Suppose that $\alpha > \frac{1+\sqrt{\delta}}{2\delta}$. If $\beta > \frac{1}{2\delta\alpha-1}$, then in any equilibrium the policy is inefficient in every period t .*

Proposition 5 shows that, provided compounding is not too slow, if ideological polarization is increasing sufficiently rapidly, this is enough to generate inefficiency in *every* period. This confirms the central insight of the baseline model, which highlights a complementarity between compounding costs and ideological polarization in producing inefficient outcomes.

This result indicates that our model may explain instances where an efficient solution to a shared problem is not merely delayed, but in fact *never* attained. Interestingly, this does not imply that policy is gridlocked. New legislation passes in each period and the common-value

²⁰See also Buisseret and Bernhardt (2017) for a model with related incentives in a spatial setting.

²¹We also restrict attention to equilibria in which continuation values are continuous. See the Appendix for further details. Focusing on stationary Markov perfect equilibria is standard in endogenous status quo bargaining models.

component of the policy path gradually moves toward the efficient level.²² It never reaches it, however, because at every date the proposer finds it optimal to exploit the effect of compounding and leave some inefficiency on the table in order to preserve the ability to extract more favorable ideological concessions in the future.²³ This dynamic pattern is reminiscent of the U.S. experience, where Congress periodically enacts sizable infrastructure packages, yet there remains a broad consensus that public funds are persistently underprovided.

5 Conclusion

Addressing policy problems often requires the agreement of multiple parties. However, bargaining parties regularly have trouble reaching an efficient solution, even on issues where they all agree that the situation will grow increasingly worse if there is a lack of action. Our model shows that agreements on these common-value policy problems are vulnerable to being distorted by disagreements on other issues precisely because they worsen over time. Furthermore, if parties anticipate becoming more entrenched on the conflict dimension in the future, then ideological infection of preferences on common-value issues is inevitable.

Our model assumes that the ideological and common-value issues are always bundled together in a single proposal, reflecting the prevalence of omnibus legislation and package deals in many contemporary legislatures. This assumption is without loss of generality in our two-dimensional environment, as an agenda setter always benefits from extracting concessions on an ideological issue by bundling it with a common-value issue. In a higher-dimensional setting, however, some issues may remain unbundled. Common-value problems will still be bundled with other issues and used for leverage, but they will not necessarily be tied to *every* ideological conflict at once, as the proposer may try to build different coalitions on different issues. Characterizing the patterns of partial bundling that arise in a general multidimensional setting is an interesting direction for future work.

The findings of our model offer both reason for optimism and cause for concern. On the hopeful side, they suggest that polarization does not necessarily stem from deep-seated disagreements but can instead arise from strategic incentives in bundled bargaining. Therefore, in contrast to explanations for congressional dysfunction rooted in electoral incentives, this opens the door to institutional reforms of the legislative process aimed at reducing polarization and improving efficiency.

²²We note that, in settings where the policy space is discrete, outcomes may remain gridlocked for long periods before any movement toward efficiency occurs because the proposer waits for compounding costs to accumulate to a sufficient level.

²³This movement towards $\hat{y}$ does not necessarily imply that the policy outcome is improving over time because the cost of inefficiency is also rising via α_t .

For instance, while omnibus legislation allows parties to move policy on purely conflictual issues, we highlight that it also leads to inefficient solutions to shared policy problems. Thus, a single-subject rule for bills, a requirement already imposed on legislation in the majority of U.S. states, would eliminate the proposer’s ability to exploit inefficiencies for strategic gain. Likewise, many countries allow the executive to exercise a line-item veto, rejecting specific provisions of a bill without vetoing it entirely. Notably, the U.S. introduced a line-item veto in 1996 precisely to curb pork-barrel spending, only for the Supreme Court to strike it down as unconstitutional two years later.²⁴

Less optimistically, our theory suggests that in an era of rapid change, growing ideological polarization, and intensifying global tensions, the notion that any issue can remain untouched by these forces may be an illusion. Even in the stark case we consider in our model—where no fundamental disagreement exists on the common-value dimension—ideological infection often proves inevitable.

²⁴Interestingly, the results of our paper offer a rationale for reducing the dimensionality of the policy space that is distinct from earlier work which proposes similar solutions, but on the basis of imposing stability and avoiding cycles (e.g., [Shepsle, 1986](#)).

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A Appendix

A.1 Proofs for Baseline Model

Recall that $U_q = u_{V1}(x_1^q) + v_1(y_1^q) + u_{V2}(x_2^*(x_1^q, y_1^q))$ denotes V 's dynamic equilibrium payoff from keeping the status quo. Thus, in the first period V accepts a proposal (x, y) if:

$$u_{V1}(x) + v(y) + u_{V2}(x_2^*(x, y)) \geq U_q,$$

and rejects otherwise.

Facing this constraint from V , in the first period player P chooses (x, y) to solve the following maximization problem:

$$\begin{aligned} \max_{x, y \in X \times Y} \quad & u_{P1}(x) + v(y) + u_{P2}(x_2^*(x, y)) \\ \text{s.t.} \quad & u_{V1}(x) + v(y) + u_{V2}(x_2^*(x, y)) \geq U_q \end{aligned} \tag{3}$$

Lemma A.1. *Any (x, y) such that $\bar{x}(x, y) > \hat{x}_2^P$ does not solve (3).*

Proof. First, we consider the case $y < \hat{y}$. Towards a contradiction, assume there exists (x, y) such that $\bar{x}(x, y) > \hat{x}_2^P$ is optimal and $y < \hat{y}$. Since $\bar{x}(x, y)$ is continuous in y there exists y' closer to $\hat{y}$ such that $\bar{x}(x, y') > \hat{x}_2^P$. Furthermore, because $u_{V1}(x) + v(y') + u_{V2}(\hat{x}_2^P) > u_{V1}(x) + v(y) + u_{V2}(\hat{x}_2^P)$ the policy (x, y') must also satisfy V 's acceptance constraint. Evaluating the objective function at (x, y') and (x, y) immediately yields $u_{P1}(x) + v(y') + u_{P2}(\hat{x}_2^P) > u_{P1}(x) + v(y) + u_{P2}(\hat{x}_2^P)$, contradicting that (x, y) solves problem (3).

Second, suppose $y = \hat{y}$ and $\bar{x}(x, y) > \hat{x}_2^P$ is optimal. This yields a dynamic payoff to P of $u_{P1}(\hat{x}_2^P) + u_{P2}(\hat{x}_2^P)$. However, the policy $(x', y) = \left(\frac{\hat{x}_1^P + \hat{x}_2^P}{2}, \hat{y}\right)$ yields a strictly greater payoff, since $\frac{\hat{x}_1^P + \hat{x}_2^P}{2}$ is the maximizer of $u_{P1}(x) + u_{P2}(x)$. Additionally, if $(\hat{x}_2^P, \hat{y})$ were acceptable to V then $(x', \hat{y})$ is also acceptable to V by $x' < \hat{x}_2^P$. Thus, (x, y) such that $y = \hat{y}$ and $\bar{x}(x, y) > \hat{x}_2^P$ cannot solve (3). $\square$

Lemma A.1 establishes that it is never optimal for the proposer to choose an (x, y) such that $\bar{x}(x, y) > \hat{x}_2^P$. Using that any optimal (x, y) is such that $\bar{x}(x, y) \leq \hat{x}_2^P$, we can then

write problem (3) as:

$$\begin{aligned} \max_{x,y \in X \times Y} \quad & u_{P1}(x) + v(y) + u_{P2}(\bar{x}(x,y)) \\ \text{s.t.} \quad & u_{V1}(x) + v(y) + u_{V2}(x) + \alpha v(y) \geq U_q \\ & u_{V2}(x) + \alpha v(y) \geq u_{V2}(\hat{x}_2^P) \end{aligned}$$

System (4) yields the KKT conditions for this problem:²⁵

$$u'_{P1}(x) + \frac{\partial \bar{x}}{\partial x} u'_{P2}(\bar{x}(x,y)) + \lambda_1 [u'_{V1}(x) + u'_{V2}(x)] + \lambda_2 u'_{V2}(x) = 0 \quad (4a)$$

$$v'(y) + \frac{\partial \bar{x}}{\partial y} u'_{P2}(\bar{x}(x,y)) + \lambda_1 [v'(y) + \alpha v'(y)] + \lambda_2 \alpha v'(y) = 0 \quad (4b)$$

$$\lambda_1 [u_{V1}(x) + v(y) + u_{V2}(\bar{x}(x,y)) - U_q] = 0 \quad (4c)$$

$$\lambda_2 [u_{V2}(x) + \alpha v(y) - u_{V2}(\hat{x}_2^P)] = 0 \quad (4d)$$

$$\lambda_1, \lambda_2 \geq 0 \quad (4e)$$

Lemma A.2 further establishes that P never chooses an (x, y) such that $\bar{x}(x, y) = \hat{x}_2^P$ unless $\hat{x}_1^P = \hat{x}_2^P$, and $y = \hat{y}$.

Lemma A.2.

- If $\hat{x}_2^P > \hat{x}_1^P$ then any (x, y) such that $\bar{x}(x, y) = \hat{x}_2^P$ cannot solve (3).
- If $\hat{x}_2^P = \hat{x}_1^P$ then any (x, y) such that $\bar{x}(x, y) = \hat{x}_2^P$ and $y < \hat{y}$ cannot solve (3).

Proof. Suppose there exists (x, y) that solves (3) such that $\bar{x}(x, y) = \hat{x}_2^P$. Thus, (x, y) must solve system (4). In particular, consider condition (4b). Letting $\bar{x}(x, y) = \hat{x}_2^P$, then $u'_{P2}(\bar{x}(x, y)) = 0$ and this condition becomes:

$$v'(y) + \lambda_1 [v'(y) + \alpha v'(y)] + \lambda_2 v'(y) = 0. \quad (5)$$

First, notice that if such an (x, y) is optimal then we must have $y = \hat{y}$. Thus, the policy will be stuck at the first-period proposal x in the second period, which implies that $x = \hat{x}_2^P$. In this case, for condition (4a) to hold requires:

$$u'_{P1}(\hat{x}_2^P) + \lambda_1 [u'_{V1}(\hat{x}_2^P) + u'_{V2}(\hat{x}_2^P)] + \lambda_2 u'_{V2}(\hat{x}_2^P) = 0. \quad (6)$$

²⁵It is straightforward to verify that the constraint qualification can only fail at points that are never optimal, thus, any maximizer must satisfy the KKT conditions.

