Policy Anchors, Voter Behavior, and the Dynamics of Party Platforms*

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Abstract

Voting is a complex task, and even well-meaning voters must use shortcuts and heuristics to make decisions. In this paper we develop a model in which voters use past platforms as reference points when evaluating parties' current proposals. We show that these *policy anchors* lead to preference reversals in voting—both within and across elections—relative to the standard model. We then incorporate policy anchors into a canonical model of electoral competition and show that anchors change policy outcomes substantively. Party platforms respond to anchors non-monotonically, such that centrist and extreme anchors both induce extreme platforms. Anchors cause policy platforms to evolve continuously where they would otherwise be largely stable, and platform cycles to necessarily be asymmetric, with large jumps outward followed by incremental movements inward. We connect these results to observed patterns in platform dynamics.

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1 Introduction

A half century of research into political behavior has established all too clearly that voters fall short of any notion of a democratic ideal. Voters are neither omniscient information processors nor perfectly rational decision makers. It does not follow from these shortcomings, however, that voters are irrational or, worse still, indifferent to political outcomes. Evaluating parties' policy programs is a difficult problem for voters, and given the cognitive limitations of the human mind, it is to be expected that even well-meaning decision makers must adopt shortcuts and rules-of-thumb in the pursuit of effective decision making (Simon, 1955).

We propose that to make sense of politics, voters use the past as a guide to cut through this complexity and evaluate current offerings. Behavioral economists have repeatedly documented that people's evaluation of their current alternatives is formed by making comparisons to the past, even when the past should be irrelevant to the decision at hand. Similarly, in our theory, voters evaluate and respond to policy platforms based, in part, upon positions that have been offered by parties in earlier elections. A voter sees a party's platform as more attractive if it is closer to her ideal policy than was the party's platform in the last election.

To provide a concrete example, the "compassionate conservatism" of George W. Bush was attractive to voters as it provided a positive contrast to the sharper-edged conservatism of an earlier era. The new policy offering can be seen by the voter as more moderate because, relative to past policies, it is. In this way, past policy positions of the parties serve as *anchors* for the voting decision.

In this paper, we formalize a theory of voting with policy anchors. The use of anchors distorts voter behavior relative to the standard model of spatial voting. Voters may support a party proposing a platform farther from their ideal, and reverse their choice across elections even if faced with identical alternatives. Incorporating policy anchors into a classic model of two-party competition, we show they produce rich dynamics that otherwise would not emerge. Within each election, party platforms respond to anchors non-monotonically. Moderate anchors induce moderate platforms, whereas centrist and extreme anchors both induce more extreme platforms. Across elections, this U-shaped reaction function causes party platforms to cycle continuously between moderate and more extreme positions in an environment where change would otherwise be infrequent. Furthermore, such cycles are *necessarily* asymmetric, with frequent incremental movements inward punctuated by occasional large jumps outward.

Our findings speak to a puzzle that has emerged in recent times. A striking finding in the empirical literature is that party platforms exhibit larger variance over time than trends in voters' underlying policy opinions (Page and Shapiro, 1992; Budge et al., 2010;

Druckman and Leeper, 2012).[1] Our model provides a rationale for this pattern. We show how frequent movements in party platforms, both small and large, emerge from a setting where the underlying distribution of voter preferences changes much less.

Policy Anchors and Constructed Preference

A long-standing perspective in the psychology of choice is that preference is constructed rather than inherent (Lichtenstein and Slovic, 2006). A key insight in this literature is that "... the true objects of evaluation and choice are neither objects in the real world nor verbal descriptions; they are mental representations" (Tversky and Kahneman, 2000, p. xiv). A core idea in this approach is that individuals, when called upon to evaluate an alternative, often rely on "reference points" that lie outside of their choice set as a useful benchmark for calibration (Kőszegi and Rabin, 2006). Individuals construct their preference anew for each decision problem using context-dependent and potentially evolving reference points.

Voting is a natural domain for preference to be constructed rather than inherent. Voting is, one may argue, substantially harder than decisions in a market because the policy space is *scale-free*. In consumer markets, for example, products come with prices that provide a common metric for comparison. In contrast, policy positions stand alone, with no natural "score" or metric for evaluation, making comparisons difficult, especially for policies on opposite sides of the ideological spectrum. What does it mean to an average voter, for example, if Republicans offer a tight immigration policy? How much should the voter value this policy? In particular, how does she compare this proposal, that may lie to her right, to the Democrats' position that is on her left? In this domain, then, it seems natural that voters may be constructing "mental representations" (Tversky and Kahneman, 2000, p. xiv) of the different policy proposals when choosing which party to support, and that they may use reference points to do so.

Our theory of voting with anchors builds on a core idea: comparing policies that lie on the same side of her ideal is easier for a voter than comparing policies on opposite sides. When a voter sees two policy alternatives, say, to her left, she can compare them directly and tell which is closer to her ideal. When the policies are on opposite sides of her ideal point, the *scale-free* nature of the policy space makes the comparison difficult and the voter turns to policy anchors for assistance.

In our model, voters use past platforms as reference points to anchor their evaluation of today's alternatives. Anchoring is thus party- and direction-specific. To evaluate the current platform of the party to her left, the voter uses that party's left wing offering in the past, and

[1] The extent of voter movement is an ongoing debate in the academic literature (e.g., Boxell et al., 2024), although a consensus holds that voters have shifted ideologically *less* than elites.

similarly for the right-wing platform.A platform that compares well to the past—in the sense that it is closer to the voter's ideal point—is evaluated more positively, and a platform that compares unfavorably will be evaluated more negatively. Voters use their ability to evaluate policies that lie in the same direction from their ideal point to facilitate comparisons of policies that lie in different directions.

This formulation is consistent with the use of reference points in psychology and economics. The literature on choice and background context (Tversky and Simonson, 1993) shows that the previous choice set influences an individual's evaluations of her current alternatives. This literature also highlights the difficulty of comparing alternatives when neither unambiguously dominates the other. Consumers have no trouble choosing between products when one dominates on all dimensions, but they struggle when each product performs better on some dimensions and worse on others. Our theory of voting with anchors follows a similar logic. We theorize that anchors matter in politics, and that they matter even in a single dimension when voters compare platforms in opposite directions from their ideal point.

Our formulation of voting also resonates with evidence from multi-candidate elections. Using a survey experiment based on US elections, Simonovits (2017) shows that adding an extreme candidate on one side of the spectrum makes a moderate candidate on the same side more appealing. In fact, when asked to place a moderate conservative on the policy spectrum, participants who were exposed to an extreme conservative placed the moderate closer to the center.[2] Exposure to an extreme liberal had the same effect on perceptions of a moderate liberal. Consistent with our theory, voters appear to rescale each side of the policy space to accommodate the extreme candidate on that side. Moreover, as we theorize, this rescaling is directional: a moderate liberal candidate is immune from comparison to an extreme conservative on the other side, and vice versa. Waismel-Manor and Simonovits (2017) and Wang and Chen (2019) provide related evidence from Israel and Taiwan, respectively.

Our theory differs from these studies in the referent that voters use to construct their preference. These studies document the directional effect of within-election comparisons in multiple candidate elections. We theorize that, when a voter evaluates only two candidates—one from each side—past platforms serve as reference points.[3]

More Detail on the Model and Our Contribution

Our contribution has two connected parts. First, we introduce a model of voting with policy anchors. We show that anchors generate two types of preference reversals, relative to the

[2]Rotter and Rotter (1966) provide related evidence on Wallace's effect on Goldwater's vote total in the 1964 U.S. presidential elections.

[3]Callander and Wilson (2006) show that the local context shapes turnout *within* an election. In contrast, we show that background context *across* elections affects vote choice.

standard spatial voting model. *Within* an election, a voter may support the party that is more distant from her in the policy space, despite preferring, all else equal, policies that are closer to her ideal point. *Across* elections, she may switch her preference over the same pair of policy positions. Although both preference reversals appear to violate spatial voting, we show they arise as its natural consequences when voters evaluate positions using policy anchors.

Our second contribution explores how policy anchors shape politics more broadly. We ask how parties respond to voters' anchoring and what this implies for platform dynamics. To answer these questions, we incorporate policy anchors into the classic model of two-party competition under uncertainty (Wittman, 1983; Calvert, 1985), extended to an arbitrary number of elections.

Voter anchoring changes parties' incentives. We show that the most moderate platforms emerge when the policy anchors are moderate, whereas centrist and extreme anchors both produce extreme platforms. Thus, the response of party platforms is U-shaped in the anchor.

Across elections, this U-shaped reaction function generates rich dynamics in party platforms. In an electoral environment that changes over time, policy anchoring induces asymmetric and cyclical patterns in platform evolution. Platforms jump outward and creep inward before restarting the process once again, even if changes to the environment are small and infrequent and platform change would otherwise be rare without anchoring. As we discuss in more detail below, these asymmetric cycles are unique to our model, distinguishing our theory from both the standard spatial models and alternative accounts of platform evolution.

It may seem surprising that a party would ever shift its platform away from the median voter, only to reverse that shift in the following election. This behavior reflects how voters use anchors, and how anchoring interacts with parties' policy motivations. In line with the reference-points literature, in our model the marginal effect of anchoring on voters' evaluations is larger when the platform and anchor are close, and flattens as they get more distant. Starting from an extreme anchor, where parties' policy motivation to move more extreme is weak, the anchor effect pushes the party to incrementally move the platform towards the median voter. As parties converge towards increasingly centrist anchors, however, policy motivations increasingly push them to shift outward. Eventually the policy effect wins out, and as the negative effect of the anchor weakens as the policy jump increases, the parties make a single large jump outward. Upon returning to an extreme platform, the incentives for incremental moderation reemerge.

Empirical Connections and Implications

An important literature studies the determinants of changes in parties' policy platforms (see Adams (2012) and Fagerholm (2016) for reviews). Empirical work based on the Comparative

Manifesto Project has documented that party platforms feature frequent movements and policy alternation across a wide variety of countries and contexts, with parties cycling between moving to the extreme and moving to the center across elections (Adams et al., 2004; Budge et al., 2010). Moreover, these movements do not smoothly track public opinion and, while the extent of voter movement is an ongoing debate in the academic literature (Boxell et al., 2024), a consensus holds that voters have shifted ideological ground *less* than have elites. Our theory of policy anchors offers a novel explanation for these stylized facts, one that is rooted in the psychology of voters and parties' strategic incentives.

The pattern observed in the United States in the three decades of the post-Reagan era illustrates the cyclical patterns predicted by our model, and the underlying logic. Democrats and Republicans have alternated between periods of movement away from each other and periods of moderation toward one another. Interestingly, many of these platform shifts were at the time framed or described, by both the media and politicians themselves, in relation to the past, suggesting a role for policy anchors. For example, the 1992 Democratic platform was described as a "third way," highlighting its departure not only from the Republican status quo, but also from the older Democratic big government approach. Similarly, as mentioned above, George W. Bush's 2000 Republican platform adopted the language of "compassionate conservatism," and contemporaneous commentary described this rhetoric as helping position Bush as a moderate in the general election by drawing a favorable comparison with the sharper-edged conservatism of an earlier era. Consistent with our theory, these examples suggest that party platforms are rarely evaluated in a vacuum, and rather comparisons to the past inform the way the electorate perceives and responds to parties' proposals.

A further benefit of our model is that we are able to derive additional empirical implications about the patterns of platform movements and their distribution. In particular, there should be an asymmetry between movements to the center and movements to the extremes. We expect small frequent movements in one direction punctuated by occasional large movements in the other. These implications are unique to our theory and, to the best of our knowledge, existing work has not investigated whether there is an asymmetry in the frequency and magnitude of changes depending on whether platforms become more moderate or more extreme.[4] These predictions thus open a particularly promising avenue for future empirical work.

[4]Walgrave and Nuytemans (2009) demonstrates that the distribution of changes to party manifestos is highly leptokurtotic, with many small changes, occasional large revisions, and few moderate changes. However, they look at changes in issue attention, rather changes in policy extremism, so their findings do not directly apply to our setting.

Other Related Ideas and Literature

As detailed above, our model builds on the reference-point literature originating with Kahneman and Tversky (1979). The key departure is what voters use as the reference point. In Kahneman and Tversky (1979) the reference point is the wealth level, and in subsequent work it is typically an exogenous benchmark or within-equilibrium conjectures. In our theory, the reference point is a party's own history. This reframes political preferences and political behavior as interlinked: past party platforms shape voter behavior, which in turn shapes future party choices, and so on.

Our approach also differs in the details. While much of the literature couples reference points with loss aversion, our model treats platform shifts away and towards voters symmetrically. This avoids hard-coding asymmetries into the model, allowing us to isolate the strategic forces underlying our asymmetric cycles.

A second difference is that voters use different reference points for different alternatives.[5] This further distinguishes our work from previous applications of reference points to politics, where voters use the same reference (typically, the status quo) to evaluate all policies or candidates (Alesina and Passarelli, 2019; Lockwood and Rockey, 2020; Grillo and Prato, 2023). This difference is crucial to the U-shaped response of party platforms to the anchors, and thus to the cyclic dynamics that we uncover. Furthermore, these existing works focus on characterizing parties' behavior within a single period, in contrast with our objective of studying platforms' long-run evolution.[6] Lockwood et al. (2024) is an exception that shows how loss aversion—voters' asymmetric reaction to gains and losses—can generate *monotonic* changes in platforms when status-quo utility is the reference point. Loss aversion plays no role in our mechanism, and our result is driven instead by the interaction of the anchor and party incentives. We show this generates platforms' *non-monotonic* response to the anchors.

Other works embed psychological insights into game-theoretic models of electoral competition, and several papers within this niche explore the dynamics of party platforms. Callander and Carbajal (2022) study the comovement of party platforms and voter preferences and explain polarization but not cycles. In contrast, we fix voters' innate preferences and show how cycles can emerge. Levy and Razin (2025) provide a model of cycles based on voter learning about the best policy. Our model is one of complete information.

Finally, our theory also differs from standard accounts of platform evolution, emphasizing

[5]The directional nature of voter evaluations in our theory is distinct from the directional theory of voting of Rabinowitz and Macdonald (1989). That theory argues that candidate evaluation is not spatial whereas our theory is firmly rooted in spatial voting.