Note that $\hat{x}_2^P > \hat{x}_2^V$, and $\hat{x}_2^P > \hat{x}_1^V$, thus the second and third terms on the LHS of (6) are weakly negative.

If $\hat{x}_2^P > \hat{x}_1^P$ then $u'_{P1}(\hat{x}_2^P) < 0$, and hence the LHS of (6) is strictly negative, which contradicts that (x, y) solves (3) and proves the first part of the lemma.

Next, consider the case where $\hat{x}_2^P = \hat{x}_1^P$. Assume there exists a solution (x, y) to (3) with $y < \hat{y}$ and $\bar{x}(x, y) = \hat{x}_2^P = \hat{x}_1^P$. Then equation (4b) becomes $v'(y) + \lambda_1(1 + \alpha)v'(y) + \lambda_2\alpha v'(y) > 0$, contradicting that (x, y) solves system (4). $\square$

Lemma 2. *On the Y dimension $\hat{y}_d^P \leq \hat{y} = \hat{y}_d^V$. On the X dimension $\hat{x}_d^V \in [\hat{x}_2^V, \hat{x}_1^V]$ and $\hat{x}_d^P \in [\hat{x}_1^P, \hat{x}_2^P]$.*

Proof. First, we prove the result for the proposer's dynamic ideal point. If $\hat{x}_2^P > \hat{x}_1^P$ then by Lemma A.2 $\bar{x}(x, y) < \hat{x}_2^P$, so the necessary condition that $(\hat{x}_d^P, \hat{y}_d^P)$ solves is:

$$u'_{P1}(x) + \frac{\partial \bar{x}}{\partial x} u'_{P2}(\bar{x}(x, y)) = 0 \quad (7)$$

$$v'(y) + \frac{\partial \bar{x}}{\partial y} u'_{P2}(\bar{x}(x, y)) = 0. \quad (8)$$

Recall that $\bar{x}(x, y)$ solves (2). The implicit function theorem then yields:

$$\frac{\partial \bar{x}}{\partial x} = \frac{u'_{V2}(x)}{u'_{V2}(\bar{x}(x, y))}, \text{ and } \frac{\partial \bar{x}}{\partial y} = \frac{\alpha v'(y)}{u'_{V2}(\bar{x}(x, y))}.$$

As such, we write (7) as:

$$u'_{P1}(x) + \frac{u'_{V2}(x)}{u'_{V2}(\bar{x})} u'_{P2}(\bar{x}(x, y)) = 0. \quad (9)$$

Note that we sometimes suppress dependence of $\bar{x}$ on the first-period policy (x, y) . Furthermore, in equilibrium, we must have $\hat{x}_2^V < x \leq \bar{x}(x, y) \leq \hat{x}_2^P$, which implies $\frac{u'_{V2}(x)}{u'_{V2}(\bar{x})} > 0$ and $u'_{P2}(\bar{x}(x, y)) \geq 0$. Therefore, $x \geq \hat{x}_1^P$ is necessary for (9) to hold, with this inequality being strict whenever $\hat{x}_2^P > \hat{x}_1^P$. Additionally, $\hat{y}_d^P \leq \hat{y}$, by $Y = (-\infty, \hat{y}]$. If instead $\hat{x}_1^P = \hat{x}_2^P$ then clearly P 's dynamic ideal point is $(\hat{x}_d^P, \hat{y}_d^P) = (\hat{x}_2^P, \hat{y})$, concluding the argument for P .

Now consider the veto player. V 's dynamic ideal point $(\hat{x}_d^V, \hat{y}_d^V)$ solves:

$$\max_{x, y} u_{V1}(x) + v(y) + u_{V2}(x_2^*(x, y)).$$

If $x_2^*(x, y) = \bar{x}(x, y)$ then $(\hat{x}_d^V, \hat{y}_d^V)$ must solve:

$$\begin{aligned} u'_{V1}(x) + u'_{V2}(x) &= 0 \\ v'(y) + \alpha v'(y) &= 0. \end{aligned}$$

Thus, $\hat{x}_d^V \leq \hat{x}_1^V$ and $\hat{y}_d^V = \hat{y}$, as required. Specifically, $\hat{x}_d^V = \frac{\hat{x}_1^V + \hat{x}_2^V}{2}$. If instead it is optimal for V to choose $(\hat{x}_d^V, \hat{y}_d^V)$ such that $x_2^*(x, y) = \hat{x}_2^P$, then the optimal choice is $\hat{x}_d^V = \hat{x}_1^V$ and $\hat{y}_d^V = \hat{y}$, but then $\bar{x}(\hat{x}_1^V, \hat{y}) = \hat{x}_1^V \neq \hat{x}_2^P$, a contradiction. $\square$

Proposition 1. *If the costs of inefficiency do not compound over time, $\alpha \leq 1$, then there is no ideological infection, $\hat{y}_d^P = \hat{y} = \hat{y}_d^V$. Furthermore, the equilibrium policy outcome is efficient, $y_1^* = \hat{y}$.*

Proof. We break the argument into two steps. Step 1 proves that if $\alpha \leq 1$ then there is no infection and step 2 proves that there is efficiency.

Step 1. To start, we prove that if $\alpha \leq 1$ then there is no ideological infection. For a contradiction, assume $\alpha \leq 1$ but $\hat{y}_d^P < \hat{y}$.

Lemma A.1 implies that if $y_d^P < \hat{y}$ then $x^*(x, y) = \bar{x}(x, y)$. Thus, from our analysis in Lemma 2, a necessary condition for P 's dynamic ideal point (x_d^P, y_d^P) is that it solves:

$$u'_{P1}(x) + \frac{u'_{V2}(x)}{u'_{V2}(\bar{x})} u'_{P2}(\bar{x}(x, y)) = 0 \quad (10)$$

$$v'(y) + \frac{\alpha v'(y)}{u'_{V2}(\bar{x})} u'_{P2}(\bar{x}(x, y)) = 0. \quad (11)$$

Using that $y < \hat{y}$, we can combine conditions (10) and (11) and rearrange to obtain that $(\hat{x}_d^P, \hat{y}_d^P)$ must satisfy:

$$\alpha = \frac{u'_{V2}(x)}{u'_{P1}(x)}. \quad (12)$$

From the proof of Lemma 2, if $\hat{x}_2^P > \hat{x}_1^P$ then $\hat{x}_d^P > \hat{x}_1^P$. Therefore, at any possible x that characterizes $\hat{x}_d^P$ we have $u'_{V1}(x) < u'_{P1}(x) < 0$ for $x > \hat{x}_1^P$. Furthermore, $u'_{V2}(x) \leq u'_{V1}(x)$. Hence, $u'_{V2}(x) < u'_{P1}(x) < 0$. However, this implies $\frac{u'_{V2}(x)}{u'_{P1}(x)} > 1 \geq \alpha$, and thus (12) cannot hold, contradicting that $\hat{y}_d^P \neq \hat{y}$. Instead, if $\hat{x}_2^P = \hat{x}_1^P$, then from the proof of Lemma 2 we have $(\hat{x}_d^P, \hat{y}_d^P) = (\hat{x}_2^P, \hat{y})$, and again there is no infection.

Part 2. Now we show that the policy outcome must also be efficient. To derive a contradiction, assume $\alpha \leq 1$ but $y_1^* < \hat{y}$. By Lemma A.1 $\bar{x}(x_1^*, y_1^*) \leq \hat{x}_2^P$. Thus, the optimal

policy proposal must solve system (4). To prove the result we now consider different cases depending on which constraints are binding.

Case 1: To start, assume $\lambda_2 > 0$. This implies that $\bar{x}(x, y) = \hat{x}_2^P$, which can only be optimal if $y = \hat{y}$ by Lemma A.2.

Case 2: Second, assume $\lambda_1 = 0$ and $\lambda_2 = 0$. By $\lambda_1 = 0$ the proposer's unconstrained optimal policy is accepted by V . Thus, by Proposition 1 $y_1^* = \hat{y}_d^P = \hat{y}$, as required.

Case 3: Finally, consider the case where $\lambda_1 > 0$ and $\lambda_2 = 0$. If $y < \hat{y}$ then solving (4a) and (4b) for λ_1 yields that (x, y) must solve:

$$\frac{1 + \frac{u'_{P2}(\bar{x})}{u'_{V2}(\bar{x})}\alpha}{1 + \alpha} = \frac{u'_{P1}(x) + \frac{u'_{P2}(\bar{x})}{u'_{V2}(\bar{x})}u'_{V2}(x)}{u'_{V1}(x) + u'_{V2}(x)}. \quad (13)$$

Rearranging condition (13), we have that any optimal (x, y) must solve:

$$\begin{aligned} u'_{P1}(x) - u'_{V1}(x) + \alpha u'_{P1}(x) - u'_{V2}(x) \\ - \frac{u'_{P2}(\bar{x})}{u'_{V2}(\bar{x})}(\alpha u'_{V1}(x) - u'_{V2}(x)) = 0. \end{aligned} \quad (14)$$

To obtain a contradiction, we show that the LHS of (14) is strictly positive.

First, we show that the last term in the LHS of (14) is always positive. To see this, note that $\frac{u'_{P2}(\bar{x})}{u'_{V2}(\bar{x})} < 0$, by $\bar{x}(x, y) \in (\hat{x}_2^V, \hat{x}_2^P]$. Thus, a sufficient condition for the last term on the LHS of (14) to be positive is that:

$$\alpha u'_{V1}(x) - u'_{V2}(x) > 0. \quad (15)$$

Clearly, in equilibrium, $x \geq \hat{x}_2^V$. Thus, if $x < \hat{x}_1^V$ then $\alpha u'_{V1}(x) > 0$ and (15) holds. Instead, suppose that $x \geq \hat{x}_1^V$. In this case, (15) rearranges to $\frac{x - \hat{x}_2^V}{x - \hat{x}_1^V} > \alpha$, which holds by assumption that $\hat{x}_2^V < \hat{x}_1^V < x$ and $\alpha \leq 1$.

Second, consider the term: $u'_{P1}(x) - u'_{V1}(x)$. By $y < \hat{y}$, this term is positive if and only if $u'_{P1}(x) \geq u'_{V1}(x)$, which holds by concavity and $\hat{x}_1^V < \hat{x}_1^P$.

Finally, to complete the argument that the LHS of (14) is strictly positive, we show that $\alpha u'_{P1}(x) - u'_{V2}(x) > 0$. This holds if and only if:

$$\alpha u'_{P1}(x) > u'_{V2}(x). \quad (16)$$

By our previous argument showing that the LHS of (15) is positive, we have $\alpha u'_{V1}(x) > u'_{V2}(x)$. Thus, a sufficient condition for (16) to hold is that $\alpha u'_{P1}(x) \geq \alpha u'_{V1}(x)$, which again follows from concavity of u_1 and $\hat{x}_1^V < \hat{x}_1^P$. Therefore, the LHS of (14) is strictly positive, contradicting that $y < \hat{y}$ is optimal. $\square$

We now prove our main results on infection and inefficiency. The analysis is based on the following three cutoffs:

$$\bar{\xi}_{bind} = 2x_0 - \hat{x}_1^P, \quad \bar{\xi}_u = \frac{(1 + \alpha)\hat{x}_1^P - 2\hat{x}_2^V}{\alpha - 1}, \quad \bar{\xi}_c = \frac{(1 + \alpha)x_0 - \hat{x}_2^V}{\alpha}.$$

The policy x_0 is the greatest ideological policy that makes V indifferent in the first period between accepting $(x_0, \hat{y})$ or keeping the status quo. It is characterized by:

$$x_0 = \frac{1}{2} \left(\hat{x}_1^V + \hat{x}_2^V + \sqrt{-2U_q - (\hat{x}_1^V - \hat{x}_2^V)^2} \right). \quad (17)$$

To start, Lemma A.3 establishes the derivatives of x_0 in α and in $\hat{x}_2^V$, facts we will use to prove later results.

Lemma A.3. $\frac{\partial x_0}{\partial \alpha} > 0$ and $\frac{\partial x_0}{\partial \hat{x}_2^V} > 0$.

Proof. Differentiating:

$$\begin{aligned} \frac{\partial x_0}{\partial \alpha} &= \frac{(y_1^q - \hat{y})^2}{2\sqrt{-2U_q - (\hat{x}_1^V - \hat{x}_2^V)^2}} > 0 \\ \frac{\partial x_0}{\partial \hat{x}_2^V} &= \frac{1}{2} \left(1 + \frac{-2(x_1^q - \hat{x}_2^V) + (\hat{x}_1^V - \hat{x}_2^V)}{\sqrt{-2U_q - (\hat{x}_1^V - \hat{x}_2^V)^2}} \right) \end{aligned}$$

Thus, $\frac{\partial x_0}{\partial \hat{x}_2^V} > 0$ if and only if:

$$\begin{aligned} &\sqrt{-2U_q - (\hat{x}_1^V - \hat{x}_2^V)^2} > 2(x_1^q - \hat{x}_2^V) - (\hat{x}_1^V - \hat{x}_2^V) \\ &\Leftrightarrow \sqrt{-2U_q - (\hat{x}_1^V - \hat{x}_2^V)^2} > 2\left(x_1^q - \frac{\hat{x}_1^V + \hat{x}_2^V}{2}\right) \\ &\Leftrightarrow -2U_q - (\hat{x}_1^V - \hat{x}_2^V)^2 > 4\left(x_1^q - \frac{\hat{x}_1^V + \hat{x}_2^V}{2}\right)^2 \\ &\Leftrightarrow -2U_q - 4\left(\frac{\hat{x}_1^V + \hat{x}_2^V}{2} - x_1^q\right)^2 - (\hat{x}_1^V - \hat{x}_2^V)^2 > 0 \\ &\Leftrightarrow -2U_q - 2(x_1^q - \hat{x}_1^V)^2 - 2(x_1^q - \hat{x}_2^V)^2 > 0, \end{aligned}$$

which holds by $-2U_q > 2(x^q - \hat{x}_1^V)^2 + 2(x^q - \hat{x}_2^V)^2$. $\square$

Next, Lemma A.4 provides an initial ordering on the cutoffs $\bar{\xi}_u$ and $\bar{\xi}_c$.

Lemma A.4. $\bar{\xi}_u > \bar{\xi}_c$.

Proof. We show that if $\hat{x}_2^P > \bar{\xi}_u$ then $\hat{x}_2^P > \bar{\xi}_c$. Assume $\hat{x}_2^P > \bar{\xi}_u$, therefore, rearranging:

$$\alpha > \frac{\hat{x}_1^P + \hat{x}_2^P - 2\hat{x}_2^V}{\hat{x}_2^P - \hat{x}_1^P}. \quad (18)$$

Additionally, rearranging, $\hat{x}_2^P > \bar{\xi}_c$ holds if and only if:

$$\alpha > \frac{x_0 - \hat{x}_2^V}{\hat{x}_2^P - x_0}. \quad (19)$$

Note that if P is constrained then $x_0 < \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$. Furthermore $\bar{\xi}_c$ is strictly increasing in x_0 . Thus, a sufficient condition for (19) to hold is that:

$$\alpha > \frac{\frac{\hat{x}_1^P + \hat{x}_2^P}{2} - 2\hat{x}_2^V}{\hat{x}_2^P - \frac{\hat{x}_1^P + \hat{x}_2^P}{2}} = \frac{\hat{x}_1^P + \hat{x}_2^P - 2\hat{x}_2^V}{\hat{x}_2^P - \hat{x}_1^P}. \quad (20)$$

Thus, (18) holds if and only if (20), and (20) implies (19), as desired. $\square$

Lemma A.5 now demonstrates that the cutoff $\bar{\xi}_c$ determines whether the equilibrium policy proposal is inefficient when the first-period constraint from the veto player is sufficiently binding that P cannot obtain its optimal efficient policy.

Lemma A.5. Assume $x_0 < \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$. $y_1^* < \hat{y}$ if and only if $\hat{x}_2^P > \bar{\xi}_c$.

Proof. First, suppose that $\hat{x}_{P2} \leq \bar{\xi}_c$. The furthest right that the proposer can move policy on the X dimension in the first period is x_0 . Thus, a necessary condition for some $y < \hat{y}$ to be optimal is that

$$u_{P1}(x_0) + v(y) + u_{P2}(\bar{x}(x_0, y)) > u_{P1}(x_0) + u_{P2}(x_0). \quad (21)$$

Rearranging the veto player's second-period constraint (1) we have that:

$$v(y) = \frac{u_{V2}(\bar{x}(x_0, y)) - u_{V2}(x_0)}{\alpha}.$$

Substituting this in for $v(y)$ and rearranging, (21) simplifies as follows:

$$\begin{aligned}
u_{P2}(\bar{x}(x_0, y)) &> \frac{u_{V2}(x_0) - u_{V2}(\bar{x}(x_0, y))}{\alpha} + u_{P2}(x_0). \\
\Leftrightarrow \alpha &> \frac{u_{V2}(x_0) - u_{V2}(\bar{x}(x_0, y))}{u_{P2}(\bar{x}(x_0, y)) - u_{P2}(x_0)} \\
\Leftrightarrow \alpha &> \frac{-(x_0 - \hat{x}_{V2})^2 + (\bar{x}(x_0, y) - \hat{x}_{V2})^2}{-(\bar{x}(x_0, y) - \hat{x}_{P2})^2 + (x_0 - \hat{x}_{P2})^2} \\
\Leftrightarrow \alpha &> \frac{(\bar{x}(x_0, y) - x_0)[(\bar{x}(x_0, y) + x_0) - 2\hat{x}_{V2}]}{(\bar{x} - x_0)[-(\bar{x} + x_0) + 2\hat{x}_{P2}]} \\
\Leftrightarrow \alpha &> \frac{\bar{x}(x_0, y) + x_0 - 2\hat{x}_{V2}}{2\hat{x}_{P2} - \bar{x}(x_0, y) - x_0}.
\end{aligned}$$

Note that $\bar{x}(x_0, y) \geq x_0$ and the RHS is strictly increasing in $\bar{x}(x_0, y)$. Therefore, a necessary condition for the above inequality to hold is that:

$$\begin{aligned}
\alpha &> \frac{2x_0 - 2\hat{x}_{V2}}{2\hat{x}_{P2} - 2x_0} \\
\Leftrightarrow \hat{x}_{P2} &> \frac{x_0 - \hat{x}_{V2}}{\alpha} + x_0.
\end{aligned}$$

Consequently, a necessary condition for $y^* < \hat{y}$ is that $\hat{x}_2^P > \bar{\xi}_c$.

Next, we prove that $\hat{x}_2^P > \bar{\xi}_c$ is also sufficient for $y^* < \hat{y}$. In particular, we will show that if $\hat{x}_2^P > \bar{\xi}_c$, then the proposer's utility is decreasing in y as $y \rightarrow \hat{y}$. Let $\hat{y} - y = \varepsilon$. Denote x_ε as the optimal X -dimension policy for the proposer to offer with an amount of inefficiency ε when the veto player's constraint binds, which it does by assumption for ε close to 0. Then, the proposer's equilibrium utility from y is

$$-(x_\varepsilon - \hat{x}_1^P)^2 - \varepsilon^2 - (\bar{x}(x_\varepsilon, \varepsilon) - \hat{x}_2^P)^2, \quad (22)$$

where

$$\bar{x}(x_\varepsilon, \varepsilon) = \hat{x}_2^V + \sqrt{(x_\varepsilon - \hat{x}_2^V)^2 + \alpha\varepsilon^2} = x + \frac{\alpha\varepsilon^2}{2(x_\varepsilon - \hat{x}_2^V)} + \mathcal{O}(\varepsilon^4)$$

with the last part following from the Taylor expansion of the square root term.