[6]Grillo (2016) considers a game where candidates can make statements about their valence, and these statements form candidate-specific reference points. However, the model features a single election, and reference points concern valence rather than policy.

pressure from activists, primaries, or changes in voter preferences, in three key respects. First, platform evolution is path-dependent: today's platforms depend on those offered in the past.[7] This intertemporal linkage is absent from standard models, in which voters' evaluations are not shaped by policy anchoring and parties simply respond to the strategic pressures of the upcoming election. Second, platforms evolve cyclically and asymmetrically, with occasional large movements in one direction punctuated by smaller, more frequent shifts in the opposite. This pattern is generally not implied when party platforms simply track voter preferences or pressures from other political actors. Third, and crucially, these dynamics do not arise from changes in voters' or activists' fundamental preferences. Voters' bliss points remain stable, and parties themselves initiate the cycle by responding to voters' path-dependent evaluations, in turn setting in motion dynamics that reshape future evaluations. Thus, our theory differs from alternative accounts in both the pattern and the mechanism of platform evolution.

2 Policy Anchors and Voter Behavior

In this section, we introduce policy anchors to the spatial theory of voting.

2.1 A Theory of Voting with Anchors

Throughout, we take the policy space to be single dimensional and given by $[-1, 1]$. Denote a generic platform at time t by p_t and the set of platforms by P_t. We consider a sequence of elections between two parties, D and R, such that $P_t = \{d_t, r_t\}$ and $P_{t-1} = \{d_{t-1}, r_{t-1}\}$. Without loss of generality, set $d_t \leq r_t$. Our focus is on how a generic voter with ideal policy $x \in [-1, 1]$ evaluates the parties at time t.

Our theory of voting with anchors builds on the idea that, because the policy space is scale-free, voters find it difficult to compare policy platforms that lie on opposite sides of their ideal point. Platforms that lie in the same direction from a voter's ideal point can be ranked ordinally, even in a scale-free space, so the voter can easily identify which is closer. But when they are in opposite directions, the voter struggles, as neither is unambiguously closer in purely ordinal terms. Comparing them requires a metric that makes distances in the two directions commensurable.

We theorize that voters respond to this problem by using parties' past platforms as policy anchors. Although a voter struggles to directly compare the current Democratic and Republican platforms when they lie on opposite sides of her ideal point, she can more eas-

[7] In Page's (2006) terminology, our cycles are *path*-dependent but not *outcome*-dependent: election outcomes depend on previous platforms, but their long-run distribution does not.

ily compare each party's current platform with its own previous platform when both lie in the same direction. Specifically, a voter contrasts the Democratic platform with the party's previous offering, and does the same for the Republican platform, as this helps her better compare the parties' current platforms to each other. A party's current platform appears more attractive when it is closer to the voter than its previous platform and less attractive when it is farther away. The voter leverages her ability to make *within-direction* comparisons to create a common evaluative frame with which to make *across-direction* comparisons.

To formalize this argument, we assume that, when faced with a choice between two platforms $d_t \leq x \leq r_t$ with anchors $d_{t-1} \leq x \leq r_{t-1}$, the voter supports D if

$$-|d_t - x| + f(\Delta d_t) > -|r_t - x| + f(\Delta r_t) \tag{1}$$

and supports R otherwise. Here, $\Delta p_t = |p_{t-1} - x| - |p_t - x|$, and thus $f(\Delta p_t)$ captures the effect of policy anchors on the voter's evaluation. This formulation follows the reference-point literature, where individuals evaluate alternatives using an intrinsic utility component, and a gain-loss component capturing the effect of comparison with the reference point. In our setting, the Euclidean distance term corresponds to the intrinsic component, while $f(\cdot)$ captures the gain-loss component.

The logic of anchors implies two immediate properties: $f(\cdot)$ is strictly increasing and $f(0) = 0$. Thus, if $\Delta p_t > 0$, the voter perceives the party's current platform more favorably, because this evaluation is benchmarked against the party's previous offering, which is less appealing; if $\Delta p_t < 0$, instead, the effect of anchoring is negative for the party. In both cases, the size of the effect increasing in the size of $|\Delta p_t|$. If a party does not shift its platform across periods, the anchor has no effect.[8]

The shape of the gain-loss evaluation. How anchors impact voter behavior depends on the shape of the gain-loss function. The first two properties we impose are for simplicity and clarity: f is twice-differentiable, and an anchor is neutral in the sense that $f(a) = -f(-a)$. This implies the boost from shifting toward the voter is exactly matched by a reduction following an equal shift away. Our results do not rely on this symmetry, which we impose to emphasize that the gain-loss asymmetry driving prospect theory plays no role in our theory.

The final property is substantive and follows from the reference-dependence literature. That literature assumes that the marginal effect of using a reference point on an individual's evaluation is stronger for small shifts away from the reference point, and diminishes as the

[8]The dependence on Δp_t implies the effect of a given movement is independent of anchor's distance from the voter. This can be generalized using *skew-symmetric* functions that allow platforms' absolute distance from the voter to affect their evaluation (Fishburn, 1982).

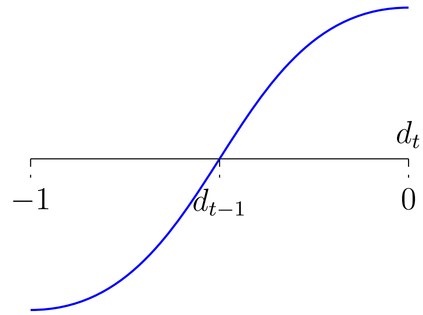


Figure 1: Anchor Function $f(\Delta d_t)$

alternative becomes more distinct from it (see, e.g., Kahneman and Tversky (1979) for theoretical arguments on this point and Abdellaoui (2000) for empirical evidence). In other words, the effect is convex for alternatives that constitute a loss relative to the reference point, and concave for gains. In our setup, this corresponds to an s-shaped f function: the marginal effect of today's platform moving away from the relevant anchor—whether toward the voter or farther from her—becomes smaller for larger movements. Formally, we assume $f''(\Delta p_t) > 0$ for $\Delta p_t < 0$ and $f''(\Delta p_t) < 0$ for $\Delta p_t > 0$, as depicted in Figure 1.

Voters on the flanks. The decision rule established by condition (1) applies to a median voter facing left-wing and a right-wing parties, and to other voters located centrally. These voters play a central swing-voter role in elections, but they are not the only voters: for those on the flanks, both platforms will lie on the same side of their ideal point.

As discussed above, a premise of our theory is that voters *can* evaluate and compare distances that lie in the same direction. Following this logic, voters on the flanks do not need, and therefore do not use, policy anchors. To a voter on the far left flank—with $x < d_t < r_t$ and both platforms to her right—the relative appeal of D over R is clear. For these voters, voting corresponds to classic spatial voting.

An edge case that applies to some moderate voters is that the current platforms lie on opposite sides of their ideal point, but past platforms do not. For example, consider a voter facing the following situation: $d_{t-1}, d_t, r_{t-1} < x < r_t$. As the voter cannot directly compare the current platforms to come to a vote decision, she will, following the logic of our theory, turn to policy anchors. She is unable, however, to use R's offering from the previous period to anchor her evaluation of r_t, as this would require performing precisely the cross-sides evaluation the voter turns to anchors for. We assume for such a voter that she uses the most recent past platform presented by the right-wing party that lies to her right, even if that goes back more than one election.[9] Notice that voters with bliss points between r_{t-1} and r_t may be using

[9] If r_t is the most extreme right-wing platform offered through t, a small number of voters cannot look to the past for anchors. We may assume that these voters use the right edge of the policy space—or, equivalently

different anchors to benchmark their evaluations. As we show below, this does not imply median decisiveness must be violated in our setting.

Do voters calculate utility? Denote the anchor used by voter with bliss point x to evaluate policy p_t by $a_x(p_t)$. The voter's decision process can be represented by the following utility function.

$$u_x(p_t|p_{t-1}) = \begin{cases} -|p_t - x| \quad + \quad \underbrace{f(\Delta p_t | a_x(p_t))}_{\text{gain-loss}} & \text{if} \quad d_t < x < r_t \\ \underbrace{-|p_t - x|}_{\text{intrinsic}} & \text{otherwise.} \end{cases} \tag{2}$$

The utility function in (2) represents voter preference without implying that voters literally calculate and aggregate each component into a quantitative measure of utility for each party. Each voter constructs her preference from the context of the choice set, and may or may not construct quantitative measures. Following the method of mathematical psychology often adopted in behavioral economics, we say that the utility function represents these preferences in the sense that its output is consistent with the behavior that a voter exhibits.

2.2 Implications for Voting Behavior

Policy anchors do not upset the basic principle of spatial voting that, all else equal, a policy platform closer to the voter is more favorable than a platform farther away. Policy anchors *do*, however, violate the standard implication of spatial voting that a voter votes for the policy platform that is closest to her ideal.

Consider a voter facing the situation depicted in Figure 2. Party R's platform is closer to the voter's ideal policy at x than is party D's. However, party R has not moved from its platform in the previous election whereas party D has moved toward the voter.

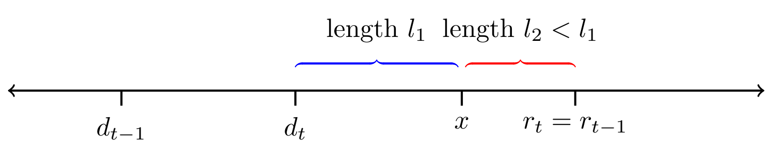


Figure 2: Within-Election Preference Reversal

in our model, R's ideal policy—to anchor their evaluation of r_t. Alternatively, they may use no anchor or the equilibrium platform from the standard Calvert-Wittman model without anchors. All these assumptions would leave our results unchanged.

D's shift makes it more appealing to the voter. If the boost from the shift is significant enough, it can induce the voter to support D over R despite D's platform being farther away. This represents a *within-election* preference reversal relative to standard spatial voting.

Within-Election Preference Reversal A voter may vote for a party other than the one with a platform closest to her ideal policy.

This effect can also generate an *across-election* preference reversal. Even if the parties do not change their platforms from one election to the next, a voter may switch from one party to the other. Figure 3 illustrates this possibility by extending the example in Figure 2 by one election. D's extreme position at $t-1$ boosts its appeal at t. After election t, however, the anchors reset and the boost disappears if parties remain at the same location. At $t+1$, D loses the voter because it is farther from her than R.

Across-Elections Preference Reversal For pairs of policy platforms that are unchanged from one election to the next, $d_{t+1} = d_t$ and $r_{t+1} = r_t$, a voter may switch which party she votes for from election t to election $t+1$.

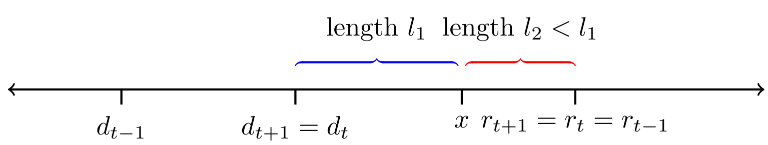


Figure 3: Across-Elections Preference Reversal

The possibility of these two forms of preference reversal is unique to our theory. It stands in sharp contrast to the predictions of standard spatial models and, as we delineate below, crucially shapes parties' incentives and the equilibrium dynamics of their platforms.[10]

3 Electoral Competition with Anchors

We turn now to how voters' use of anchors impacts politics more broadly. It is natural that those competing for voters' affection adjust and adapt to their use of anchors. To study these follow-on effects, we develop a model of electoral competition and characterize how voters' use of anchors affects the policy platforms offered by political parties. The model builds on the

[10]More precisely, what is unique is the possibility of these preference reversals in a setting where voters care *exclusively* about parties' platforms.

classic Calvert-Wittman framework with policy-motivated parties, extending it dynamically. To highlight the role of anchors, we otherwise remain faithful to their framework.

In each period $t = 1, 2, ...,$ the two parties D and R compete in an election by simultaneously offering platforms, d_t and r_t. Voters observe the proposals, cast their votes in a majority rule election, and the winning party implements its platform.

Voter ideal points are distributed over the policy space $[-1, 1]$ with a median ideal point at 0. In addition to policy, voters cares about the valence of each party. Normalizing D's valence to 0, R's valence is given by $\theta_t \sim U(-\psi, \psi)$. The valence θ_t is unknown to the parties until after they have chosen their policy platforms, but it is realized before voting occurs.

Recall that $a_x(p_t)$ denotes the anchor that a voter with ideal point x uses to evaluate platform p_t, as defined in the previous section. Thus, the median voter's utility for electing D is

$$u_0(d_t|d_{t-1}) = -|d_t| + f\big(d_t - a_0(d_t)\big) \tag{3}$$

while the voter's utility from electing R is

$$u_0(r_t|r_{t-1}) + \theta_t = -|r_t| + f\big(a_0(r_t) - r_t\big) + \theta_t. \tag{4}$$

To ensure continuity of the parties' utility, we assume that, in any period t, $d_t \leq 0 \leq r_t$: the parties are restricted to proposing platforms on 'their' side of the median's ideal point. Notice that this implies that the median voter always lies between the parties' platforms, present and past, therefore $a_0(d_t) = d_{t-1}$ and $a_0(r_t) = r_{t-1}$; that is, the median voter always uses a party's offering from the previous period to anchor her evaluation of the party's current platform. The anchors for the first election, d_0 and r_0, are exogenously given.

The parties are motivated purely by policy outcomes. The ideal policy of party D is -1 and of party R is $+1$. Their utility functions from a policy p are

$$v_D(p) = -|p+1| \quad \text{and} \quad v_R(p) = -|p-1|. \tag{5}$$

Intertemporal linkages. The key linkage across elections is the endogenous evolution of anchors: today's platforms become tomorrow's anchor points. To isolate this mechanism, we shut down other intertemporal linkages. Parties and voters are myopic and therefore do not consider how current platform choices affect future utility. We revisit and discuss the role of foresight later in Section 4.2. Valence draws are also independent across elections, precluding learning about voter preferences over time.

The electoral environment. In practice, the electoral environment evolves as issues emerge, their salience changes, and voter demographics and preferences shift. To understand the full impact of voter anchoring, we thus develop a model that allows for change to the environment. For focus and tractability, we limit this change to the valence term. Specifically, we index ψ by the time period t and we suppose that $\psi_t \in \{\psi_l, \psi_h\}$, with $\psi_l < \psi_h$, evolves as a Markov chain with a transition probability $\rho \in (0, 1)$. This means that in each period, the state switches with probability ρ and remains unchanged with probability $1 - \rho$; a small ρ, therefore, represents a very stable environment. We assume that ψ_t is drawn and publicly observed at the start of each subsequent period.

To build intuition, we begin with the case where the environment is unchanging, i.e., ψ_t is fixed in every period. We drop the time subscript from ψ when there is no room for confusion.