Plugging this into (22), we obtain

$$-(x_\varepsilon - \hat{x}_1^P)^2 - \varepsilon^2 - \left(x_\varepsilon + \frac{\alpha\varepsilon^2}{2(x_\varepsilon - \hat{x}_2^V)} + \mathcal{O}(\varepsilon^4) - \hat{x}_2^P\right)^2,$$

Once we take $\varepsilon \rightarrow 0$, we have $x_\varepsilon \rightarrow x_0$ and $\bar{x}(x_\varepsilon, \varepsilon) \rightarrow x_0$. The above can then be written

as:

$$-(x_0 - \hat{x}_1^P)^2 - \varepsilon^2 - (x_0 - \hat{x}_2^P)^2 - (x_0 - \hat{x}_2^P) \left(\frac{\alpha \varepsilon^2}{x_0 - \hat{x}_2^V} + 2\mathcal{O}(\varepsilon^4) \right) - \mathcal{O}(\varepsilon^4)$$

Dropping the non-leading ε terms, we can rewrite this as

$$-(x_0 - \hat{x}_1^P)^2 - (x_0 - \hat{x}_2^P)^2 - \varepsilon^2 - (x_0 - \hat{x}_2^P) \left(\frac{\alpha \varepsilon^2}{x_0 - \hat{x}_2^V} \right).$$

Notice that $-(x_0 - \hat{x}_1^P)^2 - (x_0 - \hat{x}_2^P)^2$ is the proposer's optimal utility from proposing the efficient policy $y = \hat{y}$. Thus, a sufficient condition to ensure that the proposer's utility is increasing in ε for ε close to 0 is that:

$$-1 - (x_0 - \hat{x}_2^P) \left(\frac{\alpha}{x_0 - \hat{x}_2^V} \right) > 0.$$

Rearranging, this reduces to

$$\hat{x}_2^P > \frac{x_0 - \hat{x}_2^V}{\alpha} + x_0 = \bar{\xi}_c,$$

as claimed. $\square$

Lemma A.6. *Assume the first-period veto constraint does not bind at P 's preferred efficient policy. $y^* < \hat{y}$ if and only if $\hat{x}_P > \bar{\xi}_u$.*

Proof. For an unconstrained proposer, write the KKT conditions with $\lambda_1 = \lambda_2 = 0$:

$$\begin{aligned} u'_{P1}(x) + \frac{u'_{P2}(\bar{x}(x, y))}{u'_{V2}(\bar{x}(x, y))} u'_{V2}(x) &= 0 \\ v'(y) + \frac{u'_{P2}(\bar{x}(x, y))}{u'_{V2}(\bar{x}(x, y))} \alpha v'(y) &= 0. \end{aligned}$$

Therefore if P chooses $y = \hat{y}$ then $x = \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$, since $y = \hat{y}$ implies $\bar{x}(x, y) = x$ (indeed this is always a solution to the KKT conditions).

That $\hat{x}_2^P > \bar{\xi}_u$ is necessary for inefficiency follows a similar argument as the proof for Lemma A.5 by replacing x_0 with $\frac{\hat{x}_1^P + \hat{x}_2^P}{2}$.

Next, we show that $\hat{x}_2^P > \bar{\xi}_u$ is also sufficient for $y^* < \hat{y}$. Let $y = \hat{y} - \varepsilon$. Consider the following maximization problem:

$$\tilde{U}(\varepsilon) = \max_x -(x - \hat{x}_1^P)^2 - \varepsilon^2 - (\bar{x}(x, \varepsilon) - \hat{x}_2^P)^2. \quad (23)$$

Hence, for a given level of inefficiency ε on the Y dimension, a solution to (23) gives P 's optimal choice of x assuming the veto player's constraint is not binding. Denote this solution by x_ε . Note that $\varepsilon = 0$ captures the case where the policy is efficient and $\frac{\hat{x}_1^P + \hat{x}_2^P}{2}$ is the optimal choice of policy on the X dimension in this case if the proposer is unconstrained. Additionally, $\tilde{U}(\varepsilon)$ is P 's actual equilibrium payoff from proposing the efficient policy for ε sufficiently small because we have assumed that P is unconstrained at $\varepsilon = 0$. Thus, to prove that choosing the efficient policy is not optimal we show that $\tilde{U}(\varepsilon)$ is increasing in ε as $\varepsilon \rightarrow 0$. Specifically, if $\frac{\partial \tilde{U}}{\partial \varepsilon} > 0$ for ε sufficiently close to 0 then P 's equilibrium payoff from proposing $y < \hat{y}$ is strictly greater than proposing $y = \hat{y}$. By the envelope theorem:

$$\frac{\partial \tilde{U}}{\partial \varepsilon} = -2\varepsilon - 2\frac{\partial \bar{x}}{\partial \varepsilon}(\bar{x}(x_\varepsilon, \varepsilon) - \hat{x}_2^P).$$

Thus, $\frac{\partial \tilde{U}}{\partial \varepsilon} > 0$ if and only if:

$$\begin{aligned} & -2\varepsilon - 2\frac{\partial \bar{x}}{\partial \varepsilon}(\bar{x}(x_\varepsilon, \varepsilon) - \hat{x}_2^P) > 0 \\ \Leftrightarrow & -1 - \frac{\alpha}{\sqrt{\varepsilon^2\alpha + (\bar{x}(x_\varepsilon, \varepsilon) - \hat{x}_2^V)^2}}[\bar{x}(x_\varepsilon, \varepsilon) - \hat{x}_2^P] > 0 \end{aligned}$$

Taking the limit $\varepsilon \rightarrow 0$ then $x_\varepsilon \rightarrow \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$ and $\lim_{\varepsilon \rightarrow 0} \bar{x}(x_\varepsilon, \varepsilon) = \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$. Then the above condition becomes:

$$-1 - \frac{\alpha}{\frac{\hat{x}_1^P + \hat{x}_2^P}{2} - \hat{x}_2^V} \left(\frac{\hat{x}_1^P + \hat{x}_2^P}{2} - \hat{x}_2^P \right) > 0 \quad (24)$$

$$\Leftrightarrow \hat{x}_2^P > \frac{(1 + \alpha)\hat{x}_1^P - 2\hat{x}_2^V}{\alpha - 1} = \bar{\xi}_u, \quad (25)$$

as claimed. □

Lemma A.7. *The veto player's constraint binds at P 's unconstrained optimal efficient policy if and only if $\hat{x}_2^P > \bar{\xi}_{bind}$.*

Proof. Recall from Lemma A.6 that if the proposer is unconstrained then its unconstrained optimal efficient policy is $x = \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$ and $y = \hat{y}$. Thus, the proposer will be constrained

whenever:

$$\begin{aligned} u_{V1}\left(\frac{\hat{x}_1^P + \hat{x}_2^P}{2}\right) + u_{V2}\left(\frac{\hat{x}_1^P + \hat{x}_2^P}{2}\right) &< U_q \\ \Leftrightarrow \frac{\hat{x}_1^P + \hat{x}_2^P}{2} &> \frac{\hat{x}_1^V + \hat{x}_2^V}{2} + \frac{1}{2}\sqrt{-2U_q - (\hat{x}_{V1} - \hat{x}_{V2})^2}, \end{aligned}$$

which rearranges to $\hat{x}_{P2} > \bar{\xi}_{bind}$, as claimed. $\square$

Lemma A.8. Define $\bar{\xi} = \min \{ \max \{ \bar{\xi}_c, \bar{\xi}_{bind} \}, \bar{\xi}_u \}$. The first-period policy is inefficient if and only if $\hat{x}_2^P > \bar{\xi}$.

Proof. We prove the result by considering each possible ordering of the cutoffs $\bar{\xi}_c, \bar{\xi}_u, \bar{\xi}_{bind}$. Recall from Lemma A.4 that $\bar{\xi}_c < \bar{\xi}_u$.

1. Case 1: $\bar{\xi}_c < \bar{\xi}_u < \bar{\xi}_{bind}$. In this case, $\bar{\xi} = \bar{\xi}_u$. We show that $y^* < \hat{y}$ if and only if $\hat{x}_2^P > \bar{\xi}_u$. Consider $\hat{x}_2^P < \bar{\xi}_u < \bar{\xi}_{bind}$. Then P 's preferences are not infected and P is not constrained, thus $y^* = \hat{y}$. Next, suppose $\hat{x}_2^P \in (\bar{\xi}_u, \bar{\xi}_{bind})$. Thus, P 's optimal efficient policy is acceptable to V and $\hat{x}_2^P > \bar{\xi}_u$ implies $y^* < \hat{y}$. Finally, if $\hat{x}_2^P > \bar{\xi}_{bind}$ then P cannot pass $(\hat{y}, \frac{\hat{x}_1^P + \hat{x}_2^P}{2})$ and $\hat{x}_2^P > \bar{\xi}_c$ yields $y^* < \hat{y}$.
2. Case 2: $\bar{\xi}_c < \bar{\xi}_{bind} < \bar{\xi}_u$. In this case, $\bar{\xi} = \bar{\xi}_{bind}$. First, if $\hat{x}_2^P > \bar{\xi}_{bind} > \bar{\xi}_c$ then P 's preferred efficient policy bundle does not satisfy V 's constraint and $\hat{x}_2^P > \bar{\xi}_c$ delivers $y^* < \hat{y}$. Second, assume $\hat{x}_2^P < \bar{\xi}_{bind} < \bar{\xi}_u$. Thus, V 's constraint does not bind at $(x, y) = (\frac{\hat{x}_1^P + \hat{x}_2^P}{2}, \hat{y})$ and $\hat{x}_2^P < \bar{\xi}_u$ implies $y^* = \hat{y}$.
3. Case 3: $\bar{\xi}_{bind} < \bar{\xi}_c < \bar{\xi}_u$. If $\hat{x}_2^P \in (\bar{\xi}_{bind}, \bar{\xi}_c)$ then the constraint is binding at $(\frac{\hat{x}_1^P + \hat{x}_2^P}{2}, \hat{y})$ and thus $y^* = \hat{y}$ by $\hat{x}_2^P < \bar{\xi}_c$. If $\hat{x}_2^P < \bar{\xi}_{bind} < \bar{\xi}_u$ then V 's constraint does not bind at P 's unconstrained optimal efficient policy and $y^* = \hat{y}$ by $\hat{x}_2^P < \bar{\xi}_u$. In either case, $\hat{x}_2^P < \bar{\xi}_c$ yields $y^* = \hat{y}$. Instead, if $\hat{x}_2^P > \bar{\xi}_c > \bar{\xi}_{bind}$ then the constraint is binding and thus $y^* < \hat{y}$ by $\hat{x}_2^P > \bar{\xi}_c$.

$\square$

Thus, to prove Proposition 2 requires characterizing which cut point determines $\bar{\xi}$ for different values of α . To do so, we define two cutoffs in α .

Definition 5.

- Let $\underline{\alpha}$ be the α that solves $\bar{\xi}_c = \bar{\xi}_{bind}$.
- Let $\bar{\alpha}$ be the α that solves $\bar{\xi}_u = \bar{\xi}_{bind}$.

We now prove that $\underline{\alpha}$, $\bar{\alpha}$ exist, are unique, and have the ordering $\underline{\alpha} < \bar{\alpha}$. We proceed via several lemmas.

Lemma A.9. $\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} > 0$, $\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} > \frac{\partial \bar{\xi}_c}{\partial \alpha}$ and $\frac{\partial \bar{\xi}_u}{\partial \alpha} < 0$

Proof. Differentiating each cutoff with respect to α yields:

$$\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} = 2 \frac{\partial x_0}{\partial \alpha} > 0, \quad (26)$$

$$\frac{\partial \bar{\xi}_c}{\partial \alpha} = \frac{1}{\alpha^2} \left(\alpha [x_0 + (1 + \alpha) \frac{\partial x_0}{\partial \alpha}] - [(1 + \alpha)x_0 - \hat{x}_2^V] \right) \quad (27)$$

$$\frac{\partial \bar{\xi}_u}{\partial \alpha} = -\frac{2(\hat{x}_1^P - \hat{x}_{V2})}{(1 - \alpha)^2} < 0, \quad (28)$$

where the inequality in (26) holds by Lemma A.3 and (28) holds by $\hat{x}_1^P > \hat{x}_2^V$. Finally, $\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} > \frac{\partial \bar{\xi}_c}{\partial \alpha}$ because:

$$\frac{\partial \bar{\xi}_c}{\partial \alpha} = \frac{1}{\alpha^2} \left(\alpha [x_0 + (1 + \alpha) \frac{\partial x_0}{\partial \alpha}] - [(1 + \alpha)x_0 - \hat{x}_2^V] \right) \quad (29)$$

$$= \frac{1}{\alpha^2} \left(\alpha(1 + \alpha) \frac{\partial x_0}{\partial \alpha} - x_0 + \hat{x}_2^V \right) < \frac{1 + \alpha}{\alpha} \frac{\partial x_0}{\partial \alpha} \quad (30)$$

$$< 2 \frac{\partial x_0}{\partial \alpha} = \frac{\partial \bar{\xi}_{bind}}{\partial \alpha}, \quad (31)$$

where (30) follows by $x_0 > \hat{x}_2^V$ and (31) by $\alpha > 1$. $\square$

Lemma A.10. $\lim_{\alpha \rightarrow \infty} \bar{\xi}_{bind} > \lim_{\alpha \rightarrow \infty} \bar{\xi}_u$ and $\lim_{\alpha \rightarrow 1} \bar{\xi}_{bind} < \lim_{\alpha \rightarrow 1} \bar{\xi}_c$.

Proof. Taking limits we have:

$$\begin{aligned} \lim_{\alpha \rightarrow \infty} \bar{\xi}_{bind} &= \infty & \lim_{\alpha \rightarrow 1} \bar{\xi}_{bind} &= 2x_0 - \hat{x}_1^P \\ \lim_{\alpha \rightarrow \infty} \bar{\xi}_u &= \hat{x}_1^P & \lim_{\alpha \rightarrow 1} \bar{\xi}_c &= 2x_0 - \hat{x}_2^V \end{aligned}$$

The result then follows immediately. $\square$

Lemma A.11. The cutoffs $\underline{\alpha}$ and $\bar{\alpha}$ exist and are unique, with $\underline{\alpha} < \bar{\alpha}$. Furthermore:

- If $\alpha > \bar{\alpha}$ then $\bar{\xi}_{bind} > \bar{\xi}_u$.
- If $\alpha \in (\underline{\alpha}, \bar{\alpha})$ then $\bar{\xi}_{bind} \in (\bar{\xi}_c, \bar{\xi}_u)$.

- If $\alpha < \underline{\alpha}$ then $\bar{\xi}_{bind} < \bar{\xi}_c$.

Proof. Lemmas A.9 and A.10 together with the intermediate value theorem yield existence and uniqueness; the orderings then follow from Lemma A.4 and the definitions of $\underline{\alpha}$ and $\bar{\alpha}$. $\square$

Lemma A.12. *If $\hat{x}_1^P$ is sufficiently large then $\bar{\xi}_c < \hat{x}_1^P$. Furthermore, $\frac{\partial \alpha}{\partial \hat{x}_1^P} > 0$*

Proof. For the first part, notice that $\bar{\xi}_c$ is not a function of $\hat{x}_1^P$. Thus the first statement follows by setting $\hat{x}_1^P > \bar{\xi}_c$. For the second statement, by the implicit function theorem:

$$\frac{\partial \alpha}{\partial \hat{x}_1^P} = \frac{1}{\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} - \frac{\partial \bar{\xi}_c}{\partial \alpha}} > 0,$$

where $\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} - \frac{\partial \bar{\xi}_c}{\partial \alpha} > 0$ by Lemma A.9. $\square$

Lemma 3. *There exists a unique $\xi_{infect} > \hat{x}_1^P$ such that ideological infection emerges if and only if $\hat{x}_2^P > \xi_{infect}$. Furthermore, ξ_{infect} is decreasing in α .*

Proof. Define $\xi_{infect} = \bar{\xi}_u$. The result then follows from Lemmas A.6 and A.9. $\square$

Proposition 2. *There exists a unique $\bar{\xi}$ such that the equilibrium policy is inefficient if and only if $\hat{x}_2^P > \bar{\xi}$. Furthermore, there exist unique $1 < \underline{\alpha} < \bar{\alpha}$ such that:*

- If $\alpha > \bar{\alpha}$, then $\bar{\xi} = \xi_{infect}$;
- If $\alpha \in (\underline{\alpha}, \bar{\alpha})$, then $\bar{\xi} \in (\hat{x}_1^P, \xi_{infect})$;
- If $\alpha < \underline{\alpha}$, then there exist conditions under which $\bar{\xi} < \hat{x}_1^P$.

Proof. Define $\bar{\xi}$ as in Lemma A.8. The ordering on $\bar{\xi}$ then follows by Lemma A.8 and A.11. For the final part, fix $\alpha < \underline{\alpha}$, then $\bar{\xi} = \bar{\xi}_c$. Hence, by Lemma A.12, we can take $\hat{x}_1^P$ sufficiently large so that $\bar{\xi}_c < \hat{x}_1^P$ and $\alpha < \underline{\alpha}$ will continue to hold. $\square$

Corollary 1. *For $\hat{x}_1^P$ sufficiently large, $\bar{\xi}$ is maximized at an intermediate value of α . Moreover, $\lim_{\alpha \rightarrow 1} \bar{\xi} < \hat{x}_1^P$ and $\lim_{\alpha \rightarrow \infty} \bar{\xi} = \hat{x}_1^P$.*

Proof. By Proposition 2, if $\alpha > \bar{\alpha}$ then $\bar{\xi} = \xi_{infect} = \bar{\xi}_u$, and L'hospital's rule yields $\lim_{\alpha \rightarrow \infty} \bar{\xi}_u = \hat{x}_1^P$. Next, the proof of Proposition 2 implies that $\bar{\xi} < \hat{x}_1^P$ for $\alpha = 1$ by $\hat{x}_1^P$ sufficiently. Finally, notice that $\bar{\xi}$ is inverse U-shaped in α for $\alpha > \underline{\alpha}$ by Lemma A.9, thus $\bar{\xi}$ is maximized at an interior α . $\square$

Corollary 2. *There exist parameter values such that, in equilibrium, $y_1^* < \hat{y}$ and $x_1^* < \hat{x}_1^P$.*

Proof. From the proof of Corollary 1 $\bar{\xi} = \bar{\xi}_c < \hat{x}_1^P$ for α sufficiently small and $\hat{x}_1^P$ large. Furthermore, inspecting $\bar{\xi}_c$, we have $x_0 < \bar{\xi}_c < \hat{x}_1^P$. Thus for α sufficiently small and any $\hat{x}_2^P > \hat{x}_1^P$ we have $y_1^* < \hat{y}$. To complete the argument recall that $x_1^* < x_0$ for any $y_1 < \hat{y}$ because x_0 is defined as P 's optimal policy paired with $y = \hat{y}$ when $(\frac{\hat{x}_1^P + \hat{x}_2^P}{2}, \hat{y})$ does not satisfy V 's constraint, which is true because $\bar{\xi}_{bind} < \bar{\xi}_c$ for $\alpha < \underline{\alpha}$ by Lemma A.11. $\square$

Lemma A.13. $\frac{\partial \bar{\xi}_u}{\partial \hat{x}_2^V} < 0$, $\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} > 0$, $\frac{\partial \bar{\alpha}}{\partial \hat{x}_2^V} < 0$, and $\frac{\partial \alpha}{\partial \hat{x}_2^V} < 0$.

Proof. Differentiating:

$$\begin{aligned}\frac{\partial \bar{\xi}_u}{\partial \hat{x}_2^V} &= -\frac{2}{\alpha - 1} \\ \frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} &= 2\frac{\partial x_0}{\partial \hat{x}_2^V} > 0.\end{aligned}$$

Clearly $\frac{\partial \bar{\xi}_u}{\partial \hat{x}_2^V} < 0$ by $\alpha > 1$ and $\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} > 0$ by Lemma A.3. Next, by the implicit function theorem:

$$\frac{\partial \bar{\alpha}}{\partial \hat{x}_2^V} = -\frac{\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} - \frac{\partial \bar{\xi}_u}{\partial \hat{x}_2^V}}{\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} - \frac{\partial \bar{\xi}_u}{\partial \alpha}} < 0$$

which follows by $\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} > 0$ and $\frac{\partial \bar{\xi}_u}{\partial \hat{x}_2^V} < 0$, and Lemma A.9 which yields $\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} > 0$ and $\frac{\partial \bar{\xi}_u}{\partial \alpha} < 0$. Now differentiating $\underline{\alpha}$:

$$\frac{\partial \alpha}{\partial \hat{x}_2^V} = -\frac{\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} - \frac{\partial \bar{\xi}_c}{\partial \hat{x}_2^V}}{\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} - \frac{\partial \bar{\xi}_c}{\partial \alpha}} < 0$$

which follows from $\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} > 0$, $-\frac{\partial \bar{\xi}_c}{\partial \hat{x}_2^V} = -\frac{1+\alpha}{\alpha} \frac{\partial x_0}{\partial \hat{x}_2^V} + 1/\alpha > 0$, and Lemma A.9 which gives $\frac{\partial \bar{\xi}_{bind}}{\partial \alpha} > \frac{\partial \bar{\xi}_c}{\partial \alpha}$. $\square$

Lemma 4.