The anchoring function. Finally, we impose the following technical conditions on the anchoring function and support of the valence shock:

Assumption 1 *For any Δp and ψ we assume: (1) $1+2f(1) < \psi$, (2) $\psi < 2$ and $f(1) < 1/2$, (3) $\frac{f''(\Delta p)}{1+f'(\Delta p)} < 1$, and (4) $f(1) < \frac{\psi}{2\left(1+f'(0)\right)}$.*

Part 1 ensures that there is always an interior probability of winning the election. Part 2 implies that the standard Calvert-Wittman equilibrium is interior to the parties' ideal points, while still being consistent with part 1. Finally, parts 3 and 4 guarantee the existence of a unique symmetric equilibrium, and that in this equilibrium the median voter is decisive.

4 Results

Policy anchors alter voters' perceptions of candidates and, as we saw in the previous section, can flip vote preference relative to the classic spatial model. Moreover, centrist voters are likely to use anchors whereas voters on the flanks do not. As a consequence, voters on the flanks may have less intense preferences over the two platforms than more moderate voters. For some platforms and realizations of the valence shock this can generate voting coalitions that are disjoint. We begin with a preliminary result showing that these possibilities do not impact the existence of symmetric equilibria where the median voter remains decisive.

Disjointed Coalitions and Median Voter Decisiveness.
Lemma 1 establishes that symmetric equilibria of our model can be characterized using the median voter. Note that, in our setting, symmetric equilibria require symmetry of the original exogenous anchors, so we assume $d_0 = -r_0$. Define the *median-decisive game* as the model where each party chooses policy to maximize its expected utility as if the election outcome is decided by the median voter's decision.

Lemma 1 *Any pair of platforms $r_t = -d_t$ is a symmetric equilibrium if and only if it is an equilibrium of the median-decisive game.*

Starting from a symmetric conjecture, with platforms equidistant from the median's ideal, this voter is decisive. This is because the anchoring effect disappears for the median ($f(d_t - d_{t-1}) = f(r_t - r_{t-1})$), and hence her preferences are no more intense than those of voters on the flanks. If a party deviates from the symmetric platform closer to the median, then the effect of anchoring reemerges and the winning coalition may be disjointed, with the party losing voters on his flank. However, under Assumption 1 part (4), the anchoring boost is not sufficiently strong to allow the deviator to gain enough voters on the opposite flank to compensate. Thus, the party's probability of winning from a deviation is lower than in the median decisive game, and we get that the symmetric equilibria of the full game coincide with those of the median decisive game.

Moving forward, we restrict attention to symmetric equilibria of the full game, and thus focus on the vote of the median voter (or "the voter"). We now explore equilibrium party platforms *within* an election before turning to platform dynamics *across* elections.

4.1 Party Platforms Within an Election

Consider the first election at $t = 1$. The voter votes for party R if $u_0(r_1|r_0) + \theta_1 \geq u_0(d_1|d_0)$, and for party D otherwise. Given her ideal point of 0, and that θ_1 is drawn uniformly over $[-\psi, \psi]$, the probability R wins the election under platforms (r_1, d_1) and anchors (r_0, d_0) is:

$$P(r_1, d_1|r_0, d_0) \equiv \frac{1}{2} - \frac{r_1 + d_1 + f(d_1 - d_0) - f(r_0 - r_1)}{2\psi}. \tag{6}$$

As the parties are policy motivated, they balance the probability of winning against the policy outcome. Party R's maximization problem is described by:

$$\max_{r_1} P(r_1, d_1|\cdot)v_R(r_1) + \big(1 - P(r_1, d_1|\cdot)\big)v_R(d_1). \tag{7}$$

Plugging (6) into this maximization problem, R's equilibrium choice must satisfy

$$\Big(\frac{-1 - f'(r_0 - r_1)}{2\psi}\Big)(r_1 - d_1) + P(r_1, d_1|r_0, d_0) = 0 \tag{8}$$

Party D's problem is analogous. Recall that we focus on symmetric equilibria, where $d_t = -r_t$ (and the initial exogenous anchors are symmetric as well, $r_0 = -d_0$). Then, (8) reduces to

$$\left(\frac{-1 - f'(r_0 - r_1)}{2\psi}\right) 2\, r_1 + \frac{1}{2} = 0 \tag{9}$$

Then, a unique symmetric equilibrium exists and is characterized as follows.[11]

Proposition 1 *There exists a unique symmetric equilibrium $r_1^* = -d_1^* \in (0, 1)$ where*

$$r_1^* = \frac{\psi}{2\Big(1 + f'(r_0 - r_1^*)\Big)}. \tag{10}$$

The effect of the anchor is evident in Equation (10). This result encompasses the canonical model without the anchor term (equivalent to the slope of $f(\cdot)$ being zero everywhere). In that case, the equilibrium in Proposition 1 reduces to the result of Calvert and Wittman.

Corollary 1 *Without voter anchoring, the equilibrium platforms are $r_{cw}^* = -d_{cw}^* = \frac{\psi}{2}$.*

The equilibrium without anchors is independent of the history. Thus, wherever the parties were previously located, they jump to the locations indicated in Corollary 1.

When voters use policy anchors, equilibrium locations depend on the history. A first observation is that anchoring provides a convergent force on party locations. To see this, recall the classic incentives of policy motivated parties: By moderating toward the median voter, a party increases its probability of winning but wins with a less attractive policy. Without policy anchoring, these forces are in perfect balance for party R at r_{cw}^*. With policy anchoring, and regardless of where the anchor is, moderating towards the median gives R a slightly larger boost in its probability of winning as the new platform will look more attractive (or less unattractive) relative to the anchor. This is evident formally in Equation (10) as the slope of $f(\cdot)$ is everywhere strictly positive. It follows that the equilibrium with anchoring is, all else equal, always more convergent than it is without anchoring.

Corollary 2 *The equilibrium platforms with voter anchoring satisfy $r_1^* \in (0, r_{cw}^*)$.*

However, it does not follow that the parties always moderate to look better to voters than they did previously. For very moderate anchors, the parties adopt platforms more extreme than their anchors, despite knowing that the comparison will be viewed poorly by voters.

[11] Uniqueness follows under Assumption 1(3), which guarantees that the left-hand side of (9) is strictly decreasing in r_1 and thus here is (at most) a unique r_1 satisfying (9). As established in the Appendix, the other conditions in Assumption 1 guarantee existence of a solution, completing the proof.

To see why, return again to the classic trade-off parties face, this time turning it around: by choosing a more extreme platform, a party lowers its probability of winning but wins with a more appealing policy. This incentive is strong when starting from a very moderate position, so even though a more extreme platform comes with the additional cost of a poor comparison to the policy anchor, it can still be worth it.

A key threshold in the logic of policy positioning in this setting is the anchor that is the fixed point of Equation (10), which we denote by $\overline{r}$. When the policy anchor for party R is at $\overline{r}$, its optimal choice is $\overline{r}$ itself. Formally, $\overline{r}$ satisfies $r_1^*(r_0) = r_0$, and is explicitly given by:

$$\overline{r} = \frac{\psi}{2\Big(1 + f'(0)\Big)}. \tag{11}$$

Recall that the anchoring function is s-shaped, i.e., f' is maximized at 0. As a consequence, $\overline{r}$ is a lower bound on the the equilibrium platforms of the right-wing party. Proposition 2 describes how the equilibrium varies in the location of the anchor around $\overline{r}$. See also Figure 4.

Proposition 2 *If:*

1. $r_0 < \overline{r}$ *then* $r_1^* > \overline{r}$*, and decreasing in* r_0*.*

2. $r_0 > \overline{r}$ *then* $r_1^* \in (\overline{r}, r_0)$*, and is increasing in* r_0 *at rate less than one.*

3. $r_0 = \overline{r}$ *then* $r_1^* = \overline{r}$*.*

When the anchor is more moderate than $\overline{r}$, the equilibrium platform is more extreme than the anchor, whereas for an anchor more extreme than $\overline{r}$ the platform is more moderate. This response is asymmetric around $\overline{r}$. For moderate anchors, the platform is substantially more extreme than the anchor, with the gap narrowing the closer the anchor is to $\overline{r}$. For extreme anchors, the platform is incrementally more moderate, with the gap widening as the anchor becomes more extreme. Thus, equilibrium platforms are non-monotonic and U-shaped in the anchor: the most extreme platforms arise from moderate and extreme anchors, while the least extreme platform arises when the anchor is $\overline{r}$ and Party R locates exactly at the anchor.

This pattern is the consequence of the decreasing marginal effect of moving away from the anchor. Small differences with the anchor lead to relatively large marginal benefits and costs whereas larger differences lead to relatively smaller marginal benefits and costs. To see the intuition for how this influences parties' platform choices, consider the case of an extreme anchor. In this case, the parties have electoral incentives to moderate towards the median voter. However, the marginal anchoring benefit gets smaller as the parties move away from

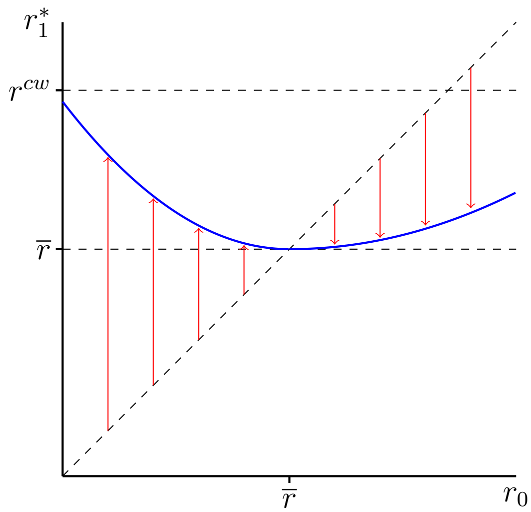


Figure 4: Equilibrium Platforms

the anchor and towards the voter, since the marginal anchoring boost is decreasing. Thus, when parties moderate away from the anchor, they do so in small incremental shifts.

Now consider a relatively moderate anchor. In this case, parties have a policy incentive to move to the extreme, away from the anchor *and* the median voter. Symmetrically to what described above, the marginal electoral cost of an outward move decreases as the platform moves farther away from the anchor. Thus, once a party begins moving outward, further movements become progressively less costly at the margin. Starting from centrist anchors, the parties then shift outward more than they otherwise would, overshooting the threshold $\overline{r}$.

The asymmetric reaction of platforms to the anchor reflects this logic, forming a single-dipped function such that centrist and extreme anchors give rise to local maxima in the equilibrium platforms. The overshooting of $\overline{r}$ starting from a moderate anchor is a key insight of our model and the driving force of cycles in party platforms, to which we now turn.

4.2 The Dynamics of Party Platforms

Over time, the policy anchors evolve with political competition. Voters use the previous positions of the parties to evaluate their current platforms, and the current platforms then become the anchors for the following election. In this section we study the dynamics in party platforms that this recursive process generates. We begin with a political environment that is unchanging (ψ_t is fixed) and then turn to the more realistic setting in which the environment itself changes over time (ψ_t may fluctuate between ψ_l and ψ_h).

4.2.1 Party Platforms in an *Unchanging* World

In an unchanging environment, the competitive incentives of the parties recur from one election to the next. Without policy anchors, equilibrium behavior is straightforward: In the first election the parties locate at d^*_{cw} and r^*_{cw} and remain there for each election thereafter.

With anchors, equilibrium behavior is very different. Even in an environment that doesn't change, party platforms continuously evolve over time. For instance, if we think of the first election as occurring de novo then voters will not have anchors to use. In this case, the parties face the classic Calvert-Wittman incentives and, following Corollary 1, they locate at d^*_{cw} and r^*_{cw}. However, once they have done so, these platforms become anchors for the second election, and the incentives of the parties change. In the second election, the parties converge incrementally closer to the median voter, with those platforms becoming the anchors for the third election, and so on ad infinitum. This process continues indefinitely with the platforms changing in every period as R converge on the stable position $\overline{r}$.

Even richer dynamics are possible if instead we think of parties starting with existing platforms the voters can use as anchors at the time of the first election (due to some unmodeled history). Namely, if the status quo platforms are centrist at the first election, the parties will react to these anchors by doing a large jump outward, over and beyond $\overline{r}$. After the initial jump, they reverse course and begin the convergence inward toward those positions. Party platforms are only ever stable if the initial anchor is precisely at the threshold $\overline{r}$.

Proposition 3 characterizes policy dynamics in an unchanging world for all possible starting platforms.

Proposition 3 *In the election at* $t = 1, 2, ...,$ *if:*

1. $r_0 > \overline{r}$ *then* $r^*_t \in (\overline{r}, r_{t-1})$ *for all* t*, and* $r^*_t \to \overline{r}$ *as* $t \to \infty$.
2. $r_0 < \overline{r}$ *then* $r^*_1 > \overline{r}$*, and dynamics thereafter follow case (1).*
3. *If* $r_0 = \emptyset$ *then* $r^*_1 = r^*_{cw}$ *and dynamics thereafter follow case (1).*
4. $r_0 = \overline{r}$ *then* $r^*_t = \overline{r}$ *for all* t.

Figure 5 illustrates a sample path of the platform dynamics. Starting from a moderate anchor, the red arrows and dashed lines trace the equilibrium path of R's platform. Initially, R makes a large jump outward, as the S-shaped anchoring function implies a decreasing marginal cost of moving away from the voter. Once the anchor resets, this low marginal cost disappears, and small inward movements yield large anchoring benefits. R creeps inward, benefiting each election from a platform that is appealing relative to the anchor. R's converges toward $\overline{r}$ but its steps get smaller at a faster rate and its platform never quite reaches $\overline{r}$.

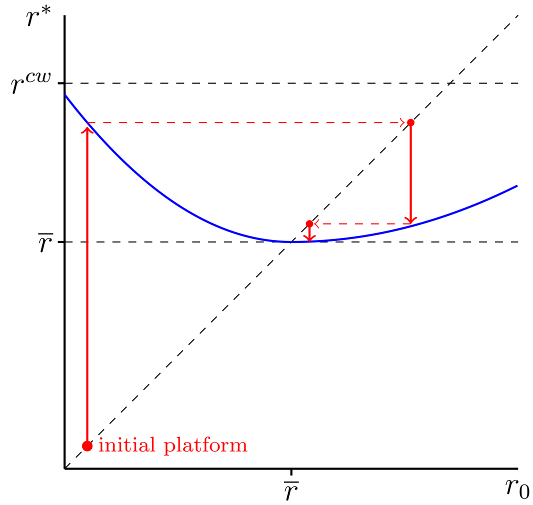


Figure 5: Equilibrium Platform Dynamics

Reversals in the direction of party platforms reflect a time-inconsistency in party incentives due to voter anchoring. From a moderate position, the usual Calvert-Wittman logic gives parties an incentive to diverge in pursuit of a better policy. Because the marginal cost of moving outward is decreasing, the parties jump out overshooting $\overline{r}$. After the anchor resets, parties moves inward in small steps to avoid the diminishing marginal benefits of a larger shift. Only at $\overline{r}/-\overline{r}$ does parties' incentive to diverge exactly balance the benefit of converging.