- If $\alpha \in (\underline{\alpha}, \bar{\alpha})$, then $\bar{\xi}$ is increasing in $\hat{x}_2^V$;
- If $\alpha > \bar{\alpha}$, then $\bar{\xi}$ is decreasing in $\hat{x}_2^V$;
- Furthermore, both $\bar{\alpha}$ and $\underline{\alpha}$ are decreasing in $\hat{x}_2^V$.

Proof. The result follows from Lemmas A.11 and A.13. $\square$

Lemma A.14. *Let $\hat{x}_2^P = -\hat{x}_2^V \equiv \hat{x}_2$ and $\hat{x}_1^P = -\hat{x}_1^V \equiv \hat{x}_1$. $\frac{\partial}{\partial \hat{x}_2} \{ \hat{x}_2 - \bar{\xi}_{bind} \} > 0$ and $\frac{\partial}{\partial \hat{x}_2} \{ \hat{x}_2 - \bar{\xi}_c \} > 0$. Furthermore, $\hat{x}_2 - \bar{\xi}_u > 0$ if and only if $\hat{x}_2(\alpha - 3) > (1 + \alpha)\hat{x}_1$.*

Proof. From Lemma A.13 we have that $\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2^V} > 0$. Therefore, $\frac{\partial \bar{\xi}_{bind}}{\partial \hat{x}_2} < 0$ which yields $\frac{\partial}{\partial \hat{x}_2} \{ \hat{x}_2 - \bar{\xi}_{bind} \} > 0$.

For the next derivative, we can write $\hat{x}_2 - \bar{\xi}_c = \frac{1+\alpha}{\alpha}(\hat{x}_2 - x_0)$. Then

$$\frac{\partial}{\partial \hat{x}_2} \{ \hat{x}_2 - \bar{\xi}_c \} \propto 1 - \frac{\partial x_0}{\partial \hat{x}_2} > 0,$$

which holds because $\frac{\partial x_0}{\partial \hat{x}_2^V} > 0$ by Lemma A.3, and hence $\frac{\partial x_0}{\partial \hat{x}_2} < 0$.

Finally, substituting $\hat{x}_2^V = -\hat{x}_2$ into $\bar{\xi}_u$ and rearranging we obtain $\hat{x}_2 - \bar{\xi}_u > 0$ if and only if $\hat{x}_2(\alpha - 3) > (1 + \alpha)\hat{x}_1$. $\square$

Proposition 3. *Let $\hat{x}_t^P = -\hat{x}_t^V \equiv \hat{x}_t$. The equilibrium policy is inefficient only if $\hat{x}_2$ is sufficiently large. Furthermore, for α sufficiently large, there exists $\xi^{pol} \in (\hat{x}_1, \infty)$ such that the equilibrium policy is inefficient if and only if $\hat{x}_2 > \xi^{pol}$.*

Proof. By an argument similar to that in Lemma A.8, we have that whether the equilibrium policy is inefficient or not is determined by one of the following conditions: (1) $\hat{x}_2 - \bar{\xi}_{bind} > 0$, (2) $\hat{x}_2 - \bar{\xi}_c > 0$, or (3) $\hat{x}_2 - \bar{\xi}_u > 0$, and which condition is the necessary and sufficient will depend on α .

We start by proving the “only if” part of the statement and argue that $\hat{x}_2$ sufficiently large is necessary for inefficiency. First, suppose that (1) or (2) are the relevant condition for inefficiency. Clearly, for $\hat{x}_2$ sufficiently close to 0 both conditions fail and, by Lemma A.14, both conditions become easier to hold as $\hat{x}_2$ increases. Second, suppose that condition (3) determines whether the equilibrium policy is inefficient. By Lemma A.14 (3) holds if and only if $\hat{x}_2(\alpha - 3) > (1 + \alpha)\hat{x}_1$. If $\alpha \leq 3$ then this condition always fails. Instead, if $\alpha > 3$, then at $\hat{x}_2 = \hat{x}_1$ the condition fails, but holds if and only if $\hat{x}_2$ is sufficiently large.

Finally, by a similar argument as for Lemmas A.11 and A.8, we obtain that for α sufficiently large the relevant condition for determining inefficiency is (3). Moreover, taking $\alpha > 3$, by Lemma A.14 there is inefficiency if and only if $\hat{x}_2 > \frac{1+\alpha}{\alpha-3}\hat{x}_1 > \hat{x}_1$, as claimed. $\square$

A.2 Extension: Turnover

Proposition 4. *There exists a unique $\hat{\rho} \in (0, 1]$ such that the proposer's preferences are infected if and only if $\rho > \hat{\rho}$. Moreover, $\hat{\rho} < 1$ if and only if $\hat{x}_2^P > \xi_{infect}$.*

Proof. The proposer's dynamic ideal point solves:

$$\max_{(x,y)} u_{P1}(x) + v(y) + \rho u_{P2}(\bar{x}_V(x, y)) + (1 - \rho)(u_{P2}(x) + \alpha v(y)).$$

Letting $a = -x$ and $b = -y$ we can rewrite P 's problem as:

$$\max_{(a,b)} u_{P1}(-a) + v(-b) + \rho u_{P2}(\bar{x}_{V2}(a, b)) + (1 - \rho)(u_{P2}(-a) + \alpha v(-b)).$$

Taking cross-partials of the objective function yields:

$$\begin{aligned} \frac{\partial^2}{\partial a \partial b} &= \rho \left(\frac{\partial^2 \bar{x}_V}{\partial a \partial b} u'_{P2}(\bar{x}_{V2}(-a, -b)) + \frac{\partial \bar{x}_V}{\partial a} \cdot \frac{\partial \bar{x}_V}{\partial b} u''_{P2}(\bar{x}_V(-a, -b)) \right) \\ \frac{\partial^2}{\partial a \partial \rho} &= \frac{\partial \bar{x}_V}{\partial a} u'_{P2}(\bar{x}_P(-a, -b)) + u'_{P2}(-a) \\ \frac{\partial^2}{\partial b \partial \rho} &= \frac{\partial \bar{x}_V}{\partial b} u'_{P2}(\bar{x}_V(-a, -b)) + \alpha v'(-b) \end{aligned}$$

We have $\frac{\partial^2 \bar{x}_{V2}}{\partial a \partial b} > 0$, $\frac{\partial \bar{x}_{V2}}{\partial a} < 0$, and $\frac{\partial \bar{x}_{V2}}{\partial b} > 0$, which yields $\frac{\partial^2}{\partial a \partial b} > 0$ and $\frac{\partial^2}{\partial b \partial \rho} > 0$. Finally, $u'_{P2}(\bar{x}) < u'_{P2}(x) = u'_{P2}(-a)$ and $\frac{\partial \bar{x}_{V2}}{\partial a} \in (-1, 0)$, thus $\frac{\partial^2}{\partial a \partial \rho} > 0$. Then the standard results on monotone comparative statics (see, e.g., [Milgrom and Shannon, 1994](#)) deliver that y^* is monotonically decreasing in ρ , and thus there exists a $\hat{\rho} \in [0, 1]$ such that $y^* = \hat{y}$ for all $\rho \leq \hat{\rho}$.

We now show that at $\rho = 0$ P is never infected, hence $\hat{\rho} > 0$. P 's problem when $\rho = 0$ is given by:

$$\max_{x,y} -(x - \hat{x}_1^P)^2 - (y - \hat{y})^2 - (x - \hat{x}_2^P)^2 - \alpha(y - \hat{y})^2.$$

The objective function is strictly concave in (x, y) and taking first-order conditions gives:

$$\begin{aligned} -2(x - \hat{x}_1^P) - 2(x - \hat{x}_2^P) &= 0 \\ -2(y - \hat{y}) - 2\alpha(y - \hat{y}) &= 0. \end{aligned}$$

Thus, $x^* = \frac{\hat{x}_1^P + \hat{x}_2^P}{2}$ and $y^* = \hat{y}$ and P 's preferences are not infected.

To conclude the proof, note that if $\hat{x}_2^P > \xi_{infect}$ then by Lemma 3 $y^* < \hat{y}$ at $\rho = 1$, and

hence $\hat{\rho} < 1$. Instead, if $\hat{x}_2^P \leq \xi_{infect}$ then $y^* = \hat{y}$ at $\rho = 1$ and because y^* is decreasing in ρ we must have $\hat{\rho} = 1$. $\square$

Figure 4 plots the results of a numerical simulation of the model. It demonstrates that the proposer's preferences can be infected even with ρ near 1/2. Figure 5 plots the equilibrium common-value policy, y^* , as a function of ρ for several different values of $\hat{x}_2^P$. It plots several numerical examples where increasing the probability of turnover increases efficiency, including when the proposer may be constrained by the veto player in the first period.

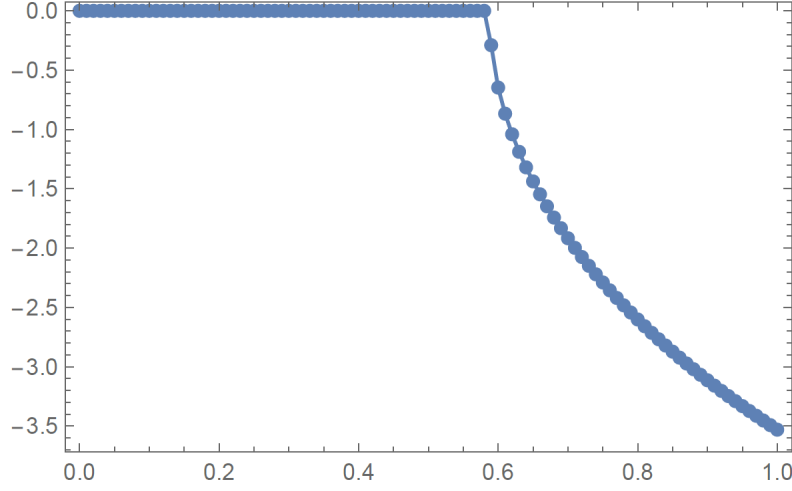


Figure 4: The figure plots the proposer's dynamic ideal point, $\hat{y}_d^P$, on Y as a function of ρ . We set the parameters at the following values: $\hat{x}_2^V = .75$, $\hat{x}_1^V = 1$, $\hat{x}_1^P = 1.25$, $\hat{x}_2^P = 12$, $\hat{y} = 0$, $\alpha = 8$, $x_1^q = 1.15$, and $y_q = -20$. P 's preferences are not infected, $y_d^P = 0 = \hat{y}$, for all $\rho < .59$, and they are infected, $y_d^P < 0 = \hat{y}$, for all $\rho \geq .59$. Note that at these choices for the parameters the proposer is unconstrained at every value of ρ . Thus, infection is necessary and sufficient for the policy outcome to be inefficient as well.