4.2.2 Party Platforms in a *Changing* World

In this section we allow for the environment itself to change over time, where the state of the world is the support of the valence shock, either ψ_l or ψ_h. The size of ψ_t affects the incentives of parties to converge or diverge. For the smaller value, ψ_l, an incremental movement toward the center increases the probability of winning by more than for the high value, ψ_h. This incentive, in turn, leads to a lower value of the threshold $\overline{r}$. We denote the level associated with each value of $\psi_t \in \{\psi_l, \psi_h\}$ by $\overline{r}_l$ and $\overline{r}_h$, where $\overline{r}_l < \overline{r}_h$.

In the benchmark spatial model without policy anchors, equilibrium behavior is once again straightforward. For each state, the equilibrium platforms are constant. The platforms shift only in a period when the state changes, and by a fixed and constant amount each time. When the state switches from ψ_l to ψ_h the platforms shift from moderate to extreme, and vice versa when the state switches back. This is depicted in Figure 6b.

With policy anchors, the platform dynamics are much richer. The platforms never stabilize. They endlessly evolve following an asymmetric pattern, with incremental shifts and moderate moves towards the median voter, interspersed with larger occasional jumps away from her. This pattern holds even if the state changes infrequently and there is little difference in the states, i.e., $\rho > 0$ and $\psi_h - \psi_l > 0$ can be arbitrarily small. In an environment that is mostly stable, voter anchoring causes platforms to change in a constant flow, cycling between moderate and extreme positions.

These continuous, asymmetric, and cyclical platform dynamics are a key contribution and distinctive prediction of our theory. As we discuss below, they are robust to altering several of our assumptions and the shape of the anchoring function, and are rooted in the effect of policy anchoring on voter behavior and, in turn, parties' incentives.

Proposition 4 describes the equilibrium behavior, distinguishing between platform change when the state changes that period and when it remains the same.

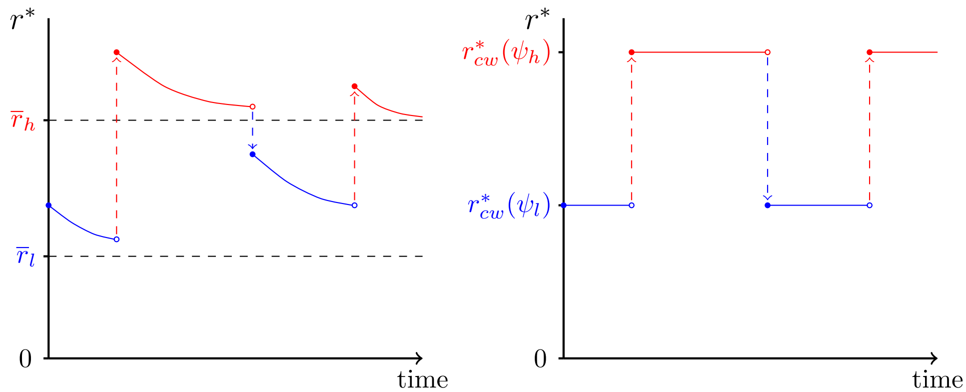
Proposition 4 *Suppose $r_0 > \overline{r}_h$. In the election at $t = 1, 2, ...$, the following holds:*

- *If the state does not change at time t, $\psi_t = \psi_{t-1}$, then $r_t^* < r_{t-1}^*$, and $|r_{t-1}^* - r_t^*|$ is increasing in r_{t-1}^*.*
- *If the state changes at time t from $\psi_{t-1} = \psi_h$ to $\psi_t = \psi_l$, then $r_t^* \in (\overline{r}_l, r_{t-1}^*)$ and $|r_{t-1}^* - r_t^*|$ is increasing in r_{t-1}^*.*
- *If the state changes at time t from $\psi_{t-1} = \psi_l$ to $\psi_t = \psi_h$ and $r_{t-1}^* < \overline{r}_h$, then $r_t^* > \overline{r}_h > r_{t-1}^*$, and $|r_{t-1}^* - r_t^*|$ is decreasing in r_{t-1}^*. If $r_{t-1}^* > \overline{r}_h$, then $r_{t-1}^* > r_t^* > \overline{r}_h$ and $|r_{t-1}^* - r_t^*|$ is increasing in r_{t-1}^*.*

Figure 6a depicts the path of platforms over time. If the state persists across elections, platforms creep inward toward the relevant threshold, $\overline{r}_l$ or $\overline{r}_h$, with convergence slowing over time. Thus, the size of the movement increases with the anchor. When the state shifts from ψ_h to ψ_l, platforms continue moving inward but make a larger jump. Likewise, when the state shifts from ψ_l to ψ_h but the anchor exceeds $\overline{r}_h$—which occurs only if the state has not remained at ψ_l for long—platforms continue moving toward the center, albeit more slowly.

Balancing this inward movement is the response when the state shifts from ψ_l to ψ_h and the anchor is below $\overline{r}_h$, which must eventually occur if the platforms have only been moving inward. When $\psi = \psi_h$ the equilibrium platform always lies above $\overline{r}_h$. Platforms therefore reverse course when the state changes to ψ_h, making a large jump away from the center. With the anchor now more extreme than $\overline{r}_h$, the process renews itself and convergence begins anew. This process thus generates stochastic, but predictable, cycles in party platforms.[12] In

[12] Figure 8 in Appendix B uses a numerical example to illustrate a possible path.



(a) With Anchoring (b) Without Anchoring

Figure 6: Platform Dynamics in a Changing World

contrast to the model without anchors (see Figure 6b), the size of the platform jumps and the amplitude of the path are not constant, instead varying with the position of the anchor.[13]

Notice that an implication of this result is that the magnitude of platform movement depends on the number of elections between a change in state, and on whether platforms continue moving in the same direction or not. Consider periods in which the inherited state is $\psi_{t-1} = \psi_l$. The longer the environment remains in ψ_l, the farther R moderates toward 0 and away from $\overline{r}_h$. Thus, when the state eventually switches to ψ_h, the anchor is far from $\overline{r}_h$ and the platform makes a very large jump outward. In this way, the asymmetric changes around $\overline{r}$ in an unchanging world manifest as asymmetric, history-dependent jumps when the environment switches from one state to the other in a changing world.

Corollary 3 *Assume $\psi_t = \psi_l$ for all periods $t \in \{1, ..., T-1\}$ and $r^*_{T-1} < \overline{r}_h$.*

- *If $\psi_T = \psi_l$ we have $r^*_T < r^*_{T-1}$. Then $|r^*_{T-1} - r^*_T|$ is decreasing in T.*
- *If $\psi_T = \psi_h$ we have $r^*_T > r^*_{T-1}$. Then $|r_{T-1} - r^*_T|$ is increasing in T.*

This result highlights that stability is elusive in our setting. If the state remains at ψ_l over a long period, then platforms will appear to be stabilizing by making smaller and smaller movements towards the center. However, it is precisely under these conditions that the platforms change most dramatically when the state eventually shifts and they reverse course.

Our last result draws empirical implications about the nature of changes in parties' policy offerings. As the parties compete over a long period of time, their equilibrium behavior will generate a stationary distribution of platforms. Proposition 5 provides predictions about changes in extremism based on this distribution.

[13] Drawing ψ_t from a continuous distribution would not alter these qualitative dynamics.

Proposition 5

- *The probability that we observe platforms become more moderate is greater than the probability they become more extreme.*
- *For δ sufficiently large, the probability that we observe platforms become more extreme by at least δ is greater than the probability we observe platforms become more moderate by at least δ.*

The model predicts that we should see platforms become more moderate more often than they become more extreme. However, large platform changes are more likely to involve movement toward the extremes. Both of these predictions differ from the Calvert-Wittman model, in which half of the platforms are at $r^*_{cw}(\psi_h)$ and half at $r^*_{cw}(\psi_l)$, and increases and decreases in extremism are therefore equally likely and equal in magnitude.

5 Extensions

5.1 Asymmetric Anchors

Our analysis thus far has fixed the parties' initial policy anchors symmetrically around the median voter's bliss point. This assumption transparently highlights how anchors shape incentives, and facilitates comparison with related work, which often focuses on symmetric equilibria. In our setting, however, a natural question is how platform dynamics change when parties begin from asymmetric anchors. Proposition 6 characterizes equilibrium behavior.

Proposition 6 *For any two anchors (r_{t-1}, d_{t-1}) there exists a unique equilibrium (r^*_t, d^*_t). Equilibrium platforms are characterized by a threshold function $\alpha : [0,1] \to [0,1]$ such that $r_t > r_{t-1}$ if $r_{t-1} < \alpha(|d_{t-1}|)$ and $r_t < r_{t-1}$ if $r_{t-1} > \alpha(|d_{t-1}|)$. An analogous characterization holds for Party D. Furthermore:*

1. *$(r^*_t, d^*_t) = (r_{t-1}, d_{t-1})$ if and only if $(r_{t-1}, d_{t-1}) = (\overline{r}, -\overline{r})$.*
2. *$r^*_t - d^*_t \geq 2\overline{r}$, with equality if and only if $(r_{t-1}, d_{t-1}) = (\overline{r}, -\overline{r})$.*
3. *$r_{t-1} - r^*_t > d^*_t - d_{t-1}$ if and only if $r_{t-1} > |d_{t-1}|$.*

Parties face similar incentives as in the baseline model: to moderate when their anchors are extreme, and move more extreme when their anchors are moderate. Unless anchors are at the stable point $(\overline{r}, -\overline{r})$, platforms move away from them. Moreover, there is an asymmetry in changes in aggregate platform extremism, $r^*_t - d^*_t$. If platforms move closer together between periods $t-1$ and t, then overall extremism decreases by *at most* $|2\overline{r}-(r_{t-1}-d_{t-1})|$. Instead, if aggregate extremism at $t-1$ is sufficiently low then platforms move farther apart in the next period, and extremism increases by *at least* this amount. Consequently, platform dynamics in an asymmetric equilibrium share similar features to those in a symmetric one: platforms evolve continuously, with large increases in extremism and smaller, more frequent decreases.

However, a crucial difference with asymmetric anchors is that platforms can move asymmetrically. In the symmetric equilibrium, either *both* parties move to more moderate positions or *both* parties become more extreme. In contrast, with asymmetric anchors there can be periods in which a party starting from an extreme anchor moves toward the median voter, while its opponent, with a very moderate anchor, shifts away.[14] Figure 7 characterizes how the anchor locations determine whether both parties move in the same direction relative to the median. Additionally, even when both parties become more extreme or more moderate, the magnitude of movement differs across them. Proposition 1 shows that the party with the initially more extreme anchor moderates more (or shifts outwards less) relative to its anchor.

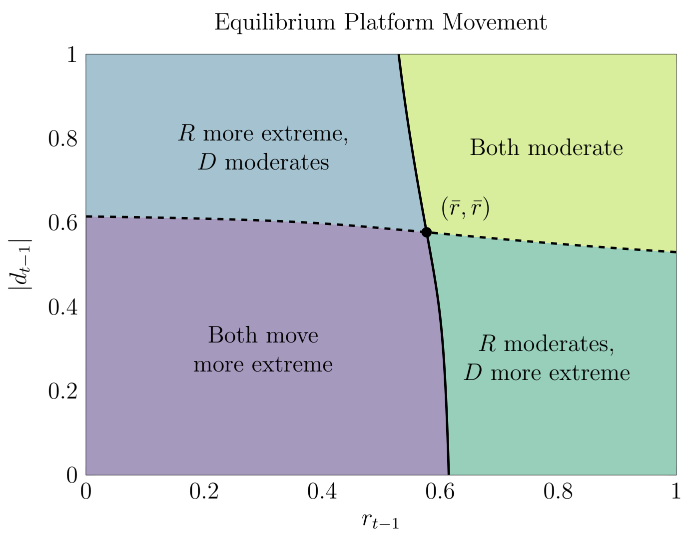


Figure 7: Asymmetric Platforms, Numerical Example

5.2 Alternative Anchoring Function

Our model assumes that the marginal effect of anchoring diminishes as platforms move further from the anchor. As discussed, this assumption is consistent with the literature on reference points. However, a natural question is whether and how altering this assumption affects our

[14]This dynamic could lead to a within-election preference reversal for the median voter on the equilibrium path, an outcome that cannot arise when platforms are always symmetric.

results. Suppose, in particular, the marginal effect of anchoring is *increasing* as platforms move away from the anchor, so that f has a *reflected s-shape*, convex for gains and concave for losses. Formally, $f''(\Delta p_t) < 0$ for $\Delta p_t < 0$ and $f''(\Delta p_t) > 0$ for $\Delta p_t > 0$. Proposition 7 describes equilibrium platforms under this alternative specification, paralleling Proposition 2.

Proposition 7 *Suppose the anchoring function is reflected s-shape. If:*

1. $r_0 < \overline{r}$ *then* $r_1^* \in (r_0, \overline{r})$, *and is increasing in* r_0 *at rate less than one.*

2. $r_0 > \overline{r}$ *then* $r_1^* < \overline{r}$, *and decreasing in* r_0.

3. $r_0 = \overline{r}$ *then* $r_1^* = \overline{r}$.

The reflected s-shaped function reverses the parties' incentives. Because the marginal cost of moving away from the median increases with the size of the shift, parties starting from a centrist anchor move outward less, undershooting $\overline{r}$. Conversely, from an extreme anchor, the marginal benefit of moving inward increases with the size of the shift, so platforms jump past $\overline{r}$. These dynamics thus invert those emerging under an S-shaped function: parties make a large jump toward the median, followed by incremental outward shifts.

However, these findings also highlight that the broader implications of voter anchoring for platform evolution remain robust. As in our baseline model, platforms move continuously despite stable core voter policy preferences, and trace asymmetric patterns, with small movements in one direction punctuated by occasional larger movements in the other.

Less regular functions, with multiple changes in concavity, would generate similar long-lasting, although more irregular, dynamics. It is only in the case of a purely linear anchoring function that platforms would monotonically converge towards the stable location $\overline{r}$.