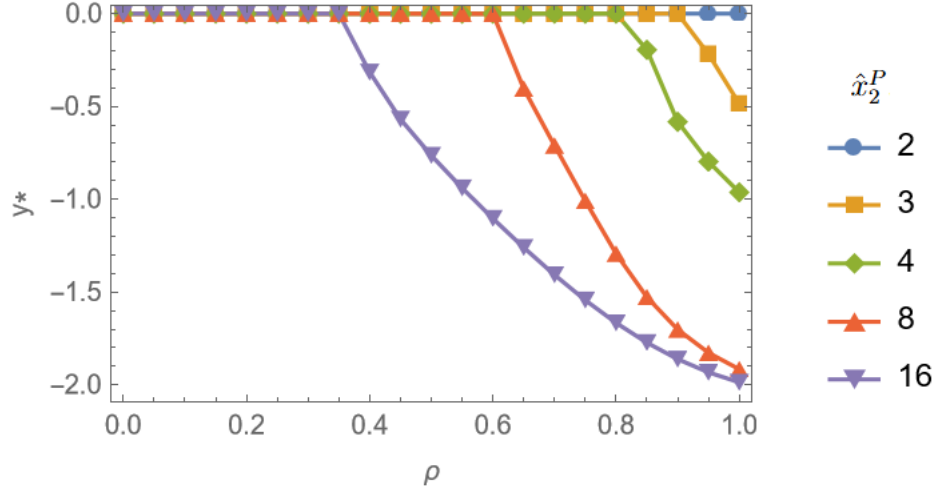


Figure 5: The figure plots the equilibrium policy outcome, y^* , as a function of ρ for different values of $\hat{x}_2^P$. We set the parameters at the following values: $\hat{x}_2^V = .75$, $\hat{x}_1^V = 1$, $\hat{x}_1^P = 1.25$, $\hat{x}_2^P = 12$, $\hat{y} = 0$, $\alpha = 8$, $x_1^q = 1.15$, and $y_q = -20$. Note that for $\hat{x}_2^P \in \{2, 3\}$ the proposer is never constrained; for $\hat{x}_2^P = 4$ the proposer becomes constrained once ρ is sufficiently low; and for $\hat{x}_2^P \in \{8, 16\}$ the proposer is constrained for every value of ρ .

A.3 Extension: Long-run Outcomes

We study stationary Markov perfect equilibria (henceforth “equilibria”). In our model, this requires that P ’s proposal in each period t only depends on the current status quo, (x_t^q, y_t^q) , and the players’ period t ideal points, $\hat{x}_t^P$ and $\hat{x}_t^V$, and the weight on the common-values dimension, α_t . Likewise, for V , a strategy is a mapping from ideal points, the weight, the status quo, and a proposed policy to a decision to accept or reject.

Given a strategy profile we can define continuation payoffs for each player as $w_t^i(x^q, y^q)$, for $i \in \{V, P\}$ (suppressing dependence on the strategy profile). Although we consider stationary strategies, we still index continuation payoffs and strategies by t to capture the changing α and ideal points of the players and reduce notation.

With this notation in hand, we say that a strategy profile constitutes an equilibrium if for all ideal points $(\hat{x}^i, \hat{y})$ for $i \in \{V, P\}$, every status quo (x^q, y^q) , and all t , the following conditions hold:

- for any proposal (x, y) , V accepts if and only if:

$$u_{Vt}(x) + v_t(y) + \delta w_t^V(x, y) \geq u_{Vt}(x^q) + v_t(y^q) + \delta w_t^V(x^q, y^q).$$

- P ’s proposal solves:

$$\begin{aligned} \max_{(x, y) \in \mathcal{P}} \quad & u_{Pt}(x) + v_t(y) + \delta w_t^P(x, y) \\ \text{s.t.} \quad & u_{Vt}(x) + v_t(y) + \delta w_t^V(x, y) \geq u_{Vt}(x^q) + v_t(y^q) + \delta w_t^V(x^q, y^q) \end{aligned}$$

Additionally, we restrict attention to equilibria in which continuation values are continuous in the state variables.

Lemma A.15. *In every period $t > 1$, the equilibrium proposal makes the veto player indifferent between accepting or rejecting.*

Proof. Suppose, toward a contradiction, that there exists some period $t > 1$ in which the veto player’s constraint is not binding at the equilibrium proposal. Then in equilibrium V must be willing to accept the proposer’s unconstrained optimal policy.

If the proposer is unconstrained in period t , then the common-value policy implemented in any earlier period $t' < t$ must be inefficient. The policy would be gridlocked at t , which implies that the equilibrium proposal $(x_{t-1}, \hat{y})$ makes V indifferent at period t . Hence, the equilibrium outcome in period $t - 1$ is some (x, y) with $y < \hat{y}$.

Consider now a deviation by the proposer in period $t - 1$ to $(x, y + \varepsilon)$, where $\varepsilon > 0$ is arbitrarily small. By our conjecture that continuation values are continuous, if the proposer is unconstrained in period t following (x, y) in period $t - 1$, then for sufficiently small ε it remains unconstrained following $(x, y + \varepsilon)$. Therefore, the continuation value in period t is unchanged by this deviation.

Moreover, the deviation strictly increases the static payoff of both the proposer and the veto player in period $t - 1$, as it reduces the cost of inefficiency. As such, the bundle $(x, y + \varepsilon)$ satisfies the constraint, and the deviation is strictly profitable for the proposer. Thus, there can exist no period $t > 1$ in which the proposer is unconstrained in equilibrium. $\square$

Lemma A.16. *An equilibrium exists.*

Proof. Lemma A.15 implies that the veto player's equilibrium continuation payoff in any period $t > 1$ is given by:

$$w_t^V(x^q, y^q) = \sum_{k=t}^{\infty} \delta^{k-t} \{ -(x^q - \hat{x}_k^V)^2 - \alpha_k(y^q - \hat{y})^2 \}.$$

Thus, in any period, the set of acceptable policies to the veto player, defined as:

$$A_t(x^q, y^q) = \left\{ (x, y) \mid -(x - \hat{x}_t^V)^2 - \alpha_t(y - \hat{y})^2 + \delta w_{t+1}^V(x, y) \geq -(x^q - \hat{x}_t^V)^2 - \alpha_t(y^q - \hat{y})^2 + \delta w_{t+1}^V(x^q, y^q) \right\}$$

is a compact, non-empty, and continuous correspondence.

Additionally, $-(x - \hat{x}_t^P)^2 - \alpha_t(y - \hat{y})^2$ is continuous and bounded since in each period either player can always obtain a payoff at least as good as the status quo. Thus, standard results in dynamic programming (see, e.g., [Stokey, Lucas and Prescott, 1989](#), Theorem 4.6) yield that there exists a unique value function that solves the proposer's problem:

$$w_t^P(x^q, y^q) = \max_{(x, y) \in A_t(x^q, y^q)} -(x - \hat{x}_t^P)^2 - \alpha_t(y - \hat{y})^2 + \delta w_{t+1}^P(x, y).$$

Moreover, the solution w_t^P is in the space of continuous functions, and thus the veto player's equilibrium continuation value is continuous, as conjectured. Recall that t indexes for the changing ideal points, so the problem is still stationary in the state variable $(x^q, y^q, \hat{x}^P, \hat{x}^V)$. $\square$

Proposition 5. *Suppose that $\alpha > \frac{1+\sqrt{\delta}}{2\delta}$. If $\beta > \frac{1}{2\delta\alpha-1}$, then in any equilibrium the policy is inefficient in every period t .*

Proof. Our approach is to conjecture that there exists an equilibrium in which P proposes the efficient policy in period t and compare this payoff to a deviation in which P instead proposes an inefficient policy in period t and then the efficient policy in period $t + 1$. To show that this deviation is profitable in every period t , and thus there does not exist an equilibrium with efficiency on the path of play, we proceed in two steps. First, we derive a sufficient condition for this deviation to be profitable in an arbitrary period t . Second, we show that this condition holds in every period t under the premise of the proposition.

First, consider the case where the veto player's constraint is binding at the proposer's optimal first-period policy.

Step 1. We identify a sufficient condition to ensure that an equilibrium where the proposer offers an efficient policy at t cannot be sustained. In particular, we will identify conditions such that a deviation to inefficiency at t followed by efficiency at $t + 1$ is strictly profitable.

Let $y - \hat{y} = \varepsilon$. Denote x_ε as the optimal X -dimension policy for the proposer to offer with amount of inefficiency ε when the veto player's constraint binds. Then, the proposer's equilibrium utility from y is:

$$-(x_\varepsilon - \hat{x}_t^P)^2 - \alpha_t \varepsilon^2 - \sum_{k=t+1}^{\infty} \delta^{k-t} (\bar{x}(x, \varepsilon) - \hat{x}_k^P)^2. \quad (32)$$

Here, $\bar{x}(x, \varepsilon)$ is the upper solution satisfying:

$$-\sum_{k=t+1}^{\infty} \delta^{k-t-1} [\alpha_k \varepsilon^2 + (x - \hat{x}_k^V)^2] + \sum_{k=t+1}^{\infty} \delta^{k-t-1} (\bar{x} - \hat{x}_k^V)^2 = 0. \quad (33)$$

Furthermore, define:

$$\Delta \equiv \frac{\frac{\varepsilon^2 \alpha^{t+1}}{1 - \alpha \delta}}{\sqrt{\left(\frac{\beta^{t+1}}{1 - \beta \delta} + \frac{x}{1 - \delta}\right)^2 + \frac{\varepsilon^2 \alpha^{t+1}}{(1 - \delta)(1 - \alpha \delta)}} + \left(\frac{\beta^{t+1}}{1 - \beta \delta} + \frac{x}{1 - \delta}\right)}.$$

We can rearrange (33) to write:

$$\bar{x}(x, \varepsilon) = x + \Delta.$$

Recall that we assume $\alpha_t = \alpha^t$ and $\hat{x}_t^P = -\hat{x}_t^V = \beta^t$. Also recall that we assume $\delta \beta^2 < 1$ and $\delta \alpha < 1$, which ensures continuation payoffs are finite. Using the identity for quadratic sums and writing $\bar{x}(x, \varepsilon) = x + \Delta$, we can decompose P 's continuation payoff starting at period $t + 1$ from the deviation (x, ε) as:

$$-\sum_{k=t+1}^{\infty} \delta^{k-t} (\bar{x}(x, \varepsilon) - \hat{x}_k^P)^2 = -\sum_{k=t+1}^{\infty} \delta^{k-t} (x - \hat{x}_k^P)^2 - 2\Delta \sum_{k=t+1}^{\infty} \delta^{k-t} (x - \hat{x}_k^P) - \Delta^2 \sum_{k=t+1}^{\infty} \delta^{k-t}.$$

Once we take $\varepsilon \rightarrow 0$, then $x \rightarrow x_t$ and $\bar{x} \rightarrow x_t$. Plugging this into (32), we obtain that P 's payoff from the deviation for ε close to 0 is

$$-(x_t - \hat{x}_t^P)^2 - \alpha_t \varepsilon^2 - \sum_{k=t+1}^{\infty} \delta^{k-t} (x - \hat{x}_k^P)^2 - 2\Delta \sum_{k=t+1}^{\infty} \delta^{k-t} (x - \hat{x}_k^P) - \Delta^2 \sum_{k=t+1}^{\infty} \delta^{k-t}.$$

Recall that the equilibrium utility from $y = \hat{y}$ is $-(x_t - \hat{x}_t^P)^2 - \sum_{k=t+1}^{\infty} \delta^{k-t} (x - \hat{x}_k^P)^2$. Thus, a sufficient condition to ensure that an equilibrium with efficiency at t cannot be sustained is that

$$-\varepsilon^2 \alpha_t - 2\Delta \sum_{k=t+1}^{\infty} \delta^{k-t} (x - \hat{x}_k^P) - \Delta^2 \sum_{k=t+1}^{\infty} \delta^{k-t} > 0. \quad (34)$$

Dividing both sides by ε^2 , letting $\varepsilon \rightarrow 0$ and dropping the non-leading terms, this reduces to

$$-\alpha_t - \frac{\alpha^{t+1}}{1 - \alpha\delta} \frac{1}{\frac{\beta^{t+1}}{1 - \beta\delta} + \frac{x_t}{1 - \delta}} \sum_{k=t+1}^{\infty} \delta^{k-t} (x_t - \hat{x}_k^P) > 0. \quad (35)$$

Using the identity for quadratic sums and our functional form assumptions, we obtain

$$\sum_{k=t+1}^{\infty} \delta^{k-t} (x_t - \hat{x}_k^P) = x_t \frac{\delta}{1 - \delta} - \delta \frac{\beta^{t+1}}{1 - \beta\delta}.$$

Plugging this into (35) and rearranging, we obtain that an equilibrium where the efficient policy is implemented at time t does not exist if:

$$x_t < \frac{1 - \delta}{1 - \delta\beta} \beta^{t+1} (2\delta\alpha - 1). \quad (36)$$

Step 2. Finally, we show that there exists a $\underline{\beta}$ such that condition (36) holds in every period t .

Let x_t^u denote the ideological policy that the proposer would pair with the efficient common-value policy if the veto player's constraint did not bind. Because implementing

$y = \hat{y}$ eliminates the proposer's future leverage, this policy is the one that maximizes

$$-\sum_{k=t}^{\infty} \delta^{k-t} (x - \hat{x}_k^P)^2.$$

Solving the first-order condition gives

$$x_t^u = \frac{1 - \delta}{1 - \delta\beta} \beta^t.$$

Note that the veto player's constraint, when binding, constrains the proposer from above. To see this, observe that the veto player's unconstrained optimal ideological policy when $y = \hat{y}$ is

$$x_t^{V,u} = -\frac{1 - \delta}{1 - \delta\beta} \beta^t.$$

The set of ideological policies that the veto player is willing to accept is an interval centered at $x_t^{V,u}$. Since $x_t^u > x_t^{V,u}$, if the veto player's constraint binds, then x_t^u lies above the upper endpoint of this interval. Therefore,

$$x_t < x_t^u = \frac{1 - \delta}{1 - \delta\beta} \beta^t.$$

Consequently, a sufficient condition for (36) is

$$\frac{1 - \delta}{1 - \delta\beta} \beta^t < \frac{1 - \delta}{1 - \delta\beta} \beta^{t+1} (2\delta\alpha - 1). \quad (37)$$

Note that $\alpha > \frac{1+\sqrt{\delta}}{2\delta}$ implies $2\delta\alpha > 1$. Hence, rearranging, (37) reduces to

$$\beta > \frac{1}{2\delta\alpha - 1}. \quad (38)$$

Next, suppose the proposer is unconstrained. Then, $x_t = x_t^u$. Applying the same deviation argument with $x_t = x_t^u$ again yields condition (38).

Finally, notice that the assumption $\alpha > \frac{1+\sqrt{\delta}}{2\delta}$ implies that $\frac{1}{2\delta\alpha-1} < \frac{1}{\sqrt{\delta}}$, so (38) is consistent with the convergence assumption that $\delta\beta^2 < 1$. Additionally, for all $\delta \in (0, 1)$ we have $\frac{1+\sqrt{\delta}}{2\delta} < \frac{1}{\delta}$. Thus, the assumption of $\alpha > \frac{1+\sqrt{\delta}}{2\delta}$ is also consistent with the convergence assumption that $\delta\alpha < 1$.

□

A.4 Different Weights on Dimensions

Here we consider the baseline model, setting $\alpha_1 = \alpha_2 = 1$, so there are no compounding costs on the Y dimension. However, we now assume that the two players put different weights on the two dimensions. Specifically, assume that the proposer's stage utility is

$$u_{Pt}(x) + \theta v(y),$$

with $\theta > 0$. As in the baseline, the veto player's stage utility is instead

$$u_{Vt}(x) + v(y).$$

Here, we show that a policy pair such that $x_1^* < \hat{x}_1^P$ and $y_1^* < \hat{y}$ can never be sustained in equilibrium. Clearly, if U_q is such that the proposer is unconstrained in the first period, the equilibrium policy must satisfy $x_1^* > \hat{x}_1^P$.

Suppose instead that U_q is sufficiently high that the proposer is constrained in the first period. To establish a contradiction, suppose that $x_1^* < \hat{x}_1^P$, $y_1^* < \hat{y}$ is an equilibrium. Recall that $\bar{x}(x_1^*, y_1^*)$ solves $u_{V2}(x) = u_{V2}(x_1^*) + v_2(y_1^*)$. Further, (x_1^*, y_1^*) must solve $u_{V1}(x_1^*) + v(y_1^*) + u_{V2}(x_1^*) + v(y_1^*) = U_q$. Let $\tilde{x}$ denote the policy that solves $u_{V1}(x) + u_{V2}(x) = U_q$. Suppose that $u_{V2}(\tilde{x}) \leq u_{V2}(x_1^*) + v(y_1^*)$. Then, it must be the case that a deviation to $(\tilde{x}, \hat{y})$ is profitable for the proposer, as this bundle is passable by definition and improves the proposer's payoff if this inequality is satisfied. Suppose instead $u_{V2}(\tilde{x}) > u_{V2}(x_1^*) + v(y_1^*)$. Then, the above equations imply that $u_{V1}(\tilde{x}) < u_{V1}(x_1^*) + v(y_1^*)$, otherwise $u_{V2}(\tilde{x}) > u_{V2}(x_1^*) + v(y_1^*)$ and $u_{V1}(\tilde{x}) + u_{V2}(\tilde{x}) = U_q$ would imply $u_{V1}(x_1^*) + v(y_1^*) + u_{V2}(x_1^*) + v(y_1^*) > U_q$. Therefore, given concavity and the assumption that $\hat{x}_{V2} \leq \hat{x}_{V1}$, the following holds: $-v(y_1^*) < u_{V1}(x_1^*) - u_{V1}(\tilde{x}) < u_{V2}(x_1^*) - u_{V2}(\tilde{x})$. Thus, $u_{V2}(\tilde{x}) < u_{V2}(x_1^*) + v(y_1^*)$, a contradiction.

Finally, note that, as in [Acharya and Ortner \(2013\)](#) and [Lee \(2020\)](#), the proposer may still want to implement an inefficient policy on the common-value dimension in this setting, $\hat{y}_d^P < \hat{y}$. To see this, consider the first-order conditions that $(\hat{x}_d^P, \hat{y}_d^P)$ must solve, assuming $\bar{x}(\hat{x}_1^P, \hat{y}) < \hat{x}_2^P$ so that P cannot just achieve its ideal point in both periods:

$$u'_{P1}(x) + \frac{u'_{P2}(\bar{x}(x, y))}{u'_{V2}(\bar{x}(x, y))} u'_{V2}(x) = 0 \quad (39)$$

$$\theta v'(y) + \frac{u'_{P2}(\bar{x}(x, y))}{u'_{V2}(\bar{x}(x, y))} v'(y) = 0 \quad (40)$$

There is always a solution to these necessary first-order conditions where $y = \hat{y}$ and x

solves $u'_{P1}(x) + u'_{P2}(x) = 0$. Indeed, if θ is sufficiently large this solution does maximize P 's payoff, as maintaining inefficiency for tomorrow is costly. However, when θ is sufficiently small, P weights the costs of inefficiency less than V and $y < \hat{y}$ can instead be better. In this case, from condition (40) we require that $\theta = -\frac{u'_{P2}(\bar{x}(x,y))}{u'_{V2}(\bar{x}(x,y))}$. When θ is small this implies that x and y are such that $\bar{x}(x,y)$ is close to $\hat{x}_2^P$. Additionally, this condition together with (39) implies that the policy on the X dimension solves $u'_{P1}(x) - \theta u'_{V2}(x) = 0$, and hence x is close to $\hat{x}_1^P$ for θ small. Thus, for θ sufficiently low the inefficient solution yields a better first-period policy on X , a better second-period policy on X , and the cost from the inefficiency is relatively negligible.

For example, suppose $u_{it}(x) = -(x - \hat{x})^2$, with $\hat{x}_1^P = -\hat{x}_1^V = 1$ and $\hat{x}_2^P = -\hat{x}_2^V = 2$. Additionally, let $v(y) = -(y - 1)^2$ and $\theta = \frac{1}{8}$. Then $(\hat{x}_d^P, \hat{y}_d^P) \approx (1.43, .058)$ which gives a dynamic payoff of $\approx -.49$, versus the best efficient policy $(x, y) = (1.5, 1)$ which yields $-.5$.