That said, it is important to emphasize that, as discussed above, our assumption of a decreasing marginal effect of anchoring is grounded in empirical findings from the literature on reference points. We therefore regard the predictions derived under this assumption and characterized in our formal results above as especially relevant for guiding empirical work.

6 Concluding Discussion and Directions for Future Work

The Dynamics of Party Platforms in Practice. In contrast with implications from classic models of electoral competition, empirical scholars often point to rich platform dynamics, with parties moving in seemingly chaotic ways, shifting back and forth between more moderate and more extreme positions (see, e.g., Budge et al., 2010). Understanding these dynamics — why and when parties make bold jumps versus incremental change, why they sometimes reverse course and other times continue moving in the same direction — is important to understanding the nature of political competition and policy outcomes.

Our model shows that a relatively small departure from the classic spatial model, that accounts for voters' difficulty in evaluating policies, yields rich implications for understanding platform dynamics. The observed frequent movement of platforms and the whipsaw pattern it features can be accounted for by voters' use of policy anchors. Additionally, the use of anchors predicts that these patterns should be asymmetric, with party platforms making frequent small shifts in one direction punctuated by occasional dramatic changes in the other, consistent with the dynamics that are also observed in public policy (Baumgartner and Jones, 1993). Moreover, our model delivers unique predictions about the exact features of these asymmetric dynamics: moves toward the center should be frequent and small, while shifts to the extremes are rarer and larger. Empirically investigating this asymmetry is a clear direction for future work.

Furthermore, our theory can help us understand the apparent mismatch between platforms' volatility and the relative stability of voters' policy opinions (Page and Shapiro, 1992; Druckman and Leeper, 2012). In our model, while voters' constructed preferences over the policy space evolve endogenously, the ideological location of their ideal points remains fixed. Our results thus resonate with observed patterns insofar as surveys of public opinion measure the latter, and not the former. For instance, survey questions that ask respondents to place their political views on a left-right scale aim to capture the location of their ideal points. Prompts asking voters to express the extent to which they approve or disapprove of different parties, instead, capture their constructed preferences. By making this distinction explicit, our theory underscores the importance of considering how public opinion is measured.

Forward-looking Parties A benefit of modeling myopic parties is that it separates the static incentives of parties from their dynamic incentives. When a party moves away from the median voter in equilibrium, it is not that it is suffering a cost today to benefit from a more attractive anchor in the future. Rather, a party shifts outward because there is an immediate policy benefit today that is worth the lower probability of winning that the shift causes. The shifts towards and away from the median voter, as well as their asymmetry, derive from voter anchoring and not forward-looking strategic behavior from the parties.

The degree that parties in practice are forward-looking is an open empirical question. Nonetheless, how anchors affect the behavior of forward-looking parties is of natural interest. A reasonable conjecture is that a qualitatively similar dynamic would emerge. Intuition suggests that the nature of platform shifts inward and outward would be exaggerated by forward-looking behavior, with the parties seeking to dynamically exploit the asymmetry in how voters respond to anchors. The parties would benefit from smoothing across time the large marginal benefits of creeping inward (Figure 5), thus slowing down convergence to the stable position, and may overdo the jumps outward to leave even more space to accrue these marginal benefits.[15] Investigating these possibilities is a natural direction for future research.

Policy Anchors in Multiparty Systems. Extending our model to multiparty elections is a natural direction for future work. Multiparty elections are particularly interesting as they offer voters additional reference points to evaluate platforms. A right-wing party's platform can be compared to the platforms of other right-wing parties as well as to the party's own past platform. This represents a merging of context-dependent voting (Callander and Wilson, 2006) with the anchoring theory of voting we develop here.[16] The combination will generate frequent movement in party platforms, including entry and exit, as parties jostle for space and seek positive contrast with the other parties and their own histories. Such patterns have been documented empirically by Budge et al. (2010) and Van de Wardt et al. (2017) across dozens of multiparty democracies. Similar dynamics may arise in two-candidate elections when candidates first compete in party primaries.

[15]See Izzo (2023) for a model where a similar dynamic incentive emerges.

[16]Simonovits (2017) demonstrates experimentally how the choice set affects preference.

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A Proofs

We present the proofs in the order that the results appear in the paper. Thus, we first prove Lemma 1, taking as given the equilibrium characterization for the median-decisive game. The proofs for the median-decisive game then follow. Recall that the anchors are symmetric, except in the case of the asymmetric extension.

Proof of Lemma 1. We prove the result via the sequence of Lemmas A1 to A4. By Proposition 1, the median-decisive model has a unique equilibrium, denote it r_m^*, it is symmetric, and characterized by:

$$r_m^* = \frac{\psi}{2[1 + f'(r_{t-1} - r_m^*)]}.$$

Let $P^T(r, d)$ denote R's probability of winning the election in the true game for a pair of platforms (r, d) and let $P^M(r, d)$ denote R's probability of winning in the median-decisive game. To prove the result we first establish the following bounds on the probability of winning.

Lemma A1

1. *If* $-d \geq f(1)$ *then* $P^T(r, d) \leq P^M(r, d)$ *for all* $r \in [0, 1]$.
2. *If* $r \geq f(1)$ *then* $1 - P^T(r, d) \leq 1 - P^M(r, d)$ *for all* $d \in [-1, 0]$.
3. *If* $-d \geq f(1)$ *and* $r \geq f(1)$ *then* $P^T(r, d) = P^M(r, d)$.

Proof. Consider part 1. To prove the result, we show that if the median prefers platform d over r then all voters with ideal policies $x < 0$ also prefer d. Consequently, if the median voter does not prefer party R over party D then R loses the election, and hence $P^T(r, d) \leq P^M(r, d)$.

Because the valence shock is common across voters, proving the result requires showing that, for all x:

$$u_x(r) - u_x(d) < u_0(r) - u_0(d). \tag{12}$$

We break the analysis into cases, based on the location of the voter's ideal point x.

1. Suppose $x \in (\max\{d_{t-1}, d\}, 0)$. In this case, x uses the same anchor as the median. As such, (12) holds if and only if:

$$\begin{aligned}
&\Leftrightarrow -|r - x| + |d - x| + f(r_{t-1} - r) - f(d - d_{t-1}) < -|r| + |d| + f(r_{t-1} - r) - f(d - d_{t-1}) \\
&\Leftrightarrow 2x < 0,
\end{aligned}$$

where the final inequality holds by $x < 0$.

2. Suppose $x < d$. Then x does not use anchors and for (12) to hold requires:

$$\begin{aligned} & -|r-x| + |d-x| \leq -|r| + |d| + f(\Delta r) - f(\Delta d) \\ & \Leftrightarrow 2d - f(r_{t-1} - r) + f(d - d_{t-1}) \leq 0. \end{aligned} \tag{13}$$

   To see that (15) holds, note that $2d - f(r_{t-1} - r) + f(d - d_{t-1}) \leq 2d - f(-1) + f(1) = 2d + 2f(1)$, which is less than or equal to 0 by the assumption that $-d \geq f(1)$.

3. Suppose $x \in (d, d_{t-1})$. Let a be the most recent platform of D such that $a < x$, and if no such platform exists let $a = -1$. Then (12) holds if and only if:

$$\begin{aligned} & -|r-x| + |d-x| + f(\Delta r) - f(\Delta a) \leq -|r| + |d| + f(\Delta r) - f(\Delta d) \\ & \Leftrightarrow -(r-x) + (x-d) - f(\Delta a) \leq -r - d - f(\Delta d) \\ & \Leftrightarrow 2x \leq f(d-a) - f(d - d_{t-1}). \end{aligned} \tag{14}$$

   To see that (14) is satisfied note that $a < d_{t-1}$ implies $f(\Delta a) > f(\Delta d)$ and $x < 0$. Thus, $2x < 0 < f(d-a) - f(d-d_{t-1})$, as required.

Thus, if $-d \geq f(1)$ then $P^T(r,d) \leq P^M(r,d)$.

The analogous argument yields part 2, the symmetric result for party D. Finally, part 3 follows immediately by parts 1 and 2.■

Lemma A2 *Fix a symmetric profile $(p, -p)$. If $r \geq p$ then $P^T(r, -p) \geq P^M(r, -p)$.*

Fix a symmetric candidate profile $(p, -p)$ with symmetric anchors $(r_{t-1}, -r_{t-1})$ and consider a deviation by party R to some $r > p$. For the median voter, the non-valence utility difference between voting for R and voting for D is

$$[-r + f(r_{t-1} - r)] - [-p + f(-p - (-r_{t-1}))] = p - r + f(r_{t-1} - r) - f(r_{t-1} - p). \tag{15}$$

We now show that every voter with ideal point $x \geq 0$ is weakly more favorable to party R than the median voter. That is, for all $x \geq 0$:

$$p - r + f(r_{t-1} - r) - f(r_{t-1} - p) \leq u_x(r) - u_x(-p) \tag{16}$$

We break the argument into cases based on the location of the voter's ideal point x.

1. Suppose $0 \leq x < \min\{r, r_{t-1}\}$, thus voter x uses the same anchor as the median. Then the non-valence utility difference between R and D is

$$[-(r-x) + f(r_{t-1} - r)] - [-(p+x) + f(-p + r_{t-1})]$$
$$= p - r + 2x + f(r_{t-1} - r) - f(r_{t-1} - p),$$

   Since $x > 0$ this is strictly larger than (15), the median voter's utility difference, and hence (16) holds.

2. Now suppose that $r_{t-1} < x < r$. Then both platforms from the previous period are to the left of the voter. Thus, r is evaluated according to platform the last offered platform that is to the right of x, or according to the edge of the policy space, 1. Denote this platform $a > x$. Instead, the left-side platform $-p$ is evaluated according to d_{t-1}. The non-valence utility difference between R and D is

$$[-(r-x) + f(a-r)] - [-(p+x) + f(-p + r_{t-1})].$$

   Subtracting the median voter's utility difference in (15) gives

$$2x + [f(a-r) - f(r_{t-1} - r)].$$

   This expression is weakly positive because $x \geq 0$, $a - r \geq r_{t-1} - r$, and f is increasing. Thus, (16) holds.

3. Finally, suppose that $x \geq r$. Both current platforms are to the left of the voter, so the voter does not use anchors to evaluate the policies. The non-valence utility difference between R and D is therefore

$$-(x-r) - [-(x+p)] = r + p.$$

   Subtracting the median voter's utility difference in (15) gives

$$r + p - [p - r + f(r_{t-1} - r) - f(d - d_{t-1})] = 2r - [f(r_{t-1} - r) - f(-p + r_{t-1})]$$

   Since $r > p$, we have $r_{t-1} - r < -p + r_{t-1}$, and hence

$$f(r_{t-1} - r) - f(-p + r_{t-1}) < 0.$$

Thus

$$2r - [f(r_{t-1} - r) - f(-p + r_{t-1})] > 0.$$

Therefore, for every voter with $x \geq 0$, the non-valence utility difference between R and D is weakly greater than it is for the median voter. Hence, for any realization of the common valence shock, if the median voter supports R, then every voter to the right of the median also supports R. Consequently, $P_R^T(r,p) \geq P_R^M(r,p)$.

Lemma A3 *The equilibrium of the median decisive game is an equilibrium of the true game.*

Proof. Fix the platforms $(r_m^*, -r_m^*)$. We show that R does not have a profitable deviation. Consider a deviation by R to some other platform r. Since $r_m^* \geq \overline{r} = \frac{\psi}{2[1+f'(0)]}$, Assumption 1 implies that $-d_m^* = r_m^* \geq f(1)$. Thus, by Lemma A1, $P^T(r, -r_m^*) \leq P^M(r, -r_m^*)$ and for any deviation r:

$$U_R^T(r, -r_m^*) \leq U_R^M(r, -r_m^*), \tag{17}$$

where U_R^T and U_R^M denote Party R's expected utilities in the true game and median-decisive game, respectively. At the symmetric profile $P^T(r_m^*, -r_m^*) = P^M(r_m^*, -r_m^*) = \frac{1}{2}$, and hence

$$U_R^T(r_m^*, -r_m^*) = U_R^M(r_m^*, -r_m^*). \tag{18}$$

Because $(r_m^*, -r_m^*)$ is an equilibrium of the median-decisive game we know that $U_R^M(r, -r_m^*) \leq U_R^M(r_m^*, -r_m^*)$ for all r. Combined with inequality (17) and equality (18), this yields:

$$U_R^T(r, -r_m^*) \leq U_R^M(r, -r_m^*) \leq U_R^M(r_m^*, -r_m^*) = U_R^T(r_m^*, -r_m^*).$$

Thus, r_m^* is a best response to $-r_m^*$ in the true model as well.

Lemma A4 *There does not exist a symmetric equilibrium with* $(r^*, -r^*) \neq (r_m^*, -r_m^*)$.

Proof. The median-decisive equilibrium r_m^* satisfies $h(r) = 0$, where $h : [0,1] \to [-1,1]$ is defined as

$$h(r) = \frac{\psi}{2\Big(1 + f'(r_{t-1} - r)\Big)} - r.$$

Note that h is strictly decreasing in r (see the proof of Proposition 1). Thus, if $p < r_m^*$ then $h(p) > 0$ and if $p > r_m^*$ then $h(r) < 0$. We break the argument into two cases.

1. Consider a candidate symmetric profile $(p, -p)$ with $p < r_m^*$. Since $h(p) > 0$ there exists a deviation $p + \epsilon$ for ϵ sufficiently small such that

$$U_R^M(p + \epsilon, -p) > U_R^M(p, -p).$$

   By Lemma A2 we have $P_R^T(p + \epsilon, -p) \geq P_R^M(p + \epsilon, -p)$. Hence,

$$U_R^T(p + \epsilon, -p) \geq U_R^M(p + \epsilon, -p). \tag{19}$$

   At the symmetric profile $(p, -p)$ we have $P_R^T(p, -p) = P_R^M(p, -p) = 1/2$, which implies $U_R^T(p, -p) = U_R^M(p, -p)$. Thus, combined with inequality (A2):

$$U_R^T(p, -p) = U_R^M(p, -p) < U_R^M(p + \epsilon, -p) \leq U_R^T(p + \epsilon, -p),$$

   contradicting that $(p, -p)$ is an equilibrium.

2. Next, consider a profile $(p, -p)$ with $p > r_m^*$. Because $p > r_m^* \geq \overline{r} > f(1)$, there exists a deviation $p - \epsilon$ with ϵ sufficiently small such that $p - \epsilon > f(1)$. Since $h(p) < 0$, this deviation is strictly profitable in the median-decisive game:

$$U_R^M(p - \epsilon, -p) > U_R^M(p, -p). \tag{20}$$

   By Lemma A1 $-d = p > f(1)$ and $p - \epsilon > f(1)$ implies $P_R^T(p - \epsilon, -p) = P_R^M(p - \epsilon, -p)$. Consequently, $U_R^T(p - \epsilon, -p) = U_R^M(p - \epsilon, -p)$, and combined with inequality (20 this yields:

$$U_R^T(p, -p) = U_R^M(p, -p) < U_R^M(p - \epsilon, -p) = U_R^T(p - \epsilon, -p). \tag{21}$$

   Thus, R has a profitable deviation from p. ■

Lemma A5 *R's probability of winning in each election is interior, $P(r_t, d_t|\cdot) \in (0, 1)$.*

Proof. First, we show $P(r_t, d_t|\cdot) < 1$. This requires

$$\begin{aligned} &\frac{1}{2} - \frac{r_t + d_t + f(\Delta d_t) - f(\Delta r_t)}{2\psi} < 1 \\ &\Leftrightarrow 0 < r + d + f(\Delta d_t) - f(\Delta r_t) + \psi \end{aligned}$$

Note that $r_t \geq 0$ and $d_t \geq -1$. Therefore, a sufficient condition for the above to hold is that:

$$\begin{aligned} &0 < 0 - 1 + f(\Delta d_t) - f(\Delta r_t) + \psi \\ &\Leftrightarrow 1 < f(\Delta d_t) - f(\Delta r_t) + \psi. \end{aligned} \tag{22}$$

Finally, notice that $f(\Delta d_t) \geq f(-1-0)$ and $f(\Delta r_t) \leq f(1-0)$. Thus, inequality (22) holds if:

$$\begin{aligned} &1 < f(-1-0) - f(1-0) + \psi \\ &\Leftrightarrow 1 + 2f(1) < \psi, \end{aligned}$$

where the second line follows from the assumption that $f(a) = -f(-a)$ and the last inequality holds by Assumption 1.

Second, we show that $P(r_t, d_t|\cdot) > 0$. This requires

$$\begin{aligned} &\frac{1}{2} - \frac{r_t + d_t + f(\Delta d_t) - f(\Delta r_t)}{2\psi} > 0 \\ &\Leftrightarrow \psi > r_t + d_t + f(\Delta d_t) - f(\Delta r_t), \end{aligned}$$

which again holds by assumption that $1 + 2f(1) < \psi$. ■

Lemma A6 *For any anchors (r_{t-1}, d_{t-1}), if (r_t^*, d_t^*) is an interior equilibrium then it solves:*

$$\Big(\frac{-1 - f'(r_{t-1} - r_t)}{2\psi}\Big)(r_t - d_t) + P(r_t, d_t|r_{t-1}, d_{t-1}) = 0 \tag{23}$$

$$\Big(\frac{1 + f'(d_t - d_{t-1})}{2\psi}\Big)(r_t - d_t) - 1 + P(r_t, d_t|r_{t-1}, d_{t-1}) = 0 \tag{24}$$

Proof. Follows by differentiating R's expected utility in (7) with respect to r_1, and likewise for the analogous problem for D. ■

Lemma A7 *Party i's expected utility is strictly concave in its own policy choice.*

Proof. We prove the result for party R, the case of party D follows symmetrically. Fixing party D's policy at d_t and differentiating yields:

$$\Big(\frac{-1 - f'(r_{t-1} - r_t)}{2\psi}\Big)(r_t - d_t) + 1 - \frac{r_t + d_t + f(d_t - d_{t-1}) - f(r_{t-1} - r_t) + \psi}{2\psi}. \tag{25}$$

Differentiating again and rearranging, the second derivative of R's expected utility is negative if:

$$f''(\Delta r_t)(r_t - d_t) < 2(1 + f'(\Delta r_t)). \tag{26}$$

In each period, $r_t, d_t \in [-1, 1]$, therefore, $r_t - d_t \leq 2$. Thus, a sufficient condition for inequality (26) to hold is that $2f''(\Delta r_t) < 2\big(1 + f'(\Delta r_t)\big)$, which holds by Assumption 1. ■

Proof of Proposition 1. We first show that any symmetric equilibrium must be interior, $r_t^* \in (0, 1)$. By the standard argument in the Calvert-Wittman setting we cannot have an equilibrium with $r_t^* = 0$. Next, suppose that $r_t^* = 1$ is an equilibrium. Fix D's policy at $d_t = -1$ and $d_{t-1} = -r_{t-1}$, and consider R's best response. A necessary condition for $r_t = 1$ to be a best response is that R's utility is increasing as $r_t \to 1$. That is,

$$\lim_{r_t \to 1} \Big(\frac{-1 - f'(\Delta r_t)}{2\psi}\Big)(r_t + 1) + 1 - \frac{r_t - 1 + f(-1 + r_{t-1}) - f(r_{t-1} - r_t) + \psi}{2\psi} > 0 \tag{27}$$

$$\Leftrightarrow \Big(\frac{-1 - f'(r_{t-1} - 1)}{2\psi}\Big)2 + 1 - \frac{\psi}{2\psi} > 0 \tag{28}$$

$$\Leftrightarrow \frac{-1 - f'(r_{t-1} - 1)}{\psi} + \frac{1}{2} > 0 \tag{29}$$

$$\Leftrightarrow \frac{\psi}{2(1 + f'(r_{t-1} - 1))} > 1. \tag{30}$$

However, $\frac{\psi}{2(1+f'(r_{t-1}-1))} < \frac{\psi}{2}$ and, by assumption, $\frac{\psi}{2} < 1$, a contradiction. Thus, there cannot be an equilibrium $r^* = 1$.

Now consider an interior equilibrium, $r_t^* \in (0, 1)$. Substituting $r_t = -d_t$ into R's first-order condition (23), we obtain the following expression:

$$2r_t\Big(-1 - f'(r_{t-1} - r_t)\Big) + \psi = 0. \tag{31}$$

Thus, if r_t^* is a symmetric equilibrium, then it must solve

$$r_t = \frac{\psi}{2\Big(1 + f'(\Delta r_t)\Big)}. \tag{32}$$

Finally, we prove that symmetric equilibrium exists and is unique. Recall from the proof

Lemma A4 that the function h is defined as:

$$h(r) = \frac{\psi}{2\Big(1 + f'(r_{t-1} - r)\Big)} - r.$$

By equation (32), if r_t^* is a symmetric equilibrium then $h(r_t^*) = 0$. We first show that there exists a solution to the equation $h(r) = 0$.

At $r = 0$ we have:

$$h(0) = \frac{\psi}{2\Big(1 + f'(r_{t-1})\Big)} > 0.$$

Instead, at $r = 1$:

$$h(1) = \frac{\psi}{2(1 + f'(r_{t-1} - 1))} - 1 < \frac{\psi}{2} - 1 < 0.$$

Furthermore, continuity of $f'(\Delta r_t)$ implies continuity of h. Thus, by the intermediate value theorem, there exists an r_t^* such that $h(r_t^*) = 0$.

Next, we demonstrate that this solution is unique. Differentiating yields

$$h'(r) = \frac{\psi}{2} \frac{f''(r_{t-1} - r)}{1 + f'(r_{t-1} - r)} - 1.$$

By Assumption 1, $\frac{\psi}{2} < 1$ and $\frac{f''(r_{t-1}-r)}{1+f'(r_{t-1}-r)} < 1$. Hence, $h'(r) < 0$ and there is only one r_t^* such that $h(r_t^*) = 0$.

Finally, we show that r_t^* is indeed R's best response to D choosing $-r_t^*$. Consider R's first-order condition with D's policy fixed at $-r_t^*$:

$$\Big(\frac{-1 + f'(r_{t-1} - r_t)}{2\psi}\Big)(r_t + r_t^*) + 1 - \frac{r_t - r_t^* + f(-r_t^* - d_{t-1}) - f(r_{t-1} - r_t) + \psi}{2\psi} = 0. \quad (33)$$

By construction r_t^* solves R's first-order condition. To conclude the argument note that, by Lemma A7, R's expected utility is concave in r_t, thus, r_t^* must be the unique solution to R's first-order condition. The symmetric argument proves that $d_t^* = -r_t^*$ is D's best response to R choosing r_t^*, as required. ■

Proof of Corollary 2. The result follows from Proposition 1, Corollary 1, and the observation that $f'(\cdot) > 0$ everywhere. ■

Lemma A8 *If $r_0 = \overline{r}$ then $r_1^* = \overline{r}$. If $r_0 < \overline{r}$ then $r_1^* > \overline{r}$. Otherwise, if $r_0 > \overline{r}$ then $r_1^* \in (\overline{r}, r_0)$.*

Proof. Recall that r_1^* must solve $h(r) = 0$, with h defined as in the proof of Proposition 1. First, let $r_0 < \overline{r}$. We show that $h(r) > 0$ for all $r < \overline{r}$ and, hence, it must be that $r_1^* \geq \overline{r}$. Notice that $\frac{\psi}{2(1+f'(r_0-r))}$ is minimized at $r = r_0$ because f is s-shaped. Therefore, for $r < \overline{r}$, $h(r) > \frac{\psi}{2(1+f'(0))} - r = \overline{r} - r \geq 0$, as required.

Finally, suppose $r_0 > \overline{r}$. We demonstrate that if r_1^* solves $h(r) = 0$ then $r_1^* \in (\overline{r}, r_0)$. Evaluating h at $r = r_0$ yields $h(r_0) = \frac{\psi}{2(1+f'(0))} - r_0 = \overline{r} - r_0 < 0$. Instead, at $r = \overline{r}$ we have $h(\overline{r}) = \frac{\psi}{2(1+f'(r_0-\overline{r}))} - \overline{r} > 0$, where the final inequality follows because $\overline{r} = \frac{\psi}{2(1+f'(0))}$ and $\frac{\psi}{2(1+f'(r_0-r))}$ is minimized at $r = r_0$ when f is s-shaped. Thus, $r_1^* \in (r_0, \overline{r})$.

To conclude, note that r_1^* is continuous in r_0 and, thus, if $r_0 = \overline{r}$ then $r_1^* = \overline{r}$. ■

Lemma A9

1. *If $r_0 < \overline{r}$ then r_1^* is decreasing in r_0. Otherwise, if $r_0 > \overline{r}$ then r_1^* is increasing in r_0 and at rate less than* 1.

Proof. Applying the implicit function theorem to (10):

$$\frac{\partial r_1^*}{\partial r_0} = \frac{\frac{-\psi f''(r_0-r_1^*)}{2(1+f'(r_0-r_1^*))^2}}{1 - \frac{\psi f''(r_0-r_1^*)}{2(1+f'(r_0-r_1^*))^2}} \tag{34}$$

By Assumption 1, $\frac{\psi}{2} < 1$ and $\frac{f''(r_0-r_1^*)}{1+f'(r_0-r_1^*)} < 1$. Thus, the denominator of (54) is positive. Therefore, $\frac{\partial r_1^*}{\partial r_0} > 0$ if and only if $f''(r_0 - r_1^*) < 0$.

By Lemma A8, $r_1^* > r_0$ if and only if $r_0 < \overline{r}$. Since f is s-shaped, this implies $f''(r_0 - r_1^*) > 0$ for $r_0 < \overline{r}$ and $f''(r_0 - r^*) < 0$ for $r_0 > \overline{r}$. Thus, $\frac{\partial r_1^*}{\partial r_0} < 0$ if $r_0 < \overline{r}$ and $\frac{\partial r_1^*}{\partial r_0} > 0$ if $r_0 > \overline{r}$, as claimed. Moreover, for $r_0 > \overline{r}$ $f''(r_0 - r^*) < 0$ implies that $\frac{\psi f''(\cdot)}{2\left(1+f'(\cdot)\right)} > 0$ and thus $\frac{\partial r_1^*}{\partial r_0} < 1$. ■

Proof of Proposition 2. Follow immediately from Lemmas A8 and A9. ■

Proof of Proposition 3. To show part 1, first note that, by Proposition 2, if $r_{\overline{t}-1}^* > \overline{r}$ at period $\overline{t}$, then $r_{\overline{t}}^* \in (\overline{r}, r_{\overline{t}-1}^*)$. Thus, the sequence of platforms $(r_{\overline{t}-1}^*, r_{\overline{t}}^*, r_{\overline{t}+1}^*, \ldots)$ is strictly decreasing and bounded below by $\overline{r}$.

We now show that $\overline{r}$ is the infimum of $(r_{\overline{t}-1}^*, r_{\overline{t}}^*, r_{\overline{t}+1}^*, \ldots)$. Suppose instead that the infimum is given by some $r' > \overline{r}$. Then the monotone convergence theorem implies that $\lim_{t\to\infty} r_t^* = r'$. However, by Proposition 2 if $r_t^* = r' > \overline{r}$ then $r_{t+1}^* < r'$. Therefore, for all $\epsilon > 0$ sufficiently small if $r_t^* = r' - \epsilon$ then $r_{t+1}^* < r'$, contradicting that $r' > \overline{r}$ is the infimum of the sequence of platforms.

Thus, $\overline{r}$ is the infimum of $(r_{\overline{t}-1}^*, r_{\overline{t}}^*, r_{\overline{t}+1}^*, \ldots)$ and the monotone convergence theorem yields $\lim_{t\to\infty} r_t^* = \overline{r}$.

Parts 2 and 3 then follow immediately from Proposition 2. ■

Proof of Proposition 4. The proof for when the state does not change, $\psi_t = \psi_{t-1}$, follows immediately by Proposition 3. Next, suppose that $\psi_{t-1} = \psi_h$ and $\psi_t = \psi_l$. First, by Proposition 3, $r_t^* \in (\overline{r}_l, r_{t-1}^*)$ for any r_{t-1}^*, as claimed. Second, note that if $\psi_{t-1} = \psi_h$ then $r_{t-1}^* > \overline{r}_h > \overline{r}_l$ by Proposition 2. Thus, Proposition 2 also yields $\frac{\partial r_t^*}{\partial r_{t-1}} < 1$, which implies that $r_t^* - r_{t-1}^*$ is increasing in r_{t-1}^*.

Similarly, if the state changes from $\psi_{t-1} = \psi_l$ to $\psi_t = \psi_h$, and $r_{t-1}^* > \overline{r}_h$, Proposition 2 implies that $r_{t-1}^* > r_t^* > \overline{r}_h$ and $r_{t-1}^* - r_t^*$ is increasing in r_{t-1}^*.

Finally, if the state changes from $\psi_{t-1} = \psi_l$ to $\psi_t = \psi_h$, and $r_{t-1}^* < \overline{r}_h$, Proposition 2 immediately implies that $r_t^* > \overline{r}_h > r_{t-1}^*$, and $r_t^* - r_{t-1}^*$ is decreasing in r_{t-1}^*. ■

The change in the state ψ_t together with the equilibrium platform r_t^* forms a Markov chain on $[0,1] \times \{\psi_l, \psi_h\}$. Abusing notation, for $\psi_j \in \{\psi_l, \psi_h\}$ let $\psi_{-j} \in \{\psi_l, \psi_h\}$ be the state not equal to ψ_j. Additionally, define $r_j^*(r)$ as the equilibrium policy when the state is ψ_j and the anchor is r. Then, for given state $(\psi_t, r_t) = (\psi, r)$ the transition kernel can be described by: (1) Draw $\psi_{t+1} \in \{\psi_l, \psi_h\}$ according to $Pr(\psi_{t+1} = \psi_j | \psi_t = \psi_j, r_t = r) = \rho$ and $Pr(\psi_{t+1} = \psi_{-j} | \psi_{t+1} = \psi_j, t_t = r) = 1 - \rho$; and (2) For $\psi_{t+1} = \psi_{j'}$ set $r_{t+1} = r_{j'}^*(r)$. Thus, from any current state (r, ψ_j), the chain can only move to one of two possible next states: $(r_l^*(r), \psi_l)$ or $(r_h^*(r), \psi_h)$, with probabilities ρ and $1-\rho$ depending on the current state. Specifically, define the transition kernel by:

$$P(r, \psi_j) = (1-\rho)\delta_{(r_j^*(r), \psi_j)} + \rho\delta_{(r_{-j}^*(r), \psi_{-j})}.$$

A stationary distribution of this Markov chain is given by a probability measure π on $[0,1] \times \{\psi_l, \psi_h\}$ such that $\pi P = \pi$.

Lemma A10 *The Markov chain $(r_t, \psi_t) \in [0,1] \times \{\psi_l, \psi_h\}$ has a stationary distribution.*

Proof. Let $\mathcal{P}([0,1] \times \{\psi_l, \psi_h\})$ be the set of Borel probability measures on $[0,1] \times \{\psi_l, \psi_h\}$. Define the mapping $T : \mathcal{P}([0,1] \times \{\psi_l, \psi_h\}) \to \mathcal{P}([0,1] \times \{\psi_l, \psi_h\})$ as $T(\mu) = \mu P$, for some probability measure μ. Thus, any fixed point of $T(\pi) = \pi$ is a stationary distribution of P. We now show that T satisfies the premises of Schauder's theorem, and thus has a fixed point.

To start, note that because $[0,1] \times \{\psi_l, \psi_h\}$ is compact, $\mathcal{P}([0,1] \times \{\psi_l, \psi_h\})$ is a non-empty, compact, and convex set under the weak* topology.

All that remains to be shown is that T is continuous. First, for any continuous function

ϕ we have

$$P\phi = (1-\rho)\phi\big(r_j^*(r), \psi_j\big) + \rho\phi(r_{-j}^*(r), \psi_{-j}).$$

Since r_l^* and r_h^* are continuous and the state only takes two values, $P\phi$ is also continuous. Thus, if $\mu_n \to \mu$ (weakly) then, for any continuous function ϕ,

$$\begin{aligned}\int_{[0,1]\times\{\psi_l,\psi_h\}} \phi dT(\mu_n) &= \int_{[0,1]\times\{\psi_l,\psi_h\}} P\phi d\mu_n \\ &\to \int_{[0,1]\times\{\psi_l,\psi_h\}} P\phi d\mu \\ &= \int_{[0,1]\times\{\psi_l,\psi_h\}} \phi dT(\mu).\end{aligned}$$

Hence, $T(\mu_n) \to T(\mu)$, and T is continuous in μ as needed. ■

Lemma A11 *Define $\pi_\psi = (\pi_l, \pi_h)$ as the marginal distribution of π over $\{\psi_l, \psi_h\}$. $\pi_l = \pi_h = 1/2$.*

Proof. The draw of $\psi_t \in \{\psi_l, \psi_h\}$ is independent of r_t and given by $Pr(\psi_{t+1} = \psi_t) = \rho$. Thus, π_ψ solves:

$$\begin{aligned}\pi_l &= \rho\pi_l + (1-\rho)\pi_h \\ \pi_h &= (1-\rho)\pi_l + \rho\pi_h.\end{aligned}$$

This implies $\pi_l = \pi_h$ and together with the requirement that $\pi_l + \pi_h = 1$ this delivers $\pi_l = \pi_h = 1/2$. ■

Proof of Proposition 5. First, we show that the probability that we observe platforms becoming more extreme is strictly less than the probability they become more moderate.

A necessary condition for platforms to become more extreme at time t is that the state changes from ψ_l to ψ_h. By Lemma A11 $\Pr(\psi_t = \psi_h) = \Pr(\psi_t = \psi_l) = \frac{1}{2}$. Thus,

$$\Pr(r_t > r_{t-1}) \le Pr(\psi_{t-1} = \psi_l, \psi_t = \psi_h) = \frac{\rho}{2}.$$

Instead, every event which switches from ψ_h to ψ_l generates an a moderating movement. Therefore,

$$Pr(x_t < x_{t-1} \ge Pr(\psi_{t-1} = \psi_h, \psi_t = \psi_l) = \frac{\rho}{2}.$$

To finish proving the result, we now show that this inequality is strict. Consider the event $\{\psi_{t-2} = \psi_h, \psi_{t-1} = \psi_l, \psi_t = \psi_l\}$. The switch from ψ_h to ψ_l yields $r_{t-1} > \overline{r}_l$. Then, because the state remains at ψ_l, we have $r_t^* < r_{t-1}^*$. Thus, this event also generates a moderating movement, which occurs with probability $\frac{1}{2}\rho(1-\rho)$. Moreover, this is disjoint from the event where $\psi_{t-1} = \psi_h$ and $\psi_t = \psi_l$. Consequently,

$$\begin{aligned} Pr(x_t < x_{t-1}) &\geq \frac{\rho}{2} + \frac{\rho(1-\rho)}{2} \\ &> \frac{\rho}{2} \geq Pr(x_t > x_{t-1}. \end{aligned}$$

Next, consider the second bullet point. Recall that, fixing the state, the platform is U-shaped in the anchor. In the stationary distribution, the most extreme platform emerges when the anchor is at $\overline{r}_l$, and the state is ψ_h. Denote this platform x^e, and let $\overline{\delta}_{out} \equiv x^e - \overline{r}_l$ be the largest outward movement we can sustain in equilibrium. Furthermore, because the size of inward movement is decreasing in the anchor, this also implies that the largest jump inward occurs when the anchor is at x^e and the state is ψ_l. Denote the resulting platform $x(x^e, \psi_l) < \overline{r}_h$, and let $\overline{\delta}_{in} \equiv x^e - x(x^e, \psi_l)$. Notice that, by Proposition 2, $\overline{\delta}_{out} > \overline{\delta}_{in}$.

If $\delta > \overline{\delta}_{out}$ then $Pr(x_t - x_{t-1} > \delta_{out}) = 0 \geq 0 = Pr(x_t - x_{t-1} > \delta_{out})$. Next, suppose $\delta \in (\overline{\delta}_{in}, \overline{\delta}_{out})$. Clearly, again we have $\Pr(x_{t-1} - x_t > \overline{\delta}_{in}) = 0$. We now show that $\Pr(x_t - x_{t-1} > \overline{\delta}_{in}) > 0$. Given Proposition 4, continuity implies that there exists a unique $x^\dagger \in \Big(\overline{r}_l, x(x^e, \psi_l)\Big)$ such that, if $x_{t-1} \in \Big(\overline{r}_l, x^\dagger\Big)$ and $\psi_t = \psi_h$, then $x_t - x_{t-1} > \overline{\delta}_{in}$. Thus, $\Pr(x_t - x_{t-1} > \overline{\delta}_{in}) = \Pr\Big(x_{t-1} \in (\overline{r}_l, x^\dagger)\Big) \times \Pr(\psi_t = \psi_h)$. In state ψ_l, r^* converges to $\overline{r}_l$. Thus, there is a finite number of periods N such that $r_{t-1} \in (\overline{r}_l, x^\dagger)$ and a subsequent switch to state ψ_h generates a large outward movement. Since every finite state history has a positive probability, this implies $Pr(x_t - x_{t-1} > \delta_{out}) > 0$, as claimed.■

Proof of Proposition 6. We break the argument into several steps.

Existence & Uniqueness. Existence of equilibrium follows straightforwardly from the Debreu-Fan-Glicksberg theorem. We now show that it is unique. To do so, we show that all equliibria must be interior and then that there is a unique interior equilibrium.

We show that party R never locates at $r = 0$ in equilibrium. That is, for any d, $\frac{\partial U_R}{\partial r}|_{r=0} > 0$.

At $r = 0$ we have

$$\frac{\partial U_R}{\partial r} = \Big(\frac{-1 - f'(r_{t-1})}{2\psi}\Big)(-d_t) + 1 - \frac{d_t + f(d_t - d_{t-1}) - f(r_{t-1}) + \psi}{2\psi} > 0$$
$$f'(r_{t-1})d_t + \psi - f(d_t - d_{t-1}) + f(r_{t-1}) > 0.$$

Note that an implication of assumption 1 is that $f'(0) < \psi$. To see this, take part 3 of the assumption

Thus, for the inequality to hold it is sufficient to show that $f(r_{t-1}) - f(d_t - d_{t-1}) + f'(r_{t-1})d_t > -f'(0)$.

If $d_t - d_{t-1} \leq r_{t-1}$ then $f(r_{t-1}) - f(d_t - d_{t-1})$, since $f' > 0$. Thus, $f(r_{t-1}) - f(d_t - d_{t-1}) + f'(r_{t-1})d_t > f'(r_{t-1})d_t \geq -f'(r_{t-1}) > -f'(0)$, where the final two inequalities follow from $d_t \in [-1, 0]$ and concavity of f for $\Delta p \geq 0$.

If instead $d_t - d_{t-1} > r_{t-1} \geq 0$ then concavity of f for $\Delta p \geq 0$ yields $f(r_t) - f(d_t - d_{t-1}) \geq f'(r_{t-1})(r_{t-1} - (d_t - d_{t-1}))$. Therefore,

$$\begin{aligned} f(r_{t-1}) - f(d_t - d_{t-1}) + f'(r_{t-1})d_t \geq & f'(r_{t-1})(r_{t-1} - (d_t - d_{t-1}) + d_t) \\ = & f'(r_{t-1})(r_{t-1} + d_{t-1}) \end{aligned}$$

and $r_{t+1} + d_{t+1} \geq -1$ with f concave for $\Delta p \geq 0$ delivers $f'(r_{t-1})(r_{t-1} + d_{t-1}) \geq -f'(0)$, as needed.

Next, we show that party R never chooses $r = 1$. We show that, for any d, $\frac{\partial U_R}{\partial r}|_{r=1} < 0$. At $r = 1$ we have

$$\frac{\partial U_R}{\partial r} = \Big(\frac{-1 - f'(r_{t-1} - 1)}{2\psi}\Big)(1 - d_t) + 1 - \frac{1 + d_t + f(d_t - d_{t-1}) - f(r_{t-1} - 1) + \psi}{2\psi} < 0$$
$$(-1 - f'(r_{t-1} - 1))(1 - d_t) + \psi - (1 - d_t + f(d_t - d_{t-1}) - f(r_{t-1} - 1)) < 0$$
$$\psi - 2 + f(r_{t-1} - 1) - f(d_t - d_{t-1}) - f'(r_{t-1} - 1)(1 - d_t) < 0.$$

Note that $\psi < 2$, $r_{t-1} - 1 < 0$ and $f' > 0$. Thus, if $r_{t-1} - 1 \leq d_t - d_{t-1}$ then the LHS is strictly negative as claimed. Suppose instead that $d_t - d_{t-1} \leq r_{t-1} - 1 < 0$. Since $f(\Delta p)$ is convex for $\Delta p < 0$ then

$$f(r_{t-1} - 1) - f(d_t - d_{t-1}) \leq f'(r_{t-1} - 1)(r_{t-1} - 1 - (d_t - d_{t-1})) = f'(r_{t-1} - 1)(d_{t-1} + r_{t-1} - 2) < 0,$$

where the final inequality follows from $f' > 0$ and $d_{t-1} + r_{t-1} \leq 1$.

Finally, we prove there is a unique interior equilibrium. By Rosen (1965) if $U_R(r, d) + U_D(d, r)$ is diagonally strictly concave then there is a unique equilibrium. A sufficient condition for it to be diagonally strictly concave is that the matrix $J(r,d) + J^T(r,d)$ is negative definite (Theorem 6 Rosen, 1965), where

$$J(r,d) = \begin{bmatrix} \frac{\partial^2 U_R}{\partial r^2} & \frac{\partial^2 U_R}{\partial r \partial d} \\ \frac{\partial^2 U_D}{\partial d \partial r} & \frac{\partial^2 U_D}{\partial d^2} \end{bmatrix}.$$

Differentiating yields

$$\frac{\partial^2 U_R}{\partial r \partial d} = -\frac{-1 - f'(r_{t-1} - r_t)}{2\psi} - \frac{1 + f'(d_t - d_{t-1})}{2\psi} \tag{35}$$

$$\frac{\partial^2 U_D}{\partial d \partial r} = \frac{1 + f'(d_t - d_{t-1})}{2\psi} - \frac{1 + f'(r_{t-1} - r_t)}{2\psi}. \tag{36}$$

Thus, $\frac{\partial^2 U_R}{\partial r \partial d} + \frac{\partial^2 U_D}{\partial d \partial r} = 0$, and

$$J(r,d) + J^T(r,d) = \begin{bmatrix} \frac{\partial^2 U_R}{\partial r^2} & 0 \\ 0 & \frac{\partial^2 U_D}{\partial d^2} \end{bmatrix}.$$

By Assumption 1 U_R is strictly concave in r and U_D is strictly concave in d. Therefore, $\frac{\partial^2 U_R}{\partial r^2} < 0$ and $\left(\frac{\partial^2 U_R}{\partial r^2}\right)\left(\frac{\partial^2 U_D}{\partial d^2}\right) - 0 > 0$. Thus, $J(r,d) + J^T(r,d)$ is negative definite, as claimed.

Threshold Characterization. We prove the result for Party R. The result for Party D follows by symmetry.

Fix the opposing party's anchor $-d_{t-1} \in [0,1]$. For notational convenience, write

$$\tilde{d}_t^* \equiv -d_t^*.$$

Thus $\tilde{d}_t^*$ is the extremity of Party D's equilibrium platform.

The equilibrium (r_t^*, d_t^*) solves the system of two first-order conditions, which can be written as

$$2r_t^* + f(\Delta_D) - f(\Delta_R) + f'(\Delta_R)L - \psi = 0, \tag{37}$$

and

$$2\tilde{d}_t^* - f(\Delta_D) + f(\Delta_R) + f'(\Delta_D)L - \psi = 0. \tag{38}$$

Since the equilibrium is interior, we have that $r_t^* \in (0,1)$. Thus, to prove there is a unique threshold in r_{t-1}, we show that $r_{t-1} - r_t^*$ is strictly increasing in r_{t-1}, that is:

$$\frac{\partial r_t^*}{\partial r_{t-1}} < 1. \tag{39}$$

By the implicit function theorem:

$$\frac{\partial r_t^*}{\partial r_{t-1}} = \frac{-1}{\frac{\partial(37)}{\partial r_t}\frac{\partial(38)}{\partial \tilde{d}_t} - \frac{\partial(37)}{\partial \tilde{d}_t}\frac{\partial(38)}{\partial r_t}} \cdot \left[\frac{\partial(38)}{\partial \tilde{d}_t}\frac{\partial(37)}{\partial r_{t-1}} - \frac{\partial(37)}{\partial \tilde{d}_t}\frac{\partial(38)}{\partial r_{t-1}}\right].$$

The denominator is strictly positive. Instead the numerator is

$$(2 + f'(\Delta_R))(2(1 + f'(\Delta_D)) - f''(\Delta_D)(r + \tilde{d})) - (f'(\Delta_R) - f'(\Delta_D))f'(\Delta_D)$$

We show it is < 0, which implies $\frac{\partial r_t^*}{\partial r_{t-1}} < 1$. For it to be negative reduces to

$$4(1 + f'(\Delta_D)) + 2f'(\Delta_R) + f'(\Delta_R)f'(\Delta_D) + [f'(\Delta_D)]^2 > f''(\Delta_D)(r + \tilde{d})(2 + f'(\Delta_R)). \tag{40}$$

Since $f'(\Delta) > 0$, a sufficient condition is that

$$f''(\Delta_D)(r + \tilde{d})(2 + f'(\Delta_R)) < 4(1 + f'(\Delta_D)). \tag{41}$$

By Assumption 1 $f''(\Delta_D) < 1 + f'(\Delta_D)$. Thus,

$$f''(\Delta_D)(r + \tilde{d})(2 + f'(\Delta_R)) < (1 + f'(\Delta_D))(r + \tilde{d})(2 + f'(\Delta_R)). \tag{42}$$

Furthermore, adding the two first-order conditions (37) and (38) together and rearranging we have

$$r + \tilde{d} = \frac{2\psi}{2 + f'(\Delta_R) + f'(\Delta_D)}. \tag{43}$$

Combining this with (42) yields

$$\begin{aligned} f''(\Delta_D)(r + \tilde{d})(2 + f'(\Delta_R)) &< \frac{2\psi}{2 + f'(\Delta_R) + f'(\Delta_D)}(2 + f'(\Delta_R))(1 + f'(\Delta_D)) && (44) \\ &< 4(1 + f'(\Delta_D)), && (45) \end{aligned}$$

where the final inequality follows by $\frac{2+f'(\Delta_R)}{2+f'(\Delta_R)+f'(\Delta_D)} < 1$ and assumption that $2\psi < 4$.

Since $r_t^* - r_{t-1}$ is continuous and strictly decreasing, there is a unique $r_{t-1} \in (0, 1)$ such that $r_t^* - r_{t-1} = 0$. Define this value by

$$\alpha(-d_{t-1}).$$

Then

$$r_t^* > r_{t-1} \quad \Leftrightarrow \quad r_{t-1} < \alpha(-d_{t-1}),$$

and

$$r_t^* < r_{t-1} \quad \Leftrightarrow \quad r_{t-1} > \alpha(-d_{t-1}).$$

The symmetric argument gives the corresponding characterization for Party D.

Parts 1 - 3. We now prove the remaining points. Recall that the parties' first-order conditions are given by

$$2r + f(\Delta d) - f(\Delta r) + f'(\Delta r)(r + \tilde{d}) - \psi \tag{46}$$

$$2\tilde{d} - f(\Delta d) + f(\Delta r) + f'(\Delta d)(r + \tilde{d}) - \psi \tag{47}$$

Adding these equations together yields

$$\big[2 + f'(\Delta r) + f'(\Delta d)\big]\big[r + \tilde{d}\big] = 2\psi \tag{48}$$

Part 1. Suppose that $d = d_{t-1}$ and $r = r_{t-1}$, then $\Delta d = \Delta r = 0$ and equation (48) gives

$$r + \tilde{d} = \frac{\psi}{1 + f'(0)} = 2\overline{r}. \tag{49}$$

Furthermore, plugging $r + \tilde{d} = 2\overline{r}$ and $\Delta r = \Delta d = 0$ into the first-order conditions gives:

$$2r + f'(0)2\overline{r} - \psi = 0 \tag{50}$$

$$2\tilde{d} + f'(0)2\overline{r} - \psi = 0. \tag{51}$$

By definition of $\overline{r}$ we have that $\psi = 2\overline{r}\big(1 + f'(0)\big)$, and substituting this into (50) and (51) we obtain $r = \tilde{d} = \overline{r}$, as required.

Next, suppose instead $r_{t-1} = -d_{t-1} = \overline{r}$. Consider the conjectured equilibrium $(r_t, d_t) = (\overline{r}, -\overline{r})$. In this case, both first-order conditions reduce to $2\overline{r}\big(1 + f'(0)\big) - \psi = 0$, which holds by definition of $\overline{r}$.

Part 2. Since $f'(\Delta)$ is maximized at $\Delta = 0$, we have $f'(\Delta r) \leq f'(0)$ and $f'(\Delta d) \leq f'(0)$.

Then by equation (48):

$$\begin{aligned} r_t^* - d_t^* &= \frac{2\psi}{2 + f'(\Delta r) + f'(\Delta d)} \\ &\geq \frac{2\psi}{2 + 2f'(0)} \\ &= 2\overline{r}. \end{aligned}$$

Thus, $r_t^* - d_t^* \geq 2\overline{r}$, and by Part 1 this holds strictly whenever $(r_{t-1}, d_{t-1}) \neq (\overline{r}, -\overline{r})$.

Part 3. To start, subtracting (47) from (46) and rearranging yields:

$$r - \tilde{d} = f(\Delta r) - f(\Delta d) - \frac{r + \tilde{d}}{2}\big(f'(\Delta r) - f'(\Delta d)\big). \tag{52}$$

Furthermore, $r_{t-1} - \tilde{d}_{t-1} = (r + \Delta r) - (\tilde{d} + \Delta d) = (r - \tilde{d}) + (\Delta r - \Delta d)$. Using (52) to substitute in for $(r - \tilde{d})$, we obtain:

$$r_{t-1} - \tilde{d}_{t-1} = \left[\Delta r + f(\Delta r) - \frac{r + \tilde{d}}{2} f'(\Delta r)\right] - \left[\Delta d + f(\Delta d) - \frac{r + \tilde{d}}{2} f'(\Delta d)\right].$$

At $\Delta r = \Delta d$ this implies $r_{t-1} - \tilde{d}_{t-1} = 0$. Thus, if $\Delta + f(\Delta) - \frac{r+\tilde{d}}{2} f'(\Delta)$ is strictly increasing in Δ then $r_{t-1} - \tilde{d}_{t-1} > 0$ if and only if $\Delta r > \Delta d$, which proves the claim. Differentiating with respect to Δ yields:

$$1 + f'(\Delta) - \frac{r + \tilde{d}}{2} f''(\Delta). \tag{53}$$

Thus, if $f''(\Delta) \leq 0$ then it is immediate that (53) is strictly positive. Instead if $f''(\Delta) \geq 0$, then for (53) to be positive requires

$$\frac{r + \tilde{d}}{2} f''(\Delta) < 1 + f'(\Delta).$$

Since $r + \tilde{d} \leq 2$, we have $\frac{r+\tilde{d}}{2} f''(\Delta) \leq f''(\Delta)$ and, by Assumption 1 part 3, $f''(\Delta) < 1 + f'(\Delta)$.

Proof of Proposition 7. In what follows, we focus on a single-voter election. We begin by establishing two preliminary results.

Lemma A12 *If $r_0 < \overline{r}$ then $r_1^* \in (r_0, \overline{r})$. Otherwise, if $r_0 > \overline{r}$ then $r_1^* < \overline{r}$. Finally, if $r_0 = \overline{r}$ then $r_1^* = \overline{r}$.*

Proof. Recall that r_1^* must solve $h(r) = 0$. First, suppose $r_0 < \overline{r}$, we show this implies that $r_1^* \in (r_0, \overline{r})$. If $r = r_0$ then

$$h(r_0) = \frac{\psi}{2(1 + f'(0))} - r_0 = \overline{r} - r_0 > 0.$$

Instead, if $r = \overline{r}$ then $h(\overline{r}) = \frac{\psi}{2(1+f'(r_0-\overline{r}))} - \overline{r}$. Because f is reflected s-shaped, $\frac{\psi}{2(1+f'(r_0-r))}$ is maximized at $r = r_0$. Therefore:

$$h(\overline{r}) = \frac{\psi}{2(1 + f'(r_0 - \overline{r}))} - \overline{r} = \frac{\psi}{2(1 + f'(r_0 - \overline{r}))} - \frac{\psi}{2(1 + f'(0))} < 0.$$

Thus, $h(r_0) > 0 > h(\overline{r})$, which implies that the unique solution to $h(r) = 0$ is given by $r_1^* \in (r_0, \overline{r})$.

Second, let $r_0 > \overline{r}$, we show that $r_1^* < \overline{r}$. At $r = 0$ we have $h(0) = \frac{\psi}{2(1+f'(r_0))} > 0$. Instead, as in the previous case, $h(\overline{r}) < 0$. Thus, $0 < r_1^* < \overline{r} < r_0$, completing the argument.

To conclude, note that r_1^* is continuous in r_0 and, thus, if $r_0 = \overline{r}$ then $r_1^* = \overline{r}$.

Lemma A13 *If $r_0 < \overline{r}$ then r_1^* is increasing in r_0. Otherwise, if $r_0 > \overline{r}$ then r_1^* is decreasing in r_0.*

Proof. Applying the implicit function theorem to (10):

$$\frac{\partial r_1^*}{\partial r_0} = \frac{\frac{-\psi f''(r_0 - r_1^*)}{2(1+f'(r_0-r_1^*))^2}}{1 - \frac{\psi f''(r_0-r_1^*)}{2(1+f'(r_0-r_1^*))^2}} \tag{54}$$

By Assumption 1, $\frac{\psi}{2} < 1$ and $\frac{f''(r_0-r_1^*)}{1+f'(r_0-r_1^*)} < 1$. Thus, the denominator of (54) is positive. Therefore, $\frac{\partial r_1^*}{\partial r_0} > 0$ if and only if $f''(r_0 - r_1^*) < 0$. We break the argument into two cases, depending on the shape of f.

By Lemma A, $r_1^* > r_0$ if and only if $r_0 < \overline{r}$. Since f is reflected s-shaped, this implies $f''(r_0 - r_1^*) < 0$ for $r_0 < \overline{r}$ and $f''(r_0 - r_1^*) > 0$ for $r_0 > \overline{r}$. Thus, $\frac{\partial r^*}{\partial r_0} > 0$ if $r_0 < \overline{r}$ and $\frac{\partial r^*}{\partial r_0} < 0$ if $r_0 > \overline{r}$, as claimed.

To conclude, we show that $|\frac{\partial r_1^*}{\partial r_0}| < 1$. Clearly $\frac{\partial r_1^*}{\partial r_0} < 1$. Finally, by Assumption 1, $\frac{\psi}{2} < 1 < 1 + f'(r_0 - r_1^*)$ and $f''(r_0 - r_1^*) < 1 + f'(r_0 - r_1^*)$, which implies $\frac{\partial r_1^*}{\partial r_0} > -1$.■
Proposition 7 then follows directly from Lemmas A and A.

B Numerical Example of Platform Dynamics

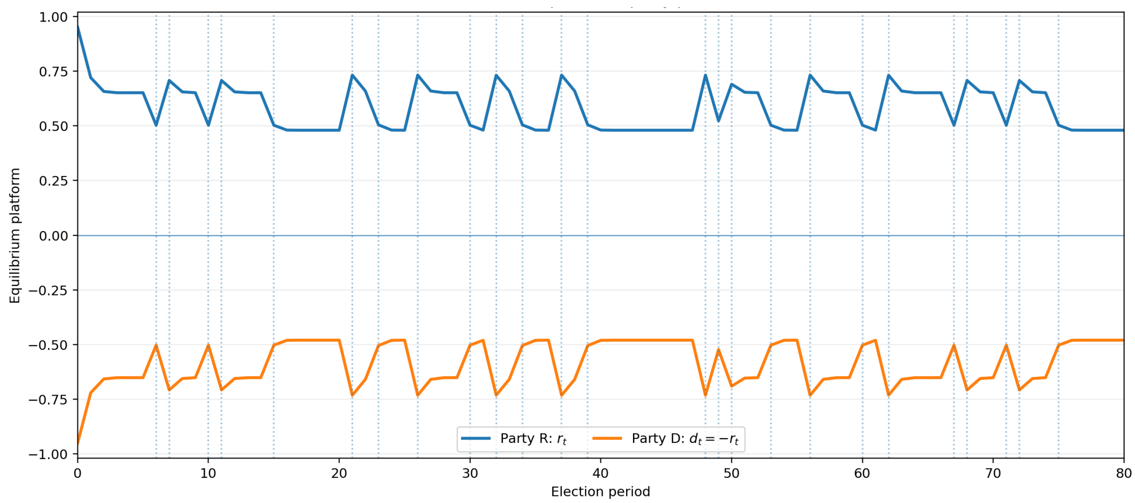

Figure 8: Figure 8 depicts a sample path of party platforms over time in a changing world. Parameters: $f(\Delta) = 0.17 \tanh(2.7\Delta), \psi_\ell = 1.4, \psi_h = 1.9$, and $\rho = .3